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Parameter-robust discretization and preconditioning of multiple-network poroelasticity equations

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The multiple-network poroelasticity theory (MPET) describes the mechanical behaviour of a poroelastic medium permeated by multiple fluid networks. The governing equations, in the quasi–stationary case, read as follows [1]: find the displacement u and network pressures p_a for $a = 1, \ldots, A$ such that

$$\begin{aligned} -\nabla \cdot (2\mu\epsilon(u) + \lambda\nabla \cdot uI) + \sum_{a=1}^{A} \alpha_a \nabla p_a = 0, \\ -s_a \dot{p_a} - \alpha_a \nabla \cdot \dot{u} + \nabla \cdot (K_a \nabla p_a) - \sum_{b \neq a} \gamma_{ba}(p_a - p_b) = 0, \end{aligned} \tag{1}$$

where λ, and μ are the Lamé constants, α_a is the Biot parameter for network a, s_a represents the compressibility, K_a is the permeability tensor, and γ_{ba} is the transfer parameter between the networks.

The aim of this talk is to present stable discretizations and robust preconditioners for the system (1). To robustly discretize (1) and to overcome locking, we introduce a new variable $p_0 = \lambda\nabla \cdot u - \sum_{a=1}^{A} \alpha_a p_a$ that leads to a new formulation with the variables u, p_0, and p_a for $a = 1, \ldots, A$. For the case $A = 2$, extending the work in [2], we propose a block diagonal preconditioner that is robust with respect to mesh refinement and variation of Lamé constants:

$$diag\left\{(-\mu\Delta)^{-1}; I^{-1}; ((s_1 + \delta\gamma + \frac{\alpha_1^2}{\lambda})I - \delta K_1\Delta)^{-1}; ((s_2 + \delta\gamma + \frac{\alpha_2^2}{\lambda})I - \delta K_2\Delta)^{-1}\right\}, \tag{2}$$

where δ is the time step. In order to obtain robustness in the whole parameter space, we will also present new formulations obtained through the linear combination of the fluid pressures p_a and the associated preconditioners.

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