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AN INNER-OUTER ITERATION APPROACH BASED ON THE GOLUB-KAHAN BIDIAGONALIZATION METHOD

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Problems of saddle point type arise in many disciplines and applications, such as constrained optimisation or mixed finite elements, and the literature thereon is vast. Here, we look at the clearly critical industrial application of the structural analysis of the containment building of nuclear power plants. This example is a problem in linear elasticity, wherein one-, two- and three-dimensional finite elements (representing the outer shell, metallic wires and the concrete block) are coupled by multi-point constraints (MPCs). The MPCs are enforced by Lagrange multipliers, which leads after discretisation to a linear system with an indefinite matrix of 2x2 block structure. Once the complexity of the structure is high and thus the augmented system large, direct solvers are no longer applicable. Classic iterative solvers however do not give satisfactory results, which explains the need for the research on new scalable iterative solvers.

With this as our motivation, we will present how a variant of the Golub-Kahan bidiagonalization (GKB) algorithm, which has been widely used in solving least-squares problems and in the computation of the singular value decomposition of rectangular matrices [1], can be used as an iterative solver for indefinite matrices described above. Let now $\mathbf{M}$ be the positive-definite (1,1)-block (e.g. the elasticity stiffness matrix) of the augmented system. For each iteration of the generalized GKB method, a linear system $\mathbf{Mz}=\mathbf{b}$ has to be solved. Our algorithm needs, for a good parameter choice and a direct solver for the inner system, only few iterations to converge and the number of iterations is independent of the problem size of the finite element discretisation. For complex applications, also the matrix $\mathbf{M}$ is large, and the inner solution step therefore requires an iterative solver. The performance of this inner-outer iterative GKB method, in terms of accuracy and computational cost, depends obviously on the inner iterative solver. In this talk, we will compare choices for it, such as preconditionned conjugate gradient and algebraic multigrid methods as preconditioner or solver, and show numerical results for a test case (generated by Code_Aster) obtained from EDF and the Stokes equation.

REFERENCES

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