

# A matrix-free implementation for $k$ -refined isogeometric analysis

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One of the distinguishing features of Isogeometric Analysis (IGA) is the possibility of using high-degree high-regularity splines (the so-called  $k$ -refinement) as they deliver higher accuracy per degree-of-freedom in comparison to  $C^0$  finite elements. Unfortunately, if the implementation is done following the approaches that are standard in the context of  $C^0$  finite elements, the computational cost increases dramatically with the spline degree  $p$ . This is true both for the formation of the linear system and for its numerical solution. As a consequence, the use of  $k$ -refinement is often unfeasible for practical problems, where quadratic or cubic splines are typically preferred.

In this talk we discuss a matrix-free implementation, recently proposed in [1] for scalar elliptic problems, which is very beneficial in terms of both memory and computational cost. In particular, the memory required is practically independent of  $p$  and the cost depends on  $p$  only mildly. Two key ingredients that contribute to achieve this result are the preconditioner discussed in [2], which is robust with respect to both the mesh size  $h$  and the spline degree  $p$ , and weighted quadrature, a novel quadrature approach presented in [3], where the number of quadrature points required is roughly independent of  $p$ .

The numerical experiments show that, with the new implementation, the  $k$ -refinement becomes appealing from the computational point of view. Indeed, increasing the degree and continuity leads to orders of magnitude higher computational efficiency with respect to standard approaches.

## REFERENCES

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