

Simple Bayesian updating using functional approximation of random fields

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This contribution is focused on inverse problems, which are generally ill-posed problems to identify parameters of the system by observing its response. The probabilistic formulation using Bayes theorem overcomes the difficulties. It leads to a unique solution describing the stochastic distribution of the parameters with incorporated information from measurements. Most of the Bayesian approaches are formulated in terms of probability measures (densities), which are usually computationally realised using sampling methods such as Markov-chain Monte Carlo. Here, we focus on a sampling-free non-linear Bayesian update of random fields, which naturally arises in the computational framework of functional or spectral approximations such as stochastic Galerkin methods using polynomial chaos expansion. This variational formulation corresponding to conditional expectation gives rise to the well-known Kalman filter when it is linearised. The full non-linear version that is also suited for the general non-linear forward problems is investigated here. Our approach builds on an approximation with simple functions leading to an approximation of the conditional expectation in a simpler form, which can be efficiently treated by a numerical algorithm. The effectiveness of the method is demonstrated by numerical examples.