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# GLOBAL SENSITIVITY ANALYSIS USING MULTILEVEL MONTE CARLO SAMPLING: APPLICATION TO OPEN-CHANNEL FLOW SIMULATIONS

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Monte Carlo (MC) methods are popular and powerful approaches for the estimation of statistical parameters, such as expectations, variances and covariances of random variables. However, it is well-known that the root mean square error $\varepsilon$ of an MC estimator converges slowly as a function of the sample size $M$, specifically $\varepsilon = \mathcal{O}(1/\sqrt{M})$ for the sample mean estimator of the expectation. This slow convergence may become a critical issue, especially if sampling involves computationally expensive operations, such as solving a (discretized) partial differential equation. Even so, MC methods may still be the most—sometimes the only—viable option for some problems involving high-dimensional uncertain data with strong nonlinearities.

Multilevel Monte Carlo (MLMC) methods were developed to improve the overall computational cost of MC sampling by introducing a sequence of so-called levels, usually corresponding to a hierarchy of numerical simulators with increasing accuracy and corresponding cost of individual simulations. Originally designed for the estimation of expectations, MLMC was recently extended to the estimation of higher-order statistical moments such as variances [2]. We focus here on the estimation of covariances, which are particularly interesting for the computation of Sobol' indices in the context of sensitivity analysis. By deriving upper bounds for the single-level and multilevel estimators, we are able to extend the adaptive MLMC algorithm introduced in [1] to the estimation of covariances. We investigate the application of this algorithm to the estimation of Sobol' indices in open-channel flow simulations using a numerical solver of the 1D shallow water equations developed at EDF R&D [3]. In this context, we examine the behavior of the algorithm and we demonstrate that significant cost reduction may be obtained.

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