6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11-15 June 2018, Glasgow, UK

A HIGH-ORDER HYBRIDIZABLE DISCONTINUOUS GALERKIN METHOD FOR GAS KINETIC EQUATION

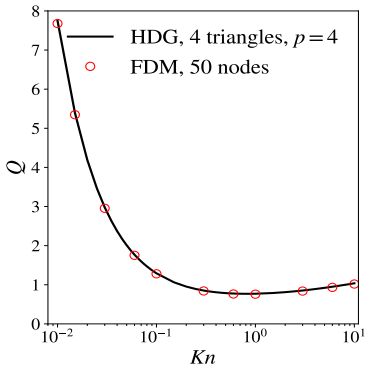
Wei Su, Peng Wang, Yonghao Zhang and Lei Wu

James Weir Fluids Laboratory, Department of Mechanical and Aerospace Engineering, University of Strathclyde, G1 1XJ Glasgow, United Kingdom

Keywords: *Boltzmann equation, rarefied gas flow, hybridizable discontinuous Galerkin*

In a wide range of applications, non-equilibrium rarefied gas flows are required to be simulated with accurate and efficient computational methods. The Boltzmann equation, which is derived from gas kinetic theory, is fundamental to describe rarefied gas dynamics using molecular velocity distribution function (VDF). The deterministic numerical simulation is an attractive approach for direct resolution of the Boltzmann equation, especially for flows at micro-scales. This method adopts a quadrature to approximate the integral with respect to molecular velocity on a discrete set of velocities. The VDF which is discrete in the velocity space but continuous in the physical space and time is solved by computational fluid dynamics methods, including finite difference, finite volume and finite element schemes. The Runge-Kutta discontinuous Galerkin (DG) method has been applied to solve the kinetic equations on arbitrary triangular mesh [1]. Although it has been shown that the DG method is more efficient than a finite volume method, the method is still expensive for steady problems. This is due to the fact of the high-dimensionality in the Boltzmann equation, as a consequence, a large number of degrees of freedom (DOF) are involved. In recent years, a new DG method called hybridizable discontinuous Galerkin (HDG) method offers an efficient way to reduce computational cost by reducing number of DOF.

Fig.1 Normalized Mass Flow Rate



In this work, we have developed a HDG method to solve the 2D Boltzmann kinetic equation. The molecular velocity space is firstly discretized with velocity abscissas. Then, the discrete VDFs and their traces are sought in the piecewise finite element space on triangles and element faces, respectively. By employing a numerical flux, which is derived from the upwind scheme, and imposing its continuity on faces, the final systems for VDF traces are obtained involving fewer DOF. Verification is carried out for a force-drive Pouseuille flow, which demonstrates its accuracy over a wide range of rarefaction (see Fig.1). The performance of this method is also evaluated.

REFERENCES

[1] W. Su, A.A Alexeenko and G. Cai. *Comput. Fluids*, Vol. **109**, pp. 123–136, 2015.