

Classical Shell Analysis in view of Tangential Differential Calculus

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The classical approach of modelling shells is to introduce a local coordinate system [1, 2]. This concept requires the introduction of co- and contra-variant base vectors and Christoffel symbols, which makes the approach less intuitive and more complex. Yet it has been the standard approach to simulate shells for the last decades.

We propose a new formulation of the linear shell theory without the explicit introduction of curvilinear coordinates. In particular, we recast the shell equations for the Kirchhoff-Love shell and Reissner-Mindlin shell in the frame of tangential operators using a global Cartesian coordinate system, which leads to a more compact and intuitive implementation. Furthermore, the requirement of a parametrized middle surface is circumvented, which enables shell analysis on implicitly defined surfaces. In [3] a rather technical procedure to recast the Kirchhoff-Love shell equations is presented.

For the numerical simulation, the derived equations are discretized with surface finite element techniques [4] using higher-order shape functions implied by Lagrange elements or NURBS patches. The approach is equivalent compared to the classical theory for curved shells with arbitrary shape. Moreover, convergence analyses for numerous test cases are performed. The numerical results show that our proposed approach is equivalent to the classical theory, yet from a different viewpoint. Higher-order convergence rates are achieved.

REFERENCES

- [1] D. Benson, Y. Bazilevs, M. Hsu and T. Hughes, Isogeometric shell analysis: The Reissner-Mindlin shell. *Comp. Methods Appl. Mech. Engrg.*, Vol. **199**, pp. 276-289, 2010.
- [2] J. Kiendl, K.U. Bletzinger, J. Linhard and R. Wüchner, R., Isogeometric shell analysis with Kirchhoff-Love elements. *Comp. Methods Appl. Mech. Engrg.*, **198**, pp. 3902–3914, 2009.
- [3] M.C. Delfour, J.P. Zolésio, Tangential Differential Equations for Dynamical Thin Shallow Shells. *J. Differential Equations*, **128**, pp. 125–167, 1996.
- [4] G. Dziuk and C. Elliott, Finite element methods for surface PDEs. *Acta Numerica*, Vol. **22**, pp. 289–396, 2013.