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Finite Volume based multigrid preconditioners for DG-SEM

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A continuing problem with high order discretizations such as discontinuous Galerkin (DG) is the performance of iterative solvers for the arising equation systems when using implicit time integration or solving steady state problems. In this talk we consider the design of multigrid preconditioners for the DG spectral element method (DG-SEM). As was shown in [2], DG-SEM methods have the SBP-SAT property in 1D. This implies that this DG discretization is equivalent to a specific finite difference discretization and another specific finite volume (FV) discretizations [1]. The meshes that these discretizations are defined on are related to the DG mesh in that they are refinements of it with additional points, respectively cells within each DG cell.

We propose to use these meshes to define approximate FV discretizations for the DG-SEM method with the purpose of defining an agglomeration multigrid method on them. This method then gives rise to a preconditioner for DG for use in Newton-Krylov schemes. It does not need geometric coarsening of the original mesh and can still be implemented completely Jacobian-free when using approximate smoothers. As smoothers, we go for preconditioned pseudo time iterations, as discussed in [3]. Numerical results demonstrate the potential of the new approach.

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