PARABOLIC MICRO PROBLEMS FOR REDUCING THE RESONANCE ERROR IN NUMERICAL HOMOGENIZATION

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There has been a growing interest in developing multiscale methods for numerical homogenization over the last two decades. These multiscale methods typically rely on the solution of microscopic problems, which are solved over representative volume elements (RVEs) of characteristic length $\delta = \mathcal{O}(\varepsilon)$, where ε is the characteristic length of the heterogeneities. A common issue is the presence of the so-called resonance error, which is proportional to the ratio ε/δ . This represents a substantial limitation for the accuracy of multiscale numerical simulations, and finding coupling strategies that improve such convergence rate is a major issue. Many techniques addressing this problem have been proposed, such as oversampling, filtering, the use of modified elliptic microscale models or the use the wave equation as microscopic model [1], but they still have computational issues. More recently, a parabolic approach has been proposed in the context of stochastic network homogenization [2]. We propose a parabolic model at the microscale as a tool for homogenizing highly oscillating coefficients in a deterministic setting. It can be shown that such an approach is equivalent to the solution of standard elliptic micro problems from the homogenization theory. However, weighted averaging in the parabolic strategy improves the convergence rate of the resonance error. Such a rate can be made arbitrarily high, in the asymptotic limit, for sufficiently smooth filters, thus allowing for a more accurate simulation for a given size of the RVE [3].

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