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TAYLOR LEAST SQUARES RECONSTRUCTION TECHNIQUE FOR MATERIAL POINT METHODS

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The material point method (MPM) [1, 2] is a computational tool for simulating problems involving large deformations. Despite its good performance in the occurrence of complex engineering studies, MPM still has some numerical shortcomings such as grid-crossing error [3] and the low spatial convergence. Methods such as the Dual Domain Material Point (DDMP) method [4] and B-Spline MPM (BSMPM) [5] smoothen the gradients of the basis functions to reduce the oscillations caused by particles crossing element boundaries thereby overcoming the problem of grid-crossing error. Moreover, Sulsky and Cong [6], and Tielen et al. [7] have shown that advanced function reconstruction techniques can significantly reduce the quadrature errors within MPM and BSMPM. However, in contrast to the original MPM approach, standard reconstruction techniques do not necessarily preserve the mass and linear momentum of the system.

In this paper, we propose a novel reconstruction technique that combines least squares approximation with local Taylor basis functions [8]. For each element, this Taylor Least Squares (TLS) technique reconstructs quantities of interest, such as stress and density, from the particle data and evaluates them at the integration points. After that, Gauss quadrature is applied to determine the nodal internal forces and velocities. If each element contains a sufficient number of Gauss points, the proposed mapping technique preserves the mass and linear momentum of the system. Thus, the TLS approximation leads to a conservative projection of the particle data and increases the accuracy of the material point methods.

We use TLS reconstruction within MPM, DDMP, and BSMPM for several time-dependent one-dimensional problems describing the deformation of one- and two-phase continua. The performance of the method is studied either qualitatively or based on the spatial errors and convergence rate, depending on the availability of an analytical solution. The obtained results show that TLS reconstruction can significantly decrease the spatial errors and improve the convergence behaviour of the considered material point methods. Its accuracy further improves for a higher number of particles per cell.

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