SMUP-UQ: very efficient uncertainty propagation for high dimensional spaces

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A novel approach and tool for high dimensional uncertainty quantification is presented here. The approach can treat large stochastic spaces with high efficiency, which even increases with the number of stochastic dimensions, that is, the number of samples required when additional dimensions are added to the problem grows sub-linearly with the number of dimensions.

The basic idea behind the developed approach is that only a small number of parameters really influence the final distribution, while all the remaining parameters can be neglected. The developed approach uses a very efficient technique to recognize the important parameters and set of correlated parameters, and the sampling to approximate the response of influence is focused on the important parameters and sets. The unimportant parameters and sets are not sampled and, therefore, the total number of required samples is drastically reduced. The efficiency of the method depends on the nature of the problem in hand, and varies with the number of effective parameters: with only a few effective parameters, the method is extremely effective and with a large number of effective parameters, the efficiency of the method drops.

The proposed approach is tested on four examples. The first two represent polynomial problems with 100 dimensions. Both examples are smooth and do not have a discontinuity in the stochastic space. The third example is the well-known borehole problem, which represents an applied example with one very important dimension. The last problem represents a high dimensional case of decaying space object, with uncertainties on the atmospheric characteristics, that normally cannot be treated with standard methods due to the cost of every single propagation.

In all test cases, the method performs well and provides an accurate representation of the final Probability Density Function (PDF). The obtained PDF is compared to PDF obtained with the Monte Carlo simulation, where the measurement metrics are the expected value and the standard deviation. The developed approach provides the results for a very small fraction of the computational cost commonly used for such high dimensional cases. Moreover, for the polynomial problems the number of samples used to interpolate the problem was smaller than 100, i.e. the number of samples were smaller than the number of dimensions.