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# VALIDATION OF DIFFERENTIATED PROGRAMS AND THEIR APPLICATION TO DATA ASSIMILATION

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Meteorological data assimilation is formulated as a large-scale nonlinear optimization problem [1]. At each search, the optimization algorithm requires the gradient of a cost function. We compute the gradient by using the adjoint program generated by AD tool TAPENADE; it differentiates in reverse mode a source computer program that simulates the atmospheric flow. We presented numerical experiments for a three-dimensional downburst. However, the gradients did not agree very well with the gradients computed with finite differences. Hence, the optimization result was not satisfactory. This is because the generated tangent and adjoint programs were not perfect.

In this paper, we identify the problems, and modify the source simulation program to solve the programs. We validate the generated differentiated programs [2]. Given a vector argument $X \in R^n$, a source computer program computes some vector function $Y = F(X) \in R^m$. The AD tool generates a new source program that, given the argument $X$, computes some derivatives of $F$; the output of the tangent program is $\dot{Y} = F'(X) \times \dot{X}$, whereas the output of the adjoint program is $\bar{X} = F'^{*}(X) \times \bar{Y}$. Introducing a function $g$ of a scalar variable $h$: $g(h) = F(X + h \times \dot{X})$, we obtain

$$\lim_{\epsilon \to \infty} \frac{F(X + \epsilon \times \dot{X}) - F(X)}{\epsilon} = g'(0) = F'(X) \times \dot{X} = \dot{Y} \tag{1}$$

Thus, we can approximate $\dot{Y}$ by running $F$ on $X$ and on $X + \epsilon \times \dot{X}$. Taking the output $\dot{Y}$ of the tangent program as the input $\bar{Y}$ of the adjoint program, we obtain

$$(\bar{X} \cdot \dot{X}) = (F'^{*}(X) \times \dot{Y} \cdot \dot{X}) = \dot{Y}^{*} \times F'(X) \times \dot{X} = \dot{Y}^{*} \times \dot{Y} = (\dot{Y} \cdot \dot{Y}) \tag{2}$$

We compute three norms; the tangent norm $(\dot{Y} \cdot \dot{Y})$ and the adjoint norm $(\bar{X} \cdot \dot{X})$ match very well, up to machine accuracy, whereas the norm obtained with Divided Differences $(F(X + \epsilon \times \dot{X}) - F(X))/\epsilon$ matches only to half the machine accuracy. Hence, the differentiated programs are correct. Using the improved adjoint program, the optimization is done for the meteorological data assimilation. Numerical experiments will be presented.

## REFERENCES

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