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NUMERICAL FORMULATION OF THREE-DIMENSIONAL BEAMS BASED ON SPATIAL INTERPOLATION OF VELOCITIES AND ANGULAR VELOCITIES

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Beam elements are still challenging due to their applicability, computational efficiency and demanding theoretical background. Here, we limit ourselves to Cosserat beam model [1], where the configuration space of a beam consists of position vectors of the line of centroids and rotations of cross-sections. The non-linear nature of spatial rotations demands special care in deriving efficient, robust and consistent numerical solution methods.

There is a number of possible ways of choosing a suitable representation and parametrization of three-dimensional rotations. An interesting possibility are the rotational quaternions. By using the quaternion algebra, we completely avoid the well known singularities of the three-component parametrizations of rotations. Additionally, rotational quaternions are used to replace both rotational matrices and rotational vectors, which simplifies equations and numerical algorithms and reduces the number of computations needed [2].

The kinetic energy is not expressed with configuration variables, but with velocities and angular velocities. The interesting fact that angular velocities are additive when expressed with respect to the moving bases can be exploited in numerical modelling. We employ velocities in fixed and angular velocities in moving frame description as the primary unknowns interpolated along the length of the beam. Therefore, a standard additive interpolation is admissible. Further, it allows very elegant and straightforward time discretization that preserves the total mechanical energy of conservative problems. Without any additional measures taken, we also satisfy the time discrete kinematic compatibility equations, which directly relate the velocities and strains.

A result of the proposed approach are simple and robust but also computationally efficient and accurate finite elements which show excellent performance for demanding problems with large displacements and rotations, finite strains and long computational times.

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