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Physically-consistent formulations for kinetic energy conservation by split convective derivatives

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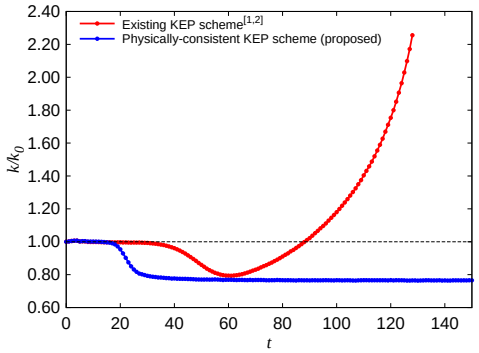

Figure 1: Time evolution of total kinetic energy (Taylor-Green vortex)

Kinetic energy preserving (KEP) formulations for compressible flow simulations are proposed here. In the existing KEP formulations[1,2], split convective derivatives are applied to the convective terms of the governing equations, and numerical stability is enhanced by preserving the kinetic energy in a discrete sense. In this study further numerical stability enhancement is achieved by designing KEP schemes such that numerical formulations satisfy the physical requirements of the governing equations; this property is called as "physically-consistent" here. Although the existing KEP formulations of Jameson[1] and Pirozzoli[2] are widely well known, their formulations are not fully physically-consistent in terms of the following two points: 1) the kinetic energy numerical fluxes calculated in the total energy equation do not satisfy the physical relation among the mass, momentum and kinetic energy, and 2) the pressure term is nonphysically calculated as the pressure per unit density in their enthalpy numerical fluxes. Regarding the first point, a physically consistent kinetic energy numerical flux is proposed instead of the inconsistent kinetic energy numerical fluxes of Refs. [1, 2]:

$$\tilde{K}_i|_{(m\pm\frac{1}{2})} = \frac{\rho|_{(m\pm1)} + \rho|_{(m)}}{2} \frac{u_i|_{(m\pm1)} + u_i|_{(m)}}{2} \frac{u_j|_{(m\pm1)} u_j|_{(m)}}{2}. \tag{1}$$

Also internal energy and pressure numerical fluxes are individually calculated instead of the enthalpy numerical fluxes of Refs. [1, 2]. In a numerical experiment (an inviscid Taylor-Green vortex), the physically-consistent KEP schemes proposed here show better numerical stability than the existing KEP schemes as demonstrated in Fig. 1 where the time evolution of the total kinetic energy is shown.

REFERENCES

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