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THE BREZIS-EKELAND-NAYROLES VARIATIONAL PRINCIPALE FOR ELASTOPLASTIC PROBLEMS

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Generalized standard materials [1] can be represented by a potential function ϕ, convex but not differentiable everywhere, naturally leading to variational methods and unconstrained or constrained optimization problems with the numerical simulations in the background. Among them, the Brezis-Ekeland-Nayroles (BEN) principle [2, 3] is based on the time integration of the sum of dissipation potential ϕ and its Fenchel polar (analogous to the Legendre polar for convex functions). Although little used in the literature, this principle is noteworthy in the sense that it allows covering the whole evolution of the dissipative system and offering an alternative to step-by-step algorithms.

Recently, Buliga et al [3] have proposed a symplectic version of the BEN principle by introducing the symplectic Fenchel polar, which allows them to generalize the Hamiltonian inclusion formalism for dissipative systems.

In the present work, this formalism is specialized to the standard plasticity in small strains and statics. We apply it to solve the classical problem of a thick tube in plane strain subjected to an internal pressure. The tube is discretized with mixed finite elements. The choice of a polynomial statically admissible stress field is guided by the aim to avoid the global (or structural) equilibrium equations in the constrained minimization problem. Only remains the local yield condition. The displacement field is chosen to provide a strain field with the same number of parameters as the one of the stress field.

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