

## MS69: Structure-preserving methods for the dynamics and optimal control of structures and multibody systems

### **Necessary Optimality Conditions for Optimally Controlled Dissipative Mechanical Systems Modelled through Fractional Derivatives.**

As it is well established, an optimal control problem is the minimization of certain cost function (depending on phase space variables and also on external control parameters) subject to forced dynamics (which is for mechanical systems obtained from the Lagrange-d'Alembert principle) and fixed initial and final conditions for the phase space variables. The Lagrange-d'Alembert principle introduces external forces which depend in general on the phase space variables (modelling dissipation, dragging, etc.) and controls as well. Based on the Pontryagin's maximum principle, the necessary optimality conditions for particular optimal control problems may be obtained by means of a constrained variational problem where the constraints (forced dynamics and endpoint conditions) are enforced through Lagrange multipliers.

In this work, we extend this theory to the case where the phase space contains fractional derivatives of the configuration variables. The advantage of this fractional variables is that, in particular cases, the dissipation forces are directly obtained from a restricted variational principle. More concretely, we obtain the necessary optimality conditions for linearly damped mechanical systems. Since the dissipation is obtained through variational calculus, we focus on external forces depending only on the control parameters (i.e. control forces). The variational discretization of this problem will produce, due precisely to its variational nature, structure-preserving numerical integrators for the optimality conditions.

We illustrate our developments through the minimum effort optimization of a controlled linearly damped harmonic oscillator.