## Comparison of Harmonic Balance methods for unilateral contact problems using the massless boundary approach

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Harmonic Balance (HB) is well-known for its numerical performance in computing periodic oscillations of nonlinear systems. However, it suffers convergence problems near discontinuities caused e.g. by impacts due to the Gibbs phenomenon. We are interested in vibrations of solids undergoing dynamical contact interactions. It is known that spatial discretization commonly leads to finite inertia of the boundary model resulting in non-physical contact oscillations. This problem can be overcome by redistributing or removing the boundary mass [1].

For a start, we consider the chain of oscillators with unilateral constraints depicted in Fig. 1a, where gray and white oscillators represent inner and boundary part. The inner dynamics of the model without boundary mass are consistent with that with sufficiently small boundary mass, see Fig. 1b (even if Newton's impact law is considered). For the massless boundary model, three Alternating-Frequency-Time HB methods are compared. In the elastic variant (EL), HB is applied to only the inner coordinates, leading to a penalty-like formulation. The boundary coordinates are explicitly accounted for in the (DL) variant, where the rigid contact is resolved using the Dynamic Lagrangian formulation [2]. The (AL) variant is similar, but the contact constraints are resolved in sub-iterations within every iteration of the HB solver [3]. As illustrated in Fig. 1c, the EL method performs well only when the boundary stiffness  $k_b$  is not much larger than the inner stiffness  $k_i$ , whereas both DL and AL methods are rather robust.





## REFERENCES

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