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Geometrically-exact time-integration mesh-free schemes for advection-diffusion problems derived from optimal transportation theory

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We present a particle method for advection-diffusion problems based on the Optimal Transport Method (OTM), where the density of the diffusive species is approximated by Dirac measures [1]. In alternative to traditional schemes formulated in linear spaces, relying on the optimal transport theory the method hybridizes elements of a Galerkin approximation with those of an updated Lagrangian approach. The time discretization of the diffusive step is based on the Jordan-Kinderlehrer-Otto (JKO) variational principle [2]. The JKO functional characterizes the evolution of the density as a competition between the Wasserstein distance (which penalizes departures from the initial conditions) and entropy (which tends to spread the density and it make uniform over the domain), and is regarded as a functional of an incremental transport map which rearranges the density over the time step. Remarkably, the resulting update is geometrically exact with respect to advection and volume. The JKO functional is discretized in space using one discretization for the density and another for the incremental transport map. Exploiting the structure of the Euler-Lagrange equations, which are linear in the density, we may treat said density as a measure and approximate it as a collection of Diracs. Since the transport maps carries Jacobian information, its interpolation requires more regularity and specifically we resort to the max-ent meshfree conforming interpolation scheme proposed in [3], that supplies converging approximations in general $W^{1,p}$ spaces. This research has been developed in collaboration with Michael Ortiz and Livio Fedeli.

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