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SELF-BALANCING ELECTRIC MOTORCYCLE MODELLING AT LOW SPEED: PRELIMINARY RESULTS

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Abstract. In the autonomous driving the main challenge is vehicle stabilization with electronic control systems - even when vehicle stops - which can enhance riders safety. A mathematical model which captures its main dynamics is needed for control system design, but such models has not been thoroughly investigated at low speed.

In the work a validated model of the motorcycle dynamics has been derived with the specific goal of a model simple but able to capture all the dynamics relevant to the capsize motion of two-wheeled vehicles. For these purposes, the work presents a 4 degrees of freedom model dynamically similar to an inverted pendulum that considers both rear and front wheel driving torques instead of rear driving torque and steering one. Moreover, steering axis is initially set on a strictly positive steering angle and is kept constant over time. Front wheel driving torque actuation with a rotated steering axis helps balancing when steering torque is not actuated. The analytical equations of motion are given by the Lagrangian approach: the result is a nonlinear second order ODE system in four unknowns - roll and yaw angles and rear contact point coordinates.

The analytical model has been then validated by FastBike, a computer simulation multibody software for dynamic analysis of two wheeled vehicles which includes five bodies, a nonlinear tyre model and nine degrees of freedom. The software has been suitably modified for low speed range. In validation process a controller has been designed using the analytical model and then applied to the multibody one: the simulations show a good match in roll and yaw angles comparisons. This indicates that the analytical model captures the main dynamics and can be used for model-based control design.

1 INTRODUCTION

In recent years technology advances in automotive industry open the way to motorcycle autonomous driving and rider safety systems which main challenge is the enhancement of vehicle stability also at low speed or when the rider stops, for example during a red traffic light. As such, it is of growing importance to devise control oriented models of the bike dynamics to be employed for control design purposes. However, motorcycle dynamics are more complex than four-wheeled vehicles one because in-plane and out-of-plane dynamics are coupled, [1]. Therefore, the derivation of control-oriented models and the design of model based control systems is not trivial.

In scientific literature the stability and dynamics of bicycles or motorcycles has been studied by many researchers [2, 3, 4]: in [5] is presented a review on dynamic modelling of single-track-vehicles. Some simple second-order dynamic models are presented in [6] to study the balance stability of a bicycle. On the other hand, some researchers have studied the motorcycle dynamics using multi-body approach [7, 1, 8, 9] which are not suitable for control system design due to their complexity. On autonomous motorcycle modelling, Yi et al. [10, 11] proposed a 5th order mathematical model based on constrained Lagrangian: the model includes also the steering dynamics and for control design the model is then linearised. Moreover, in these papers a trajectory tracking and stability control for agile manoeuvres using steering angular velocity and rear thrust as control inputs is presented. The control works fine at low speed too. The stability is achieved by steering control and gyroscopic actuators. Similarly, in [12] a set of four second order equations of motion based on Lagrangian approach has been derived: the steer torque applied by the rider to the handlebar is the only control input. A control-oriented motorcycle analytical model is presented in [13]. The model considers both longitudinal and lateral forces exerted by the tires and has as inputs the steering torque and the front and rear wheel torques. Recently, electrification of vehicle propulsion is also applied to motorcycles allowing the birth of all-wheel driven motorcycles. The research has investigated whether this feature helps a better management of vehicle stabilization. Yang and Murakami [14] proposed an electric motorcycle model, where there are two steering actuators and two driving ones. When the motorcycle moves with normal or high speed, both front and rear steering motors can be effectively controlled by swaying to keep the balance; when it stops or moves with slow speed, the front and rear steerings are rotated in the same direction and the self-balancing is achieved by driving motors similar to Segway stabilization control. In [15, 16] control strategies that increase the stability of the motorcycle by acting only on driving and braking torques have been presented. These strategies take into account the rider intentions and are applied for cornering stability. In mentioned articles the front wheel torque is only brake one, whereas in this paper authors want to take advantage of driving front wheel torque. Summarizing, all aforementioned studies involves in some way either the steering actuation or only the rear driving torque or in other cases a braking not driving front wheel torque or multibody approaches, not suitable for control design. Moreover, in most of cases works are carried out at medium and high speed range.

In the paper a validated model of the motorcycle dynamics has been derived with the specific goal of a model simple but able to capture all the dynamics relevant to the capsize motion. In fact, these are the requirements for designing a model-based self-balancing control method for the vehicle system, even when it stops or moves slowly (0.1 - 1 m/s). Moreover, it is worth underlining that the work would also find out whether front wheel torque can help in some way bike stabilization when the steering handlebar cannot be actuated. Going in this direction, the paper presents a 4 degrees of freedom (DoF) model dynamically similar to an inverted pendulum that considers both rear and front wheel driving torques instead of rear driving torque and steering one. Steering axis is initially set on a strictly positive steering angle and is kept constant over time. The idea is to reproduce a configuration similar to Segway or wheelchair which are stable at low speed [17, 18]. In fact, when the steering axis is rotated up to its maximum positive angle and then locked, front wheel driving torque actuation should help motorbike balancing even if steer torque is not available. In the presented model the analytical equations of motion are obtained starting from the Lagrangian approach: the result is a nonlinear second order ODE system. Based on this model a controller has been designed and then tested on a multibody software.

The rest of the paper is organized as follows. Section 2 is devoted to the mathematical model of the two wheel drive electric motorcycle. Section 3 presents a stability control design and numerical simulation results for mathematical model validation. Finally, concluding remarks are described in Section 4.

2 MOTORCYCLE DYNAMICAL MODEL

The riderless motorcycle model has two parts: a rear frame and a front steering assembly. Model assumptions are: 1. the contact of the tread and the ground is point-contact, thus the thickness of tires is supposed to be ignored; 2. the frame of the motorbike is regarded as a point mass; 3. the ground is flat and the vertical motion is neglected (no suspension motion); 4. there is no side sliding when the motorcycle is running; 5. front contact point and instantaneous rotation axis do not change when the lean angle changes; 6. steering angle is positive and constant over time. Notice that these assumptions are not so restricted in the viewpoint of motorcycle low speed.

Figure 1 shows a schematic diagram of the model. Let P and Q denote rear and front contact point, respectively. To identify the motorcycle in a generic configuration and derive the model, two different reference frames have been adopted (see Fig.1):

- the inertial reference frame $\Sigma = (Oxyz)$: a right-handed time-invariant reference frame fixed in the space;

- the body reference frame $S = (Px'y'z')$: a reference system fixed in the rear contact point of the main frame of the motorcycle with the z'-axis parallel to the vehicle vertical axis and pointing downwards; the x'-axis indicates the forward direction and the y'-axis completes a right-handed frame. The reference frame origin P has coordinates $(x_0, y_0, 0)^T$ with respect to the inertial one.

In a generic configuration the rear frame is no more parallel to x, but forms an angle θ,

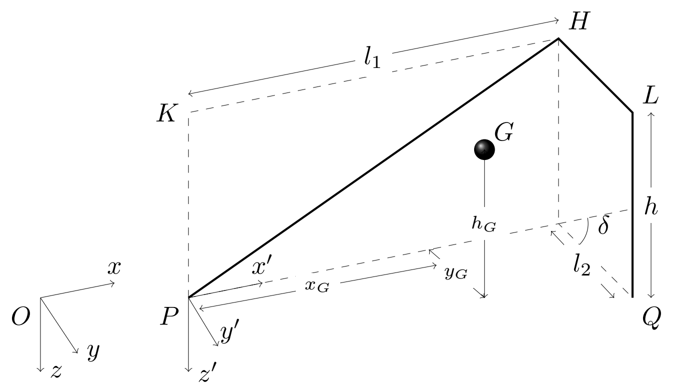


Figure 1: Motorcycle model scheme in trivial configuration with reference frames used to derived the model.

named *yaw angle* and taken about the vertical z-direction. Moreover, the *roll angle* α is the one that motorcycle's rear plane makes with the vertical one. We take α positive when the bike leans to the right according to right hand rule (see Fig. 2).

Let $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ and $(\mathbf{i_S}, \mathbf{j_S}, \mathbf{k_S})$ be the unit vector sets for the two coordinate systems respectively and $R(\theta)$ and $R(\alpha)$ the rotation matrices:

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}. \tag{1}$$

It is straightforward to obtain that

$$\begin{bmatrix} \mathbf{i_S} \\ \mathbf{j_S} \\ \mathbf{k_S} \end{bmatrix} = [R(\theta)R(\alpha)]^T \begin{bmatrix} \mathbf{i} \\ \mathbf{j} \\ \mathbf{k} \end{bmatrix}. \tag{2}$$

The model has 4 degrees of freedom: the x and y position of the contact point between the rear tire and the road expressed in the inertial reference frame; the yaw angle θ and the roll angle α. Rear and front assembly directions differ by the *steering angle* δ. In this paper this angle do not change over time because we assume that the handlebar is locked over time as specific model feature. So our model can be considered as a single body one. The input variables of the model are the rear wheel torque T_r and the front one T_f: both of them can be positive or negative and used as driving torque as well as braking one. Once again it is worth highlighting the innovative presence of an electric motor in the front wheel hub of motorcycle to use for stabilization, in addition to the rear one. Moreover, lateral tire forces have been also included in the model. In what follows, the symbols c_θ, s_θ and t_θ stand for $\cos\theta$, $\sin\theta$ and $\tan\theta$, respectively.

2.1 Mathematical model derivation

The equations of motion are given by Lagrange's equations:

$$\frac{d}{dt}\frac{\partial L(q,\dot{q})}{\partial \dot{q}} - \frac{\partial L(q,\dot{q})}{\partial q} = Q_q, \tag{3}$$

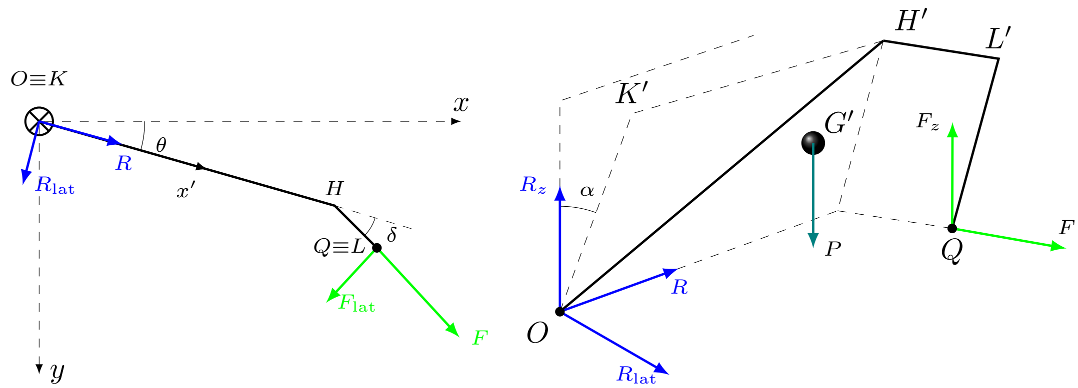


Figure 2: Two Degrees of Freedom of the proposed model: yaw angle θ on the left and roll angle α on the right. Figure also shows forces involved in the model.

where $L(q, \dot{q}) = T(q, \dot{q}) - V(q)$ is the Lagrangian function, $T = T(q; \dot{q})$ is the kinetic energy, $V = V(q)$ is the potential energy, $Q_q = [Q_x \; Q_y \; Q_\alpha \; Q_\theta]^T$ is the vector of the generalized external forces and $q = [x \; y \; \alpha \; \theta]^T$ is the generalized coordinates vector.

2.1.1 Model Lagrangian function

The kinetic quantities needed to compute kinetic and potential energy are the mass centre velocity and the system angular velocity. Let G be the mass centre of the body (see Fig. 1). In its local coordinate system we have $G_S = (x_g, 0, h_g)$. Thus, the mass centre velocity with respect to the inertial frame Σ is obtained by differentiating the expression of its inertial position with respect to time:

$$\begin{aligned}\mathbf{v_G} =& (\dot{x} - h_g c_\alpha s_\theta \dot{\alpha} - (h_g s_\alpha c_\theta + x_g s_\theta)\dot{\theta})\mathbf{i} + (\dot{y} + h_g c_\alpha c_\theta \dot{\alpha} + (x_g c_\theta - h_g s_\alpha s_\theta)\dot{\theta})\mathbf{j} + \\ &+ (h_g s_\alpha \dot{\alpha})\mathbf{k}.\end{aligned} \tag{4}$$

On the other hand, the angular velocity of system is

$$\omega_S = \dot{\alpha}\mathbf{i_S} + \dot{\theta}\mathbf{k_S} \tag{5}$$

in the local reference frame.

Now, the kinetic energy of a rigid body is the sum of its kinetic energy associated to the movement of the centre of mass and the kinetic energy associated to the movement of the particles relative to the centre of mass, that is,

$$T = T_{\text{trasl}} + T_{\text{rot}} = \frac{1}{2} m \mathbf{v_G}^2 + \frac{1}{2}\langle \omega_S, I_G \omega_S \rangle \tag{6}$$

where m is the body mass, I_G its inertia tensor in the local reference frame and $\langle\, , \rangle$ indicates the scalar product. Substituting the kinematics quantities (4) and (5) in (6), the kinetic energy terms become

$$\begin{aligned} T_{\mathrm{trasl}} =&\frac{1}{2}m\left[h_g^2\dot{\alpha}^2+\left(h_g^2s_\alpha^2+x_g^2\right)\dot{\theta}^2-2\dot{x}\left(h_gc_\alpha s_\theta\dot{\alpha}+(h_gs_\alpha c_\theta+x_gs_\theta)\dot{\theta}\right)+\right. \\ &\left.+2h_gx_gc_\alpha\dot{\alpha}\dot{\theta}+2\dot{y}\left(h_gc_\alpha c_\theta\dot{\alpha}+(x_gc_\theta-h_gs_\alpha s_\theta)\dot{\theta}\right)+\dot{x}^2+\dot{y}^2\right], \end{aligned} \tag{7a}$$

$$T_{\mathrm{rot}} = \frac{1}{2}(I_{\mathrm{xx}}\dot{\alpha}^2+2I_{\mathrm{xz}}\dot{\alpha}\dot{\theta}+I_{\mathrm{zz}}\dot{\theta}^2), \tag{7b}$$

and the potential energy is

$$V = mgG_z = mgh_g\cos\alpha. \tag{8}$$

Using the expressions (7) and (8), the Lagrangian function L of the motorcycle is

$$L = T_{\mathrm{trasl}} + T_{\mathrm{rot}} - V. \tag{9}$$

2.1.2 Contact point modelling and generalized external forces

The potential term $V(q)$ of Lagrangian function is the potential associated to external conservative forces such as the gravity force. On the other hand, the non-conservative external forces (e.g. friction forces) contribute to generalized forces term $Q_q = \sum_h \mathbf{F}_h \frac{\partial P_h}{\partial q}$, where P_h is the application point of the force $\mathbf{F}_h$. The external active forces acting on the body are the conservative gravity force $\mathbf{P}$ applied at the centre of mass G and the rear and front wheel thrusts $\mathbf{R}$ and $\mathbf{F}$ applied at the rear and front contact points P and Q, respectively. In the local frame the last two forces can be written as $\mathbf{R} = R\mathbf{i}$ and $\mathbf{F} = F\cos\delta\mathbf{i} + F\sin\delta\mathbf{j}$ where $R = T_rr_r$ and $F = T_fr_f$ (r_r and r_f are the rear and front wheel radius). The model also includes tyre forces. These forces are generated at the contact patch between tire and road and are the consequence of the sliding of the tread rubber on the asphalt surface. For this reason, forces can be calculated using wheel kinematics and in particular the velocity of the contact point (see Fig. 2). In this work it has been adopted a linear tire model - the simplest available - where all equations are linearised with respect to a straight running configuration. Let N_i, $i = r, f$ be the tire static load. In the considered linear tire model the lateral force has the roll and slip angles contributions: $F_{\mathrm{lat}} = (k_\alpha\alpha + k_\lambda\lambda)N$ with λ the slip angle, k_α and k_λ the roll and cornering stiffness, respectively. However, the slip angle contribution is smaller than the roll angle one at low speed and for this reason this second term is neglected here [7, 20]. Longitudinal slips are ignored as well. So, tire forces are reduced to

$$F_y = k_\alpha\alpha N_f \tag{10}$$

$$F_z = -N_f. \tag{11}$$

Similar formulas hold for rear tire forces. Notice that lateral forces are friction ones.

If p denotes the length of contact line, which does not change over time because the steering angle is constant, then calculating $Q_q = \sum_h \mathbf{F}_h \frac{\partial P_h}{\partial q}$ the generalized force terms are

$$Q_x = Rc_\theta + F\cos(\theta+\delta) - F_y\sin(\theta+\delta) - R_y s_\theta; \quad (12a)$$
$$Q_y = Rs_\theta + F\sin(\theta+\delta) + F_y\cos(\theta+\delta) + R_y c_\theta; \quad (12b)$$
$$Q_\alpha = 0; \quad (12c)$$
$$Q_\theta = p(F\sin\delta + F_y\cos\delta). \quad (12d)$$

Finally, applying (3) the Lagrange's equations of motion of the model are:

$$m(\ddot{x} - h_g c_\alpha s_\theta \ddot{\alpha} - (h_g s_\alpha c_\theta + x_g s_\theta)\ddot{\theta} + h_g s_\alpha s_\theta \dot{\alpha}^2 - 2h_g c_\alpha c_\theta \dot{\alpha}\dot{\theta} + \\ +(h_g s_\alpha s_\theta - x_g c_\theta)\dot{\theta}^2) = Q_x; \quad (13a)$$
$$m(\ddot{y} + h_g c_\alpha c_\theta \ddot{\alpha} - (h_g s_\alpha s_\theta - x_g c_\theta)\ddot{\theta} - h_g s_\alpha c_\theta \dot{\alpha}^2 - 2h_g c_\alpha s_\theta \dot{\alpha}\dot{\theta} - \\ -(h_g s_\alpha c_\theta + x_g s_\theta)\dot{\theta}^2) = Q_y; \quad (13b)$$
$$- mh_g c_\alpha s_\theta \ddot{x} + mh_g c_\alpha c_\theta \ddot{y} + \left(h_g^2 m + I_{\mathrm{xx}}\right)\ddot{\alpha} + (h_g m x_g c_\alpha + I_{\mathrm{xz}})\ddot{\theta} - \\ - mh_g^2 s_\alpha c_\alpha \dot{\theta}^2 = mgh_g s_\alpha; \quad (13c)$$
$$- m(h_g s_\alpha c_\theta + x_g s_\theta)\ddot{x} + m(x_g c_\theta - h_g s_\alpha s_\theta)\ddot{y} + (mh_g x_g c_\alpha + I_{\mathrm{xz}})\ddot{\alpha} + \\ + (m(h_g^2 s_\alpha^2 + x_g^2) + I_{\mathrm{zz}})\ddot{\theta} - mh_g x_g s_\alpha \dot{\alpha}^2 + h_g^2 m s_{2\alpha}\dot{\alpha}\dot{\theta} = Q_\theta, \quad (13d)$$

that is a non-linear ODE system which depends on the front and rear longitudinal forces.

3 MODEL VALIDATION AND SIMULATION RESULTS

The mathematical model presented in Section 2.1.2 is an analytical model - dynamically similar to an inverted pendulum - tuned to capture the coupling between longitudinal variables (both rear and front driving and braking torques) and capsize mode. The model should be employed for control design purposes - in case of this work for vehicle stabilization. For this reason it has been developed a simple model of only four degrees of freedom that has to be validated by a more complex and complete motorcycle model.

In order to achieve these aims, a control strategy has been designed using the analytical model and then applied to FastBike$^{\mathrm{RT}}$, a computer simulation software for real-time dynamic analysis of motorcycles (distribuited by Dynamotion [21]) used for model validation. More detailed on the software are presented in Section 3.2.

3.1 Control design

The control system wants to test whether vehicle can be self-balanced by wheel torques when its initial velocity is zero, the steering axis is locked at a positive angle and the steering handlebar can not be actuated. This describes the motorcycle initial configuration after the rider leaves it.

For control design the non-linear second order ODE system (13) of the equations of motion with generalized coordinates $\mathbf{q} = [x\ y\ \alpha\ \theta]^T$ has been reshaped into a first order one defining the state vector $X = [x\ y\ \alpha\ \theta\ \dot{x}\ \dot{y}\ \dot{\alpha}\ \dot{\theta}]^T$. By recasting, it has been obtained

Table 1: Numerical values of the motorcycle model parameters

Symbol [SI]	**Definition**	**Value**
p [m]	wheelbase	1.416
r_f [m]	front wheel radius	0.347
r_r [m]	rear wheel radius	0.318
x_G [m]	CoM G x-axis local coordinate	0.745
h_G [m]	CoM G height	0.601
δ [deg]	steering angle	40
m [kg]	motorcycle mass	130.5
g [m/s^2]	gravity acceleration	9.806
I_{xx} [kgm^2]	inertia tensor term	8.268
I_{xz} [kgm^2]	inertia tensor term	0.19
I_{zz} [kgm^2]	inertia tensor term	21.025
N_f [N]	front tire load	678.69
N_r [N]	rear tire load	600.69
k_α [1/rad]	rolling stiffness	0.8
α_0 [deg]	initial roll angle	4

the following state space representation

$$X = A(X) + B(X)u \tag{14}$$

where $u = [T_f \; T_r]$ is the input vector. Notice that the steering torque is not a control input. Moreover, both matrix A and B depend on the state vector X that means the system is non linear. As input vector shown, in the paper only wheel torques can be used as control inputs to achieve stabilization that is possible due to the rotated steering axis: this model feature can not be removed. Specifically, for a preliminary study only front wheel torque has been chosen as system input, whereas the rear one has been set identically zero during simulations.

Motorcycle is balanced when it has a null roll angle. Thus, roll angle α is the controlled variable with set-point $\alpha_s = 0$. A PID feedback control scheme has been chosen and designed as vehicle stability control strategy for its easiness in implementation: the control gains are $K_P = 1.5\text{e}5$, $K_I = 3\text{e}5$ and $K_D = 9\text{e}3$.

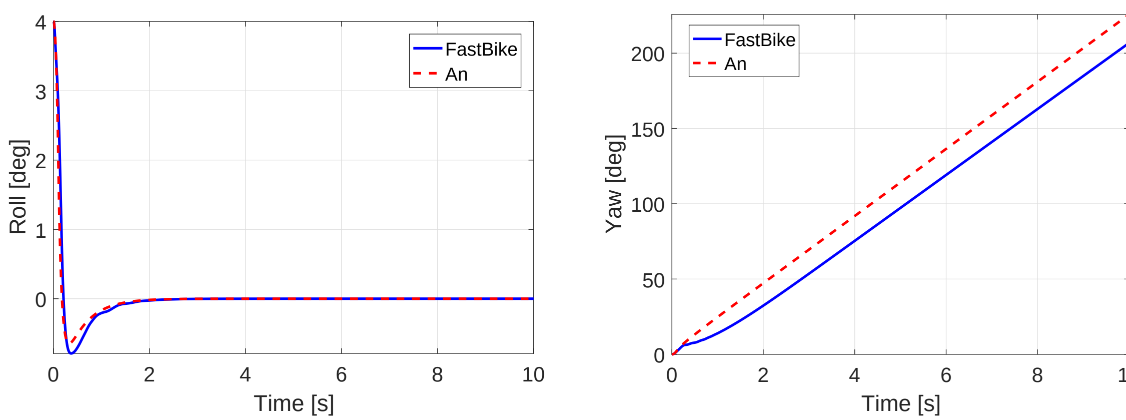
As it will be explained in Section 3.2, the multibody model used for validation process has three control inputs: rear and front wheel torques and the steering one, but in this study case the steering axis is locked over time at 40°. Thus, to reproduce this model feature it has been added a further PI controller on the steering torque using the steer error $e = \delta - 40°$ (δ is the steering angle) with gains $K_P = 150$ and $K_I = 1\text{e}3$.

3.2 Model validation

To validate the presented model and analyse whether it captures roll vehicle dynamics, a equal set of a real motorcycle parameters has been used in both analytical and multibody model. They are reported in Table 1.

As stated before, the comparison is made with a computer simulation software for real-time dynamic analysis of two wheel vehicles, called FastBike^{RT} and distributed by Dynamotion [21]. The software has been suitably modified for low speed range where the model should be validated. Its multibody model includes five bodies - the rear assembly, the rider, which is rigidly attached to the rear assembly, the front steering assembly, the rear and the front wheels - three control inputs - front and rear wheel torques and steering one - and has 9 degrees of freedom - longitudinal, lateral and vertical motion, roll, yaw and pitch angle, steering rotation and rear and front wheel spin. Moreover, the software accounts the deformability of tyres using a nonlinear tyre model. On the other hand, the presented mathematical model and controller are implemented in Matlab/Simulink.

For comparison in validation process, the roll angle is set equal to $\alpha = 4°$ at the beginning of simulation and the PID feedback controller of front wheel torque - designed by the analytical model - has been tested in the multibody software applying the same design parameters.



(a) Roll angle

(b) Yaw angle

Figure 3: Comparison of roll and yaw angle simulation responses to PID front wheel torque control input: model (red dashed line) and simulator (blue solid line).

The designed front wheel controller wants to test whether motorcycle can be self-balanced by only electric wheel motors when vehicle has null initial velocity and locked steering axis. Remember that steering torque is not actuated. In Figure 3(a) both red and blue lines go to zeros in less than 2 seconds that it means the controller stabilizes the vehicle giving a positive preliminary result for motorcycle stabilization without the use of steering torque. Moreover, roll angle time history of the two models has an impressive good match: the same controller can stabilize the motorcycle multibody model of the software - which is a more complex model - behaving in a similar way with respect to the response of the analytical one. This indicates that the analytical model captures the main vehicle roll dynamics, predominant with respect to the other out-of-plane modes at low speed. Also in the yaw angle response both models reproduce the same linear behaviour, as reported in Figure 3(b), even though in FastBike it is slightly smaller.

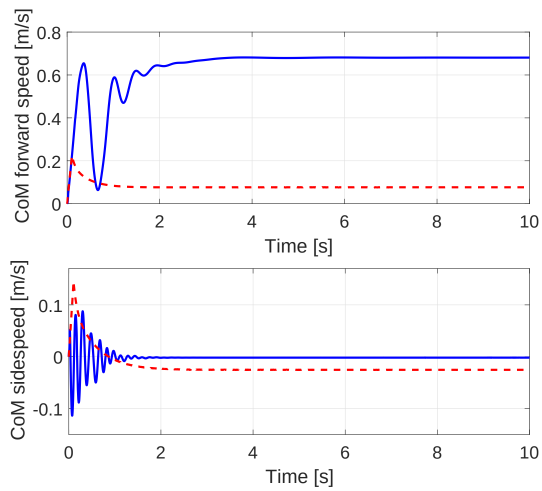
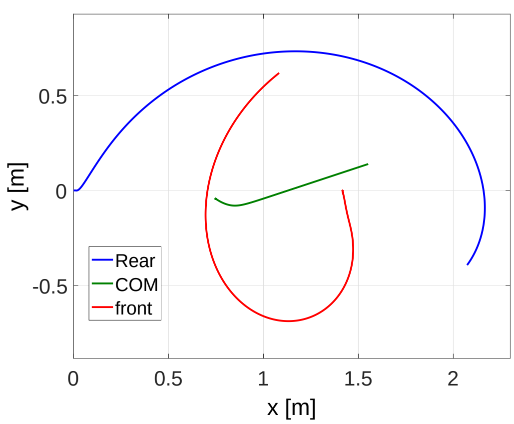
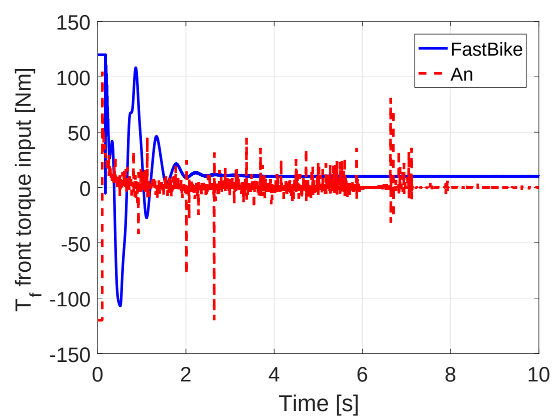


Figure 4: Simulation results of longitudinal (above) and lateral (below) centre of mass velocity: model (red dashed line) and simulator (blu line).

The mathematical model has been developed for simulations at low speed: Figure 4 shows vehicle forward velocity keeps low (less than 0.7 m/s in absolute value) during the whole control action in both models, remaining in the work assumptions.



(a) Motorcycle track of analytical model: rear contact point (blue), centre of mass (green) and front contact point (red)



(b) Front torque control input: model (red dashed line) and simulator (blu line)

Figure 5: Motorcycle trajectory (left) and control signals (right).

In Figure 5(b) it can be seen the two model control signals have a comparable magnitude and remain below the physical limit imposed by the problem ($T_f = 120$ Nm) during the whole simulation time. In addition, the motorcycle track of PID control system applied to the analytical model is reported in Figure 5(a): rear contact point track shows that vehicle moves on a curve.

4 CONCLUSIONS

In the present paper, a four degrees of freedom control-oriented mathematical model for an autonomous two wheel drive electric motorcycle has been presented. The study has been carried out under the hypotheses the steering handlebar cannot be actuated, both rear and front wheel driving and braking torques are available and vehicle moves slowly (0.1-1m/s). The model has been derived with the specific goal of a model as simple as possible but able to capture all the dynamics relevant to active stability control of two-wheeled vehicles.

The model validation with a multibody software has highlighted a good match of the balancing variable (the roll angle) and this indicates that the presented analytical model captures the main dynamics of capsize motion. The availability of non linear equations represents an advantage with respect to the classical Jacobian linearization approach commonly used in the literature. The model can be employed with advanced non linear model-based control system design and analysis tools and it is also suitable for MIMO control strategies taking into account both rear and front torques.

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