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A GOAL-ORIENTED MODEL REDUCTION TECHNIQUE FOR PARAMETRIC FLUID-STRUCTURE PROBLEMS

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Abstract. The computation of the sound power level resulting from the vibration of submerged structures is a CPU time-consuming process in the naval industry, due to both geometrical complexity and strong coupling with the fluid. The goal is here to obtain a parametric reduced-order model (ROM), yielding a fast and accurate approximation of the radiated sound power level, when the parameters and the fluid loading vary. The proposed approach includes the following ingredients: (i) the goal-oriented formulation of the problem at hand; (ii) the reduction of both primal and dual vibroacoustic problems through reduced basis techniques; (iii) the use of a leave-one-out cross-validation (LOOCV) process in the offline step for the selection of the truncated representations; (iv) the reduction of the hydrodynamic loading. It is shown, on a simple test case involving a submerged structure with variabilities of the structural parameters, that the approach enables to obtain an accurate approximation of the quantity of interest. The LOOCV technique moreover limits the offline CPU time and yields a cheap error estimator. The approach is versatile and can take into account frequency-dependent damping. The CPU time gain compared to the full model evaluations is of several orders of magnitude, opening the way to new design strategies in the naval industry.

1 INTRODUCTION

The computation of the sound power level resulting from the vibration of submerged structures is a CPU time-consuming process in the naval industry, on account on the large size of the underlying problems, their geometrical complexities, as well as the strong coupling with the fluid. The finite element discretization of such a forced vibration problem usually results in a linear matrix system with millions of degrees of freedom. The matrices at hand are sparse, complex-valued and frequency-dependent for dissipative materials. The resulting system is typically ill-conditioned and not Hermitian. It has to be solved at each frequency of interest, ideally on a wide frequency range. In the design process, the choice of the materials and their characteristics, if known, may moreover evolve, so that numerous computations are theoretically required with new values of the parameters. The use of dissipative materials furthermore complicates the numerical modeling and computation of the problem since they are very sensitive to non-mastered operational conditions. A robust design should take into account all these variabilities and uncertainties, and their impacts on the quantities of interest should be quantified. However in industrial practice, the large CPU-time requirement drastically limits the number of shots performed. Analyzes on coarse frequency grids and with only few parameters values are usually performed. This prevents the rigorous computation of the impact of the variabilities, which may ultimately affect the robustness and optimality of the design.

Reduced Order Models (ROMs) are nowadays able to represent complex systems with few degrees of freedom at the cost of a moderate loss of accuracy. The crucial point to obtain a reliable parametric ROM is to build a reduced trial space that spans most of the physics at hand, over the whole parameter space of interest. Obviously, the reduced basis has to be of much lower dimension than the discretized full model, for a very fast online evaluation of the solution. In the vibroacoustic domain, most common reduction techniques are based on Krylov subspace reduction [1], more precisely with the second-order Arnoldi algorithm [2, 3], and on modal projection techniques [4, 5, 6]. These approaches are able to efficiently solve problems involving viscoelastic materials and submerged structures. The multidimensional character of the parameter space constitutes here an additional challenge. The extension of the classical approaches to multi-parametric problems may not be straightforward and may still involve ad hoc and non-automatic procedures, at the risk of inaccurate estimations of the quantities of interest.

A model reduction technique is developed and evaluated in this paper, so as to make tractable analyzes on very fine frequency grids, as well as to handle the complexity of a relatively high-dimensional parameter space, for damped and submerged structures. The proposed approach may be seen as an extension of [7], with here a goal-oriented formulation of the problem at hand. More precisely, the quantities of interest are the point radiated pressure level and the sound pressure level, respectively a linear output and a quadratic one. The goal-oriented feature comes from both a primal-dual Reduced-Basis formulation inspired by the work [8], and the development of a cheap error estimator on the output, based on a leave-one-out-cross-validation (LOOCV) process in the offline step. Furthermore, the problem is formulated by taking into account the variabilities the material parameters and the hydrodynamic loading [9, 10].

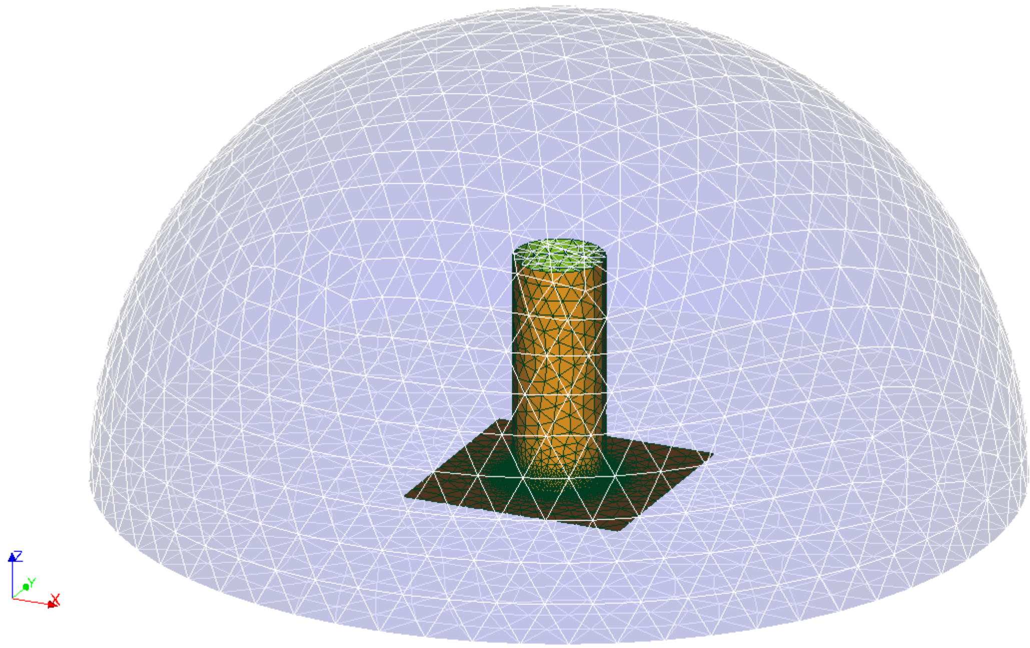

Figure 1: Illustration of the mesh of the studied case. The structure Ω_s is made of a circular cylindrical shell of finite length and a flat clamped plate. The structure is loaded on its cylindrical part and is submerged in a fluid domain Ω_f. An impedance is applied on the spherical boundary Γ_∞ to mimic the influence of an infinite fluid domain.

2 FULL MODELS

We consider a structure Ω_s submerged in a fluid domain Ω_f, see for instance Figure 1. The structure is made of an elastic or a viscoelastic material, with a frequency-dependent damping. The fluid medium is assumed at rest with an acoustic behavior and interacts with the body on a boundary $\tilde{\Gamma}$. An impedance, corresponding to the BGT condition of order 1 [11], is applied on the spherical boundary Γ_∞ to mimic the influence of an infinite fluid domain. Numerous formulations can be found in the literature to model the structure-acoustic interaction problem. We choose here the symmetrical (displacement, pressure, potential) formulation [12, 13]; the proposed methodology can nevertheless be straightforwardly applied to other formulations. Under these assumptions, the finite element discretization of the harmonic structure-acoustic interaction problem yields the following matrix system [7, 14]:

$$\left[-i\omega^3\mathbf{I} - \omega^2\mathbf{M} + \mathbf{K}(\omega)\right]\mathbf{x} = \mathbf{f} \tag{1}$$

with ω the frequency, $\mathbf{I}$ the impedance matrix resulting from the impedance boundary condition, $\mathbf{M}$ the coupled mass matrix, $\mathbf{K}$ the coupled stiffness matrix, $\mathbf{x} \in \mathbb{C}^N$ the structure-acoustic solution (with N the number of degrees of freedom) and $\mathbf{f}$ the loading. As for the quantities of interest, more precisely the local pressure (linear output s^l) and the sound power (quadratic output s^q), they take respectively the following form:

$$
\begin{aligned}
s^l &= \mathbf{l}^T\mathbf{x} \\
s^q &= \bar{\mathbf{x}}^T\mathbf{S}\mathbf{x}
\end{aligned} \tag{2}
$$

with $\mathbf{l}$ a vector extracting the pressure at a defined location from the solution $\mathbf{x}$, and $\mathbf{S}$ a real-valued symmetrical matrix related to the numerical integration of the pressure degrees of freedom on Γ_∞.

The main goal of the present paper is to be able to evaluate quickly and accurately the outputs when the system parameters vary. The parameters set $\mathcal{D} \subset \mathbb{R}^P$ is now defined and a point in this set is denoted $\mu \equiv \{\mu_1, \cdots, \mu_P\}$. The variations of the physical parameters are considered through the parameterized quantities $\omega(\mu)$, $E(\mu) \equiv E(\omega(\mu), \mu)$ the Young modulus, $\eta(\mu) \equiv \eta(\omega(\mu), \mu)$ the loss factor and $\mathbf{f}(\mu)$. The parameterized primal full model can now be written:

$$
\mathbf{A}(\mu)\mathbf{x}(\mu) = \mathbf{f}(\mu) \tag{3}
$$

where the matrix $\mathbf{A}(\mu)$ and the vector $\mathbf{f}(\mu)$ are explicitly given by:

$$
\begin{aligned}
\mathbf{A}(\mu) &= -i\omega^3(\mu)\mathbf{I} - \omega^2(\mu)\mathbf{M} + E(\mu)[1 + i\eta(\mu)]\mathbf{K}_s + \mathbf{K}_f \\
\mathbf{f}(\mu) &= \sum_{i=1}^{N_f} \gamma_i(\mu)\mathbf{f}_i
\end{aligned} \tag{4}
$$

with $\mathbf{K}_s$ the structural part of the stiffness matrix obtained with an unitary Young modulus and $\mathbf{K}_f$ its acoustic part. The affine form of the hydrodynamic loading $\mathbf{f}(\mu)$ may come from a hydrodynamic ROM built with Proper Generalized Decomposition [9] or Proper Orthogonal Decomposition [10], through a special focus on the reduction of the fluid flow pressure.

3 REDUCED-BASIS APPROXIMATION

3.1 Definitions and low-rank approximations

We choose the space-parameter decomposition of order N_p for the primal solution under the following monolithic low-rank approximation:

$$
\mathbf{x}_{N_p}(\mu) = \sum_{n=1}^{N_p} \alpha_n(\mu)\Phi_n = \Phi^{N_p}\alpha^{N_p}(\mu) \tag{5}
$$

with $\alpha_n(\mu) \in \mathbb{C}$, $\Phi_n \in \mathbb{C}^N$, $\alpha^{N_p}(\mu) = \left\{\alpha_1(\mu), \cdots, \alpha_{N_p}(\mu)\right\}^T \in \mathbb{C}^{N_p}$ and $\Phi^{N_p} = \left[\Phi_1, \cdots, \Phi_{N_p}\right] \in \mathbb{C}^{N \times N_p}$. The introduction of Eq. (5) into the primal full model Eq. (3) yields the primal residual $\mathbf{r}_{N_p}(\mu) = \mathbf{b}(\mu) - \mathbf{A}(\mu)\mathbf{x}_{N_p}(\mu)$. We now introduce the dual full models, inspired by the work [8]:

$$
\begin{aligned}
\bar{\mathbf{A}}^T(\mu)\mathbf{y}^l(\mu) &= \mathbf{l} \\
\bar{\mathbf{A}}^T(\mu)\mathbf{y}^q_{N_p}(\mu) &= \mathbf{S}(\mathbf{x}(\mu) + \mathbf{x}_{N_p}(\mu))
\end{aligned} \tag{6}
$$

with $\mathbf{y}^l$ and $\mathbf{y}^q_{N_p}$ the dual solutions related respectively to the linear and the quadratic outputs. As for the N_d-rank approximations of the dual solutions, they are chosen under the form:

$$\begin{aligned} \mathbf{y}^l_{N_d}(\mu) &= \sum_{n=1}^{N_d} \beta^l_n(\mu)\Psi^l_n = \Psi^{l,N_d}\beta^{l,N_d}(\mu) \\ \mathbf{y}^q_{N_p,N_d}(\mu) &= \sum_{n=1}^{N_d} \beta^q_n(\mu)\Psi^q_n = \Psi^{q,N_d}\beta^{q,N_d}(\mu) \end{aligned} \tag{7}$$

with, for $j = l$ or q, $\beta^j_n(\mu) \in \mathbb{C}$, $\Psi^j_n \in \mathbb{C}^N$, $\beta^{j,N_d}(\mu) = \left\{\beta^j_1(\mu), \cdots, \beta^j_{N_d}(\mu)\right\}^T \in \mathbb{C}^{N_d}$ and $\Psi^{j,N_d} = \left[\Psi^j_1, \cdots, \Psi^j_{N_d}\right] \in \mathbb{C}^{N\times N_d}$. We finally define the low-rank approximations of the outputs Eq. (2) as:

$$\begin{aligned} s^l_{N_p,N_d} &= \mathbf{l}^T\mathbf{x}_{N_p}(\mu) + (\bar{\mathbf{y}}^l_{N_d}(\mu))^T\mathbf{r}_{N_p}(\mu) \\ s^q_{N_p,N_d} &= (\bar{\mathbf{x}}_{N_p}(\mu))^T\mathbf{S}\mathbf{x}_{N_p}(\mu) + \Re\left\{(\bar{\mathbf{y}}^q_{N_p,N_d}(\mu))^T\mathbf{r}_{N_p}(\mu)\right\} \end{aligned} \tag{8}$$

The first terms of the above right-hand sides can be evaluated with only the low-rank approximation of the primal solution, and the second terms, which can be considered as corrective terms, require the computation of the primal residuals and the low-rank approximations of the dual solutions.

3.2 Offline building of the parametric primal and dual ROMs

Let us assume that appropriate trial space bases $\Phi^{N_p} \in \mathbb{C}^{N\times N_p}$ and $\Psi^{j,N_d} \in \mathbb{C}^{N\times N_d}$ (for $j = l$ or q) are already known. The building of such bases constitutes the subject of Section 3.3. The primal and dual ROMs are here classically obtained by performing a Galerkin projection of the full model Eq. (3) and Eq. (6) respectively:

$$\begin{aligned} \bar{\Phi}^T_m\mathbf{A}(\mu)\mathbf{x}_{N_p}(\mu) &= \bar{\Phi}^T_m\mathbf{f}(\mu), \quad \text{for} \quad m = 1\cdots N_p \\ (\bar{\Psi}^j_m)^T\bar{\mathbf{A}}^T(\mu)\mathbf{y}^l_{N_d}(\mu) &= (\bar{\Psi}^j_m)^T\mathbf{l}, \quad \text{for} \quad m = 1\cdots N_d \\ (\bar{\Psi}^j_m)^T\bar{\mathbf{A}}^T(\mu)\mathbf{y}^q_{N_p,N_d}(\mu) &= 2(\bar{\Psi}^j_m)^T\mathbf{S}\mathbf{x}_{N_p}(\mu), \quad \text{for} \quad m = 1\cdots N_d \end{aligned} \tag{9}$$

These equations can be more explicitly written under an online-efficient form by using Eq. (4). We can now quickly evaluate the low-rank approximations $\mathbf{x}_{N_p}(\mu)$, $\mathbf{y}^l_{N_d}(\mu)$ and $\mathbf{y}^q_{N_p,N_d}(\mu)$ by solving respectively the above primal and dual ROMs.

It can then be easily shown with all the above definitions, that the outputs approximations Eq. (8) satisfy the following properties:

$$\begin{aligned} s^l(\mu) - s^l_{N_p,N_d}(\mu) &= \left[\bar{\mathbf{y}}^l(\mu) - \bar{\mathbf{y}}^l_{N_d}(\mu)\right]^T\mathbf{A}(\mu)\left[\mathbf{x}(\mu) - \mathbf{x}_{N_p}(\mu)\right] \\ s^q(\mu) - s^q_{N_p,N_d}(\mu) &= \Re\left\{\left[\bar{\mathbf{y}}^q_{N_p}(\mu) - \bar{\mathbf{y}}^q_{N_p,N_d}(\mu)\right]^T\mathbf{A}(\mu)\left[\mathbf{x}(\mu) - \mathbf{x}_{N_p}(\mu)\right]\right\} \end{aligned} \tag{10}$$

which are the expected 'quadratic' convergence properties [15].

Algorithm 1 RB-LOOCV algorithm

Input: parameterized full model $\mathbf{A}(\mu)$ and $\mathbf{f}(\mu)$, $\varepsilon < 1$, N_{loocv}
Output: Trial bases Φ^{N_p} and Ψ^{j,N_d},

Initialization: set $\Phi^0 = \emptyset$, $\Psi^{j,0} = \emptyset$, $m = 1$, $e_{loocv} = 1$
Generate a random sampling of N_{max} values of the parameter $\mu \in \mathcal{D}$

$$\mathcal{S}_{N_{max}} = \left\{\mu^1, \cdots, \mu^{N_{max}}\right\} \in \mathcal{D}^{N_{max}}$$

while $m \leq N_{max}$ **and** $e_{loocv} > \varepsilon$ **do**
  Solve the primal full model Eq. (3)

$$\mathbf{A}(\mu^m)\mathbf{x}(\mu^m) = \mathbf{f}(\mu^m)$$

  Enrich the primal space basis

$$\Phi^m = \left[\Phi^{m-1}, \text{Gram-Schmidt}(\mathbf{x}(\mu^m))\right]$$

  Solve the dual full model Eq. (6)

$$\bar{\mathbf{A}}^T(\mu^m)\mathbf{y}^l(\mu^m) = \mathbf{l} \quad \text{or} \quad \bar{\mathbf{A}}^T(\mu^m)\mathbf{y}^q_{N_p}(\mu^m) = 2\mathbf{S}(\mathbf{x}(\mu^m))$$

  Enrich the dual space basis

$$\Psi^{j,m} = \left[\Psi^{j,m-1}, \text{Gram-Schmidt}(\mathbf{y}^l(\mu^m) \text{ or } \mathbf{y}^q_{N_p}(\mu^m))\right]$$

  if $m > N_{loocv}$ **then**
    Evaluate the mean error e_{loocv} of the output Eq. (8) with LOOCV on the sampling

$$\mathcal{S}_{N_{loocv}} = \left\{\mu^{m-N_{loocv}}, \cdots, \mu^m\right\} \in \mathcal{D}^{N_{loocv}}$$

  end if
  $m \leftarrow m + 1$
end while

3.3 Offline building of the trial space bases

A critical aspect to build an accurate parametric ROM lies in the reduced trial basis: the trial space has to be rich enough to cover most of the physics of the problem at hand, and low dimensional enough to significantly reduce the CPU time of the ROM evaluation. We follow here the classical reduced basis approach to build the trial space bases for the primal and dual problems. More precisely, the low dimensional trial subspaces are defined by $\mathcal{X}_{N_p} = \text{span}\{\mathbf{x}(\mu^i), i = 1, \cdots, N_p\}$, $\mathcal{Y}^l_{N_d} = \text{span}\{\mathbf{y}^l(\mu^i), i = 1, \cdots, N_d\}$ and $\mathcal{Y}^q_{N_d} = \text{span}\{\mathbf{y}^q_{N_p}(\mu^i), i = 1, \cdots, N_d\}$ for respectively the primal problem, the dual one related to the linear output and the dual one related to the quadratic output. In practice, the modified Gram-Schmidt orthogonalization process is used to build Φ^{N_p}, Ψ^{l,N_d} and

Ψ^{q,N_d}. In the case treated here, we take $N_p = N_d$ and an unique random sampling in $\mathcal{D}^{N_p}$ is chosen to create the discrete set of parameters μ^i for $i = 1, \cdots, N_p$ for both the primal and dual problems.

Another crucial aspect concerns the stopping criterion for the size of the space bases. Since the matrix $\mathbf{A}(\mu)$ is neither hermitian nor positive definite in standard vibroacoustic formulations, usual error estimators, for instance based on the coercivity constant, may not be used. Inspired by machine learning techniques, a leave-one-out-cross-validation (LOOCV) process is developed here to control the accuracy of the ROM as a function of the approximations ranks. More precisely, the accuracy of the output is computed and used as stopping criterion, which hence contributes to the goal-oriented feature of the approach. To limit the CPU time requirement for the error estimator, the LOOCV technique is performed only on the N_{loocv} last added samples (with $N_{loocv} = 10$ or 15 for instance). The resulting methodology is explicitly given in Algorithm 1.

4 ILLUSTRATIONS ON A SIMPLE CASE

The ROM is evaluated on the parametric vibroacoustic problem illustrated in Figure 1. On this simple case, the solicitation is considered as homogeneous on the cylindrical part of the structure along the $\mathbf{x}$ axis. The approach is implemented within the open source finite element industrial software code_aster [16] through the development of scripts written in Python language. The finite element mesh is made of ten-nodes tetrahedral elements for the fluid and seven-nodes triangular elements for the shell, generated with the preprocessing open source software Salome [17].

The minimum and maximum frequencies of interest are respectively $\omega_{min} = 2\pi \times 10$ rad.s^{-1} and $\omega_{max} = 2\pi \times 4000$ rad.s^{-1}. More generally, $P = 4$ variable parameters are considered here: the frequency ω, the Young's modulus E and the frequency-dependent loss factor η (which depends itself on 2 parameters, η_1 and η_2, through the linear relation $\eta = \eta_1 + (\eta_2 - \eta_1)\omega/\omega_{max}$). They are explicitly expressed as functions of the uniform random variable $\mu \equiv \{\mu_1, \mu_2, \mu_3, \mu_4\} \in (0,1)^4$ by:

$$\begin{aligned}
\omega(\mu) &= \omega_{min} + (\omega_{max} - \omega_{min})\mu_1 \\
E(\mu) &= 2.1 \times 10^{11} \left[1 + 0.1(2\mu_2 - 1)\right] \\
\eta_1(\mu) &= 0.01 \left[1 + 0.5(2\mu_3 - 1)\right] \\
\eta_2(\mu) &= 0.03 \left[1 + 0.5(2\mu_4 - 1)\right]
\end{aligned} \tag{11}$$

The linear and quadratic outputs obtained with the ROM, Eq. (8) with $N_p = 80$, are first compared to those computed from the full model in Figure 2, on a uniform random sampling made of 200 values of $\mu \in (0,1)^4$. A perfect match can be observed, which visually proves the ROM accuracy.

The convergence properties of the outputs are illustrated, as functions of the rank of the approximations, on Figure 3. More precisely, the RMS errors on the linear and quadratic outputs, with and without the corrective terms, are plotted. The reference curves are obtained with the expensive holdout approach, on a random sampling of 200 values. The RMS errors computed via the LOOCV technique (our error estimator with $N_{loocv} = 15$),

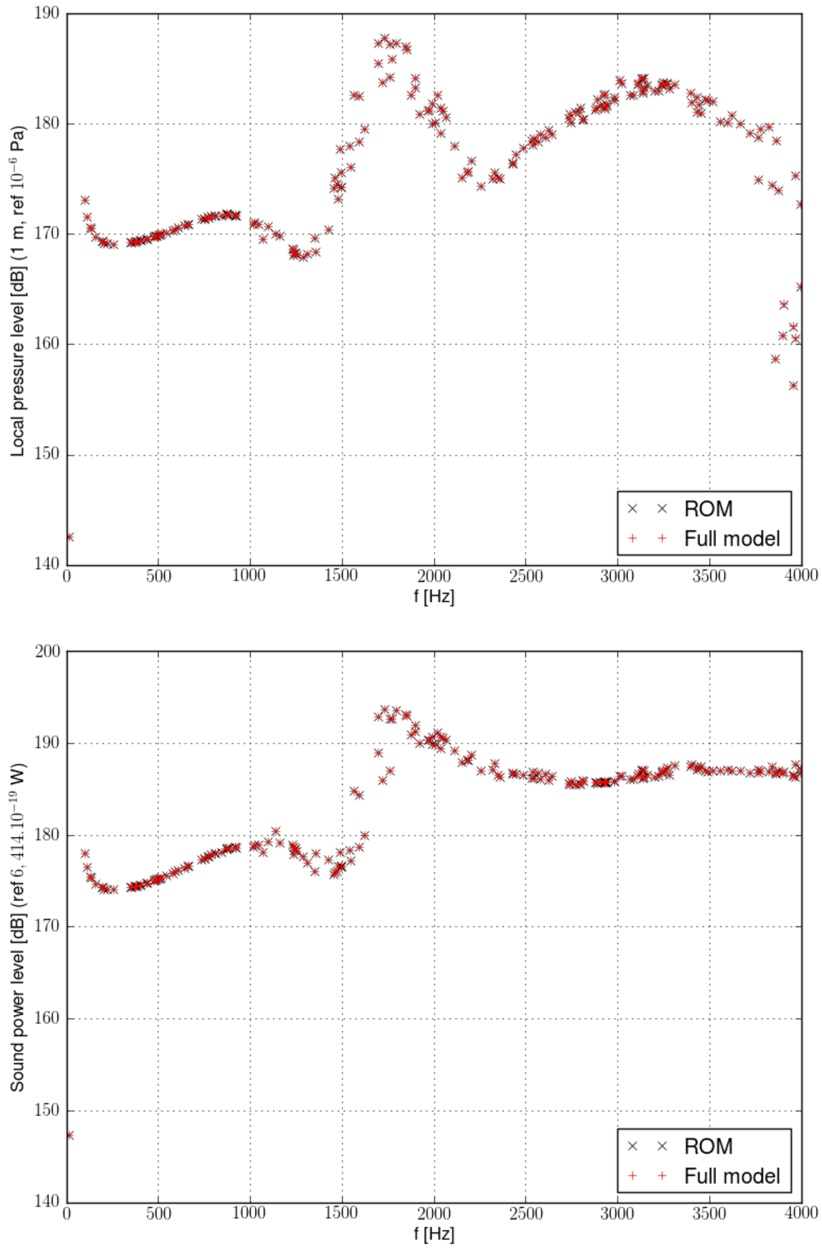

Figure 2: Comparisons of the outputs (linear output above and quadratic output below) computed with the ROM (80-rank approximations) and the full model, on a uniform random sampling in $(0,1)^{4\times200}$.

as well as their polynomial regression curves, are also displayed. It can be seen that the related curves are relatively close to those obtained with the holdout approach; the LOOCV technique therefore constitutes in this context an appropriate error estimator and enables to save CPU time during the offline step. It can also be seen that the corrective terms enable to obtain more accurate estimations of the outputs, as expected. The accuracy gain is more pronounced on the linear output than on the quadratic one.

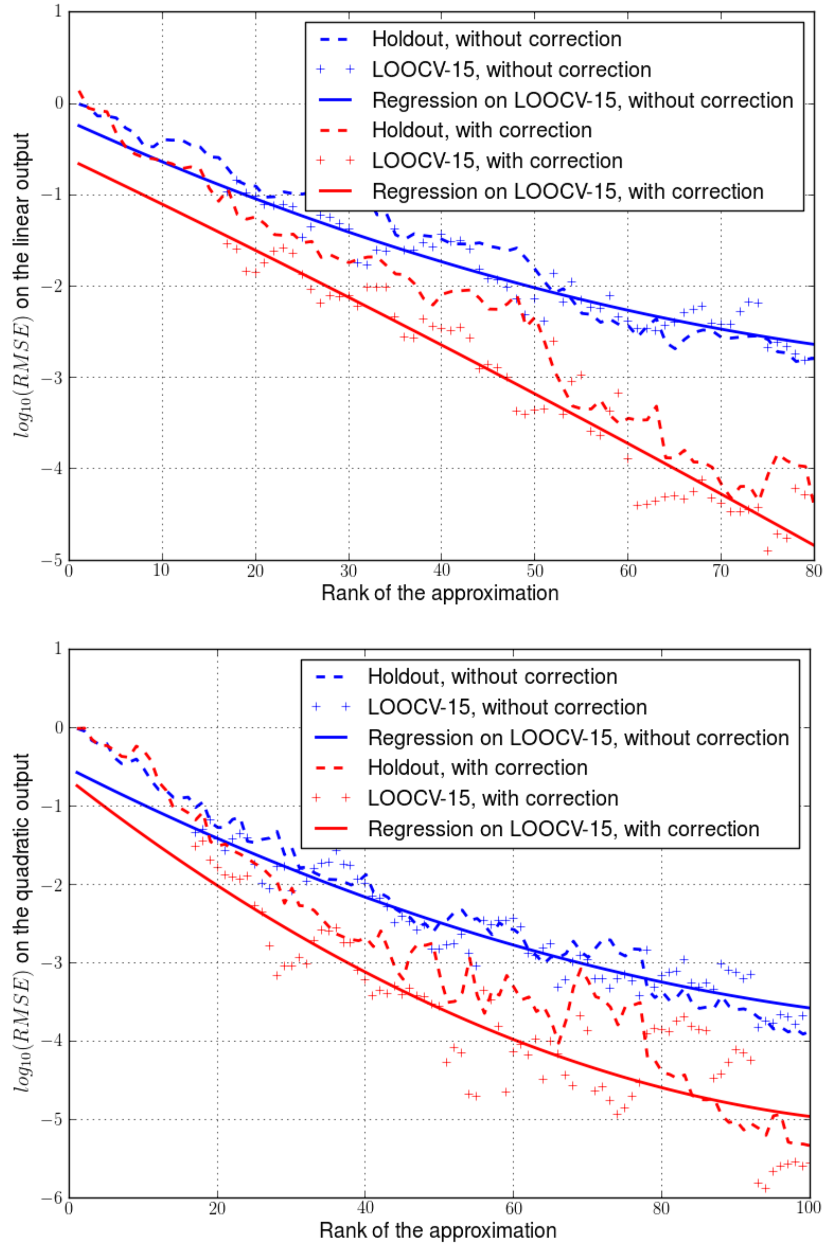

Figure 3: Convergence of the outputs as a function of the rank of the approximations.

This can be explained by a slower convergence of the reduced basis approximation on the dual problem related to the quadratic output, due to its parameter-dependent right-hand side.

5 WORK IN PROGRESS

A goal-oriented reduced basis approach is developed in this paper to estimate accurately and quickly the local pressure level and the sound power level resulting from the vibration of submerged structures, when the material parameters are varied. It is shown that the LOOCV technique can be used as a cheap error estimator in the offline step.

Taking into account the variability of the hydrodynamic loading is a work in progress. The fluid loading $\mathbf{f}(\mu)$ has to be formulated under the affine form Eq. (4). To this end, we use a POD-ROM technique based on the Navier-Stokes equations [10]. POD bases are first precomputed at different values of Reynolds numbers (Re), and an advanced interpolation based on the Grassmann manifold and its tangent space [18, 19] is performed, to obtain a basis at a new value of Re. For instance, the first POD pressure modes, related to the case considered here and for $Re = 10000$, are illustrated in Figure 4. Once the new basis is obtained, the POD-ROM can be solved to obtain the new temporal coefficients (or the frequency ones by FFT). For illustration, the wall pressure obtained with the POD-ROM is compared to the full model curve, at one time, in Figure 5.

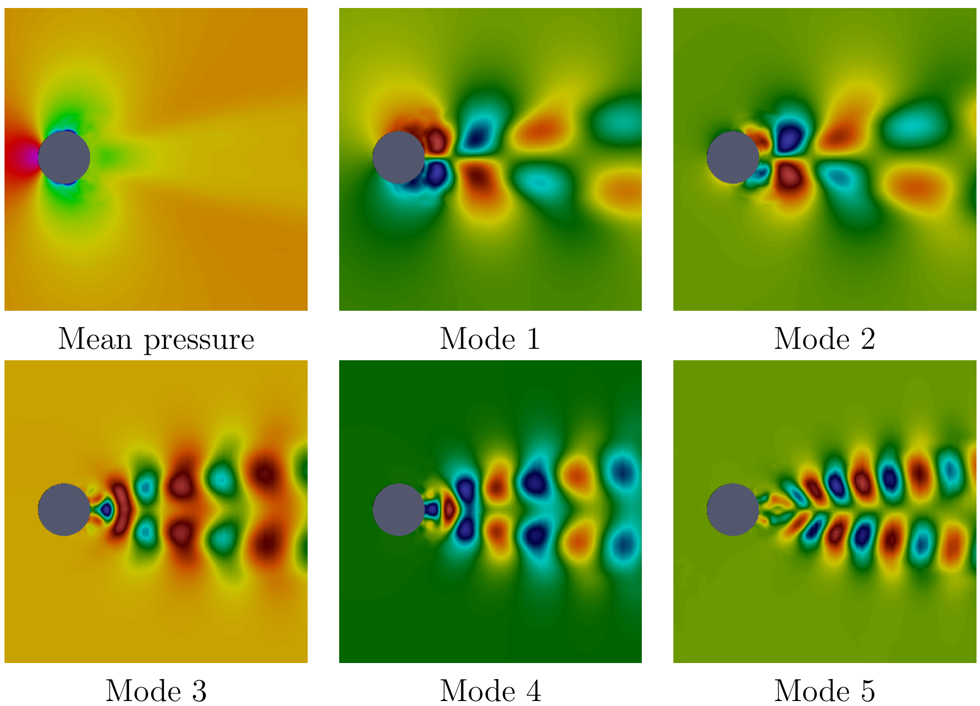

Figure 4: Mean pressure field and POD pressure modes for the hydrodynamic loading at $Re = 10000$.

The computation of the flow parameters influence on the vibroacoustic outputs Eq. (2) constitutes an ongoing research. Further studies will also include the exploitation of the low-rank approximations for sensitivity and quantification of uncertainties studies, including both structural and flow parameters variabilities.

6 ACKNOWLEDGEMENTS

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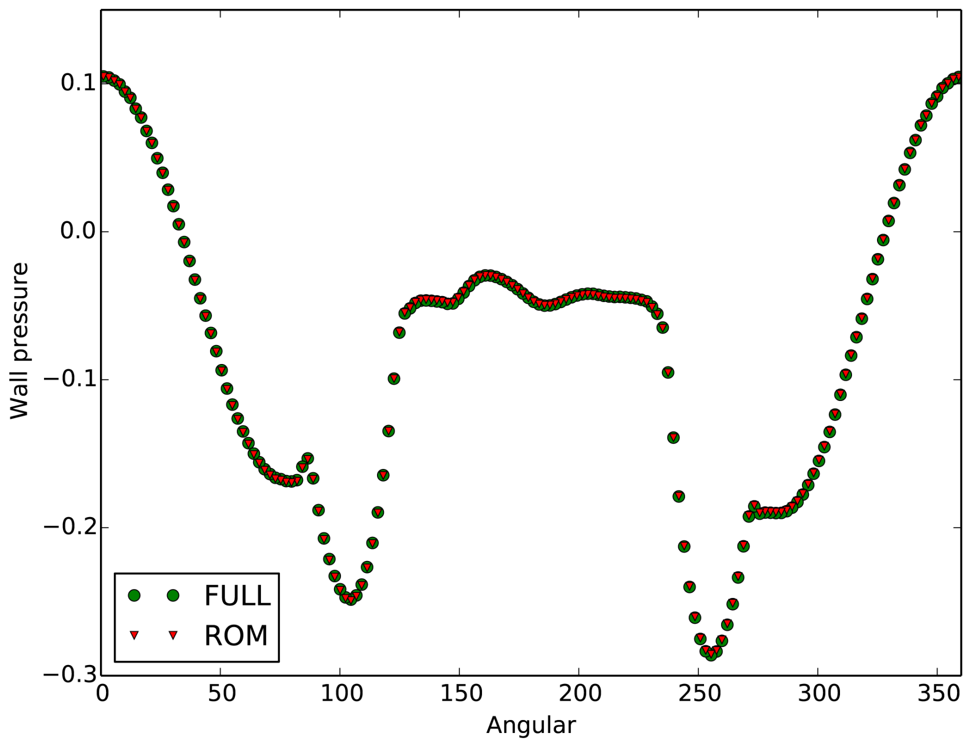

Figure 5: Wall pressures for $Re = 10000$ at one time, obtained with the POD-ROM approach and the full model.

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