6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11–15 June 2018, Glasgow, UK

# ACOUSTICS SIMULATION IN THE PRESENCE OF MOVING INTERFACES IN MULTIPHASE FLOWS

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**Key words:** Multiphase Flow, Acoustics, Surface Tension, Moving Interfaces

**Abstract.** Numerical simulations can help to understand the mechanisms of sound producing sources. In this work a numerical scheme for calculating surface tension dominated multiphase flows coupled with acoustics is presented. The Volume-of-Fluid (VOF) method is used to calculate the incompressible multiphase flow. Surface tension is taken into account by the Continuum Surface Force (CSF) method. An Expansion about Incompressible Flow (EIF) approach using the Linearized Euler Equations (LEE) is employed for the acoustics. The LEE are valid in each phase independently of the material parameters, but not at the interface. When the interface moves, unphysical acoustic sources arise. An approach which suppresses the sources in the interface region is applied. Comparing the new approach with the unchanged LEE in a moving drop test case, it is shown that the parasitic sound vanishes. An elliptic drop oscillating in a resting background flow serves as a second test case, in which the moving interface has to produce sound. A good agreement under 2.5 % between the frequencies of the oscillation and the acoustic pressure emitted by the drop is achieved.

## 1 INTRODUCTION

Sound produced by multiphase flow can be observed in our daily life, for example breaking waves, sloshing in a car tank or a water drop falling into a water pool. To understand the mechanism of the sound producing sources, numerical simulations can be helpful. Direct noise computations (DNC) solve the compressible Navier-Stokes equations (NSE) for the aerodynamic and the acoustic field simultaneously [4]. In case of low Mach numbers, flow and acoustics represent a multiscale problem for which DNC is not efficient. The cause for the inefficiency are different spatial and temporal resolution requirements [25]. Different approaches are available to make noise computation more practicable.

One approach is the acoustic/viscous splitting technique [10]. The compressible quantities are decomposed into a superposition of an incompressible part with an acoustic perturbation (Expansion about Incompressible Flow, EIF). With this method the acoustic sources are derived from the incompressible NSE. Shen and Sørenson [19] modified the original method from Hardin and Pope [10] by changing slightly the basic decomposition of the variables. Further Seo and Moon [18] employed linearized compressible perturbation equations for the acoustic computations. Different solver strategies have been used for the disciplines, e.g. Ali et al. [2] computed the flow field with a finite-volume solver and the acoustic field with a discontinuous Galerkin approach.

A variety of approaches for numerical simulations of multiphase systems exist. Cano [7] divides the methods for finite-volume solvers into interface-tracking and interface-capturing methods. Capturing the interface can be done by front tracking techniques, level-set methods, marker and cell (MAC) techniques or the volume-of-fluid (VOF) method. The VOF method is the most popular one because of its mass conservation and the good handling of topology changes. Difficulties such as sharpness of the interface were tackled with additional improvements, e.g. by a coupling with the level-set method [15]. In particular, an accurate interface representation is necessary if surface tension is involved. The continuum surface force model (CSF) by Brackbill et al. [6] models the surface tension as a continuous effect across the interface. With the CSF model surface tension can be approximated in the momentum equations of the incompressible NSE with one additional term.

Single-phase flow and acoustics have been analyzed thoroughly over the last decades whereas there is only a few work additionally taking multiphase flow into account. Munz [14] replaced the incompressible solution with the solution of the compressible NSE at zero Mach number which takes into account additional physical effects as variable density or temperature gradients. The sound generation and propagation in two phases were simulated by Tajiri et al. [21] with the help of a finite difference lattice Boltzmann method.

In this paper we use a finite-volume framework solving the incompressible NSE extended by the VOF method for the multiphase flow and the linearized Euler equations with a high-resolution scheme for the acoustics. Unphysical acoustic sources arise when simulating moving interfaces. We introduce a solution to this problem by adding an interface condition to the source term calculation.

The paper is organized as follows: the main equations for multiphase flows, surface tension and acoustic wave generation and propagation are described in Section 2, following in Section 3 by the numerical implementation and models. More details on the acoustic problem in the interface region and its solution is given in Section 4. Two test cases with moving interfaces and the new approach for the suppression of unphysical acoustic pressure sources are shown in Section 5. Finally, Section 6 resumes the outcome of this paper.

## 2 GOVERNING EQUATIONS AND BASIC ASSUMPTIONS

The equations for calculating acoustics in a finite-volume multiphase framework are presented in this section. The compressible NSE for Newtonian fluids [17] are the basis for the fluid dynamics and the acoustics:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_i)}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)\right] - \frac{\partial p}{\partial x_i} + f_i, \tag{2}$$

with the density $\rho$, the velocity vector $u_i$, the pressure $p$, the external forces $f_i$, the time $t$ and the Cartesian coordinates $x_i$. Applying the CSF model [6], one gets the volume forces consisting of the gravitational force and the surface tension:

$$f_i = \rho g_i + \sigma \kappa n_i |\nabla \alpha^m|, \tag{3}$$

with the gravitational acceleration $g_i$, the constant surface tension $\sigma$ and the unit normal vector to the multiphase interface $n_i$. The volume-fraction $\alpha^m$ is defined as the fraction of the volumetric amount of one phase $m$ to the entire volume [11]. In this paper only two phases are considered so that $m = 2$. A transport equation for the volume-fraction variable is added to evolve the multiphase flow:

$$\frac{\partial \alpha^m}{\partial t} + u_i \frac{\partial \alpha^m}{\partial x_i} = 0. \tag{4}$$

Using the one-fluid formulation [16] only one flow field ($u_i = u_i^1 = u_i^2$) has to be solved in which the volume-fraction distinguishes the phases in the Navier-Stokes equations through the material parameters. As part of the one-fluid formulation the tangential and normal velocity components at the interface are set equal on both sides. The result of those assumptions is that there is no slip at the interface and that mass transfer through the interface is not taken into account.

When surface tension is considered, the curvature $\kappa$ and the unit normal vector $n_i$ in equation (3) need to be determined. For instance, in a two-dimensional case ($x_1 = x$, $x_2 = y$) the curvature is described by:

$$\begin{aligned} \kappa = -\nabla \cdot n_i &= -\nabla \cdot \frac{\nabla \alpha}{|\nabla \alpha|} \\ &= -\frac{\alpha_{xx}\alpha_y^2 + \alpha_{yy}\alpha_x^2 - 2\alpha_x\alpha_y\alpha_{xy}}{(\alpha_x^2 + \alpha_y^2)^{\frac{3}{2}}}, \end{aligned} \tag{5}$$

in which the first and second spatial derivatives of the volume-fraction are labeled with the direction in the subscript.

To derive the acoustic equations the density, the velocity and the pressure are split into an incompressible (*inc*) part and an acoustic perturbation (*ac*) [10].

$$\rho = \rho^{inc}(x_i) + \rho^{ac}, \tag{6}$$

$$u_i = u_i^{inc} + u_i^{ac}, \tag{7}$$

$$p = p^{inc} + p^{ac}. \tag{8}$$

Inserting (6)-(8) into the compressible Navier-Stokes equations (1)-(2) one obtains a system of equations for the acoustic density and velocity. The equation for the acoustic pressure is derived with the speed of sound $c = \sqrt{(\partial p/\partial \rho)_S}$, the time derivative of the pressure splitting (8) and the acoustic density equation (9). Further insight can be gained in [3, 9, 12]. The LEE for the acoustic quantities including the source term read as:

$$\frac{\partial \rho^{ac}}{\partial t} + \rho^{inc}\frac{\partial u_i^{ac}}{\partial x_i} + u_i^{inc}\frac{\partial \rho^{ac}}{\partial x_i} = 0 \tag{9}$$

$$\frac{\partial u_i^{ac}}{\partial t} + u_j^{inc}\frac{\partial u_i^{ac}}{\partial x_j} + \frac{1}{\rho^{inc}}\frac{\partial p^{ac}}{\partial x_i} = 0 \tag{10}$$

$$\frac{\partial p^{ac}}{\partial t} + c^2\rho^{inc}\frac{\partial u_i^{ac}}{\partial x_i} + u_i^{inc}\frac{\partial p^{ac}}{\partial x_i} = -\frac{\partial p^{inc}}{\partial t}. \tag{11}$$

Note that the density $\rho^{inc}$ and the speed of sound $c$ as well as the viscosity $\mu^{inc}$ depend on their location due to the multiphase system.

## 3 NUMERICAL METHODS

If one of the phases moves, the transport equation of the volume fraction (4) has to be discretized and solved properly. For the temporal discretization a Crank-Nicolson method and for the spatial discretization the high-resolution scheme M-CICSAM are used [24].

For the curvature estimation in equation (3) an exact and fast method is desirable. In this paper a hybrid model [26] for the calculation of the spatial derivatives is chosen. The first spatial derivatives are calculated with a convolution technique [1] and the second derivatives are approximated with a standard finite differencing scheme. The convolution of the volume-fraction distribution is done with a $K_8$ kernel [26]. Since the second derivatives of the kernel need bigger support or rather more computational time it is advantageous to use here a simpler method.

The acoustic equations (9)-(11) in flux formulation read as

$$\frac{\partial U}{\partial t} + \frac{\partial F_i}{\partial x_i} = Q, \tag{12}$$

with the variable vector

$$U = [\rho^{ac}, u_i^{ac}, p^{ac}]^T, \tag{13}$$

the fluxes $F_i$, exemplary shown for $F_1$:

$$F_1 = \begin{bmatrix} u_1^{inc}\rho^{ac} + \rho^{inc}u_1^{ac} \\ u_1^{inc}u_1^{ac} + \frac{p^{ac}}{\rho^{inc}} \\ u_1^{inc}u_2^{ac} \\ u_1^{inc}u_3^{ac} \\ c^2\rho^{inc}u_1^{ac} + u_1^{inc}p^{ac} \end{bmatrix} \tag{14}$$

and the source term

$$Q = [0, 0, 0, 0, -\frac{\partial p^{inc}}{\partial t}]^T. \tag{15}$$

The LEE is transformed into a local coordinate system with the coordinate $\xi$ which is normal to the cell face. According to that, a one-dimensional problem results for each cell face:

$$\frac{\partial U_\xi}{\partial t} + A\frac{\partial U_\xi}{\partial \xi} = 0. \tag{16}$$

In (16) the Jacobi matrix $A$ is defined as

$$A = \begin{bmatrix} u_\xi^{inc} & \rho^{inc} & 0 \\ 0 & u_\xi^{inc} & 1/\rho^{inc} \\ 0 & c^2\rho^{inc} & u_\xi^{inc} \end{bmatrix} \tag{17}$$

and the variable vector $U_\xi$ as

$$U_\xi = [\rho^{ac}, u_\xi^{ac}, p^{ac}]^T. \tag{18}$$

Due to locally changing impedance $Z(x_i) = \rho(x_i)c(x_i)$ the underlaying media is referred to as layered media [13]. Solving a general Riemann problem in this media, a high-resolution scheme is applied for the acoustic fluxes using Godunov's method in combination with the Lax-Wendroff method and a Van-Leer limiter [22].

The source term in (15) is the connection between the NSE and the LEE. Only the influence of the flow on the acoustics is considered, so that the local change of pressure causes an acoustic effect. Taking into account multiphase computations this procedure induces unphysical acoustic sources, which is explained in the next section.

## 4 PROBLEM WITH MOVING MULTIPHASE INTERFACES

With the methods described in the previous section it was shown that an acoustic wave hitting a fixed inclined fluid/fluid interface gets refracted and reflected with under 1 % error [20]. When the interface moves, a problem arises. Consider a stationary drop with fluid 1 (e.g. water) in a resting fluid 2 (e.g. air), see Figure 1. If gravitational force is neglected, then equation (2) simplifies to an equilibrium between the pressure derivative and the surface tension at the interface with

$$\frac{\partial p}{\partial x_i} = \sigma\kappa n_i |\nabla\alpha|. \tag{19}$$

This so called jump condition expresses the change of pressure over the interface and is analytically described by the Young-Laplace equation [5]

$$\Delta p = \sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{20}$$

with the principal radii of curvature $r_1$ and $r_2$. If the drop and the surrounding fluid have the same velocity equation (20) still holds. Therefore when the drop moves through a cell $C$, this cell will notice a change of pressure from one time step to the next. Regarding to the source term definition (15), cell $C$ will excite an acoustic source, although in the whole domain no sources should arise. The values of the pressure and the source term

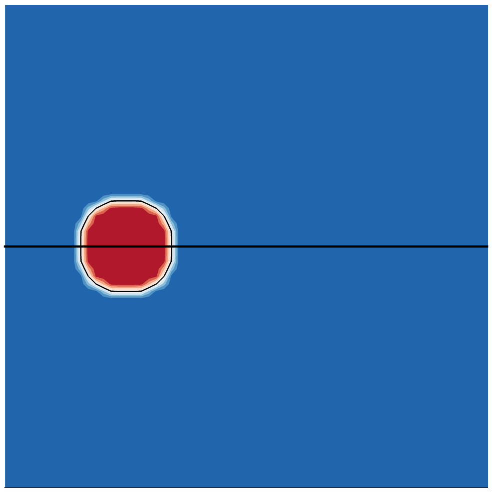

**Figure 1**: Initial concentration distribution with isoline for $\alpha = 0.5$ and horizontal middle line

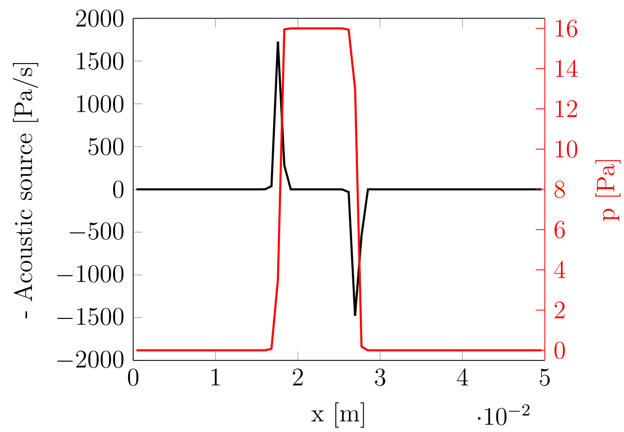


**Figure 2**: Pressure and acoustic source along the horizontal middle line at $t = 0.1\,\mathrm{s}$

along the horizontal line in Figure 1 are shown in Figure 2, in which one can see two peaks of the source term in the interface region between the two fluids. Because of the two peaks the drop incorrectly emits sound.

Since the LEE are valid in each phase but not in the region in between, a solution to overcome this numerical problem is to suppress the sources at the interface. The one-directional routine in the inhouse solver FASTEST [8] is demonstrated in Figure 3. At first the incompressible flow field is computed from the NSE followed by the multiphase advection. With the information of the concentration field one knows in which cells the interface is located and is then able to set the source term in those cells to zero.

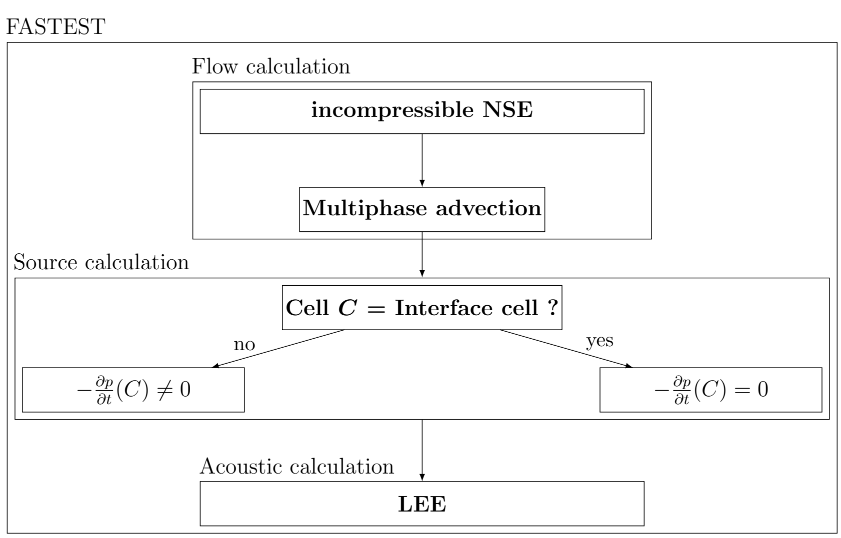


**Figure 3**: Schematic one-dircetional routine

## 5 TEST CASES AND RESULTS

In this section two test cases are shown to validate the method described in Section 4. The first test is a water drop (fluid 2) inside a duct filled with air (fluid 1) that leads to a density ratio of $\rho_1/\rho_2 = 0.001$ and a viscosity ration of $\mu_1/\mu_2 = 0.01$. A uniform flow field with a constant velocity in $x$-direction $u = 0.1\,\mathrm{m/s}$ is set. The time step size is $\Delta t = 2 \times 10^{-5}\,\mathrm{s}$ and the simulation time is $T = 0.2\,\mathrm{s}$. The duct has a length in both directions of $0.05\,\mathrm{m}$ and the uniform cell size is $\Delta h = 7.8125 \times 10^{-4}\,\mathrm{m}$. The drop radius is $r = 0.005\,\mathrm{m}$ and the drop starts at $(x, y) = (0.0125, 0.025)$. Referring to equation (20) with constant surface tension $\sigma = 0.08\,\mathrm{N/m}$, the exact pressure jump is $\Delta p = 16\,\mathrm{Pa}$. To focus on the moving interface the curvature and the pressure jump is kept constant and moreover the speed of sound for both fluids is set to $c = 1\,\mathrm{m/s}$. Beside the inlet on the left side and the outlet on the right side, for the top and the bottom of the domain symmetric boundary conditions are applied. All four boundary conditions act like acoustic outlets. Since the constant drop and the background fluid move simultaneously

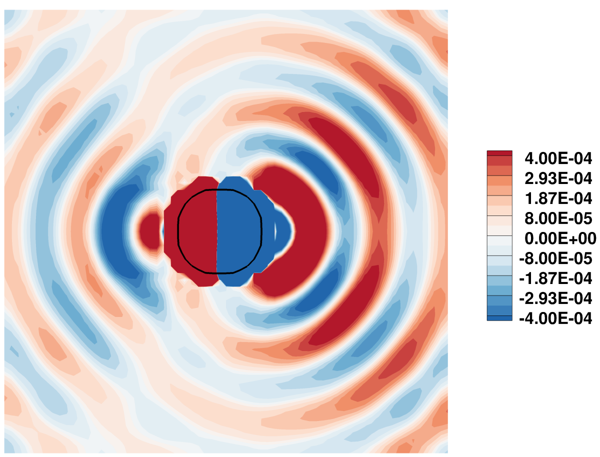


**Figure 4**: Acoustic pressure at $t = 0.1177\,\mathrm{s}$ with concentration isoline for $\alpha = 0.5$

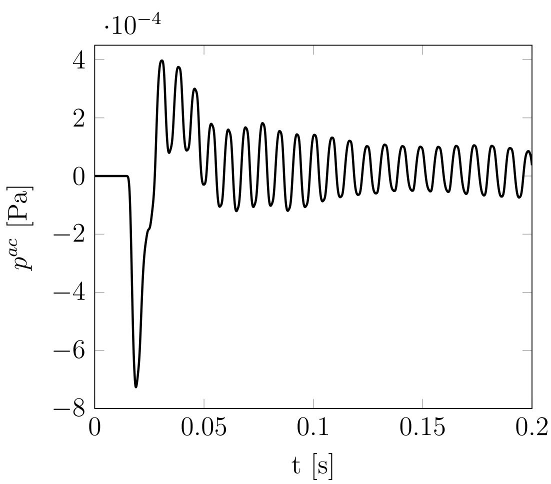


**Figure 5**: Acoustic pressure at point $P$

no acoustics should be produced. In Figure 4, the acoustic pressure contour plot is shown at time $t = 0.011\,77\,\mathrm{s}$. The sources at the front and the rear of the drop produce acoustic waves which are emitted into the domain and inside the drop. Waves inside the drop travel until the opposite side of the interface where they are nearly completely reflected due to the material parameters of the fluids. Exemplary taking the acoustic pressure at point $P = (0.025, 0.045)$ over the simulation time, one can see in Figure 5 that the magnitude is in a range of $10^{-4}$.

If the simulation is repeated without the sources of the interface the outcome is as expected. The magnitude of the acoustic pressure in point $P$ is about $10^{-17}$, see Figure 6. Comparing the two simulations, the regime with the suppressed sources appears as a line in Figure 7. For this test case suppressing the sources in the interface region leads to negligible acoustic pressure in the domain.

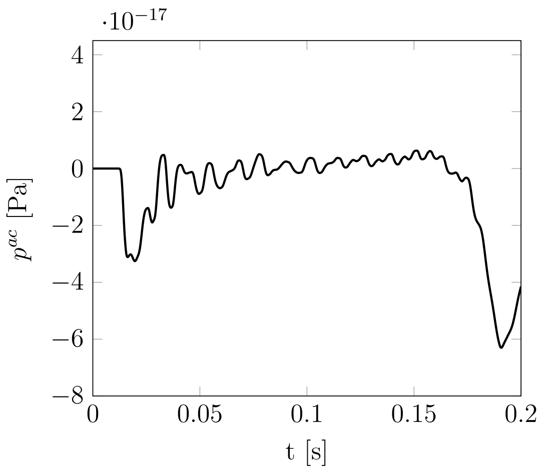


**Figure 6**: Acoustic pressure at point $P$ without interface sources

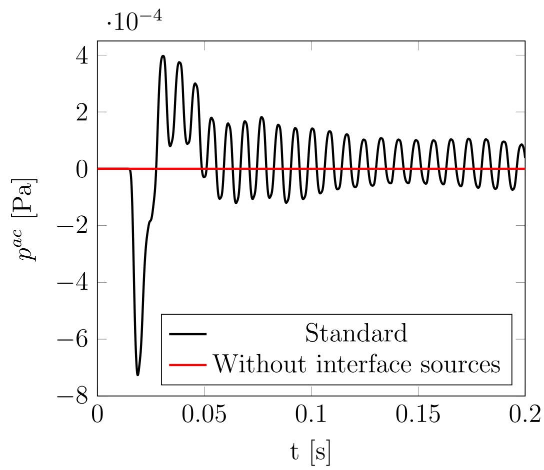


**Figure 7**: Comparison of acoustic pressure at point $P$

While in the first test case a moving interface should not produce sound, the moving interface in the second test case has to do so. Consider an elliptic drop with two semi-axes of $a = 0.005\,\mathrm{m}$ and $b = 0.003\,\mathrm{m}$ which is located in the center of the domain. The lengths and discretization are the same as in the first test case. Due to the acting surface tension the ellipsoid tries to transform into a circle and overshoots into an ellipsoid with the semi-axes exchanged. The background fluid is at rest and only the elliptic drop oscillates around his equivalent circle shape. It is expected that the acoustic pressure emitted from the ellipsoid has the same frequency as the oscillation itself. The simulation parameters are: $\Delta t = 3 \times 10^{-7}\,\mathrm{s}$, $\sigma = 0.08\,\mathrm{N/m}$, $\rho_1/\rho_2 = 1$, $\mu_1/\mu_2 = 1$ and $T = 0.15\,\mathrm{s}$. The surface tension is determined with the method described in Section 3 with four cells used for the kernel of the convolution method. Figures 8 and 9 show the acoustic pressure contour

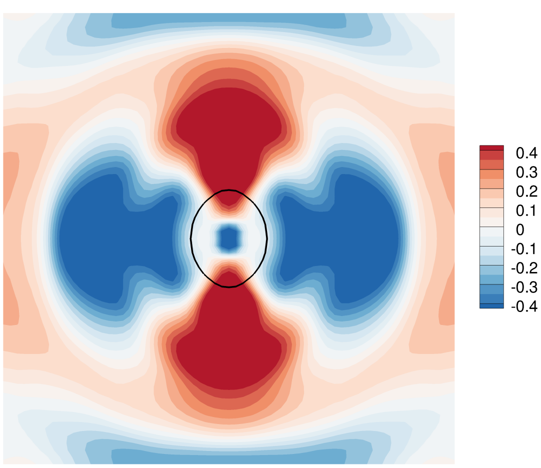


**Figure 8**: Acoustic pressure at maximum deflection in $y$-direction with concentration isoline for $\alpha = 0.5$

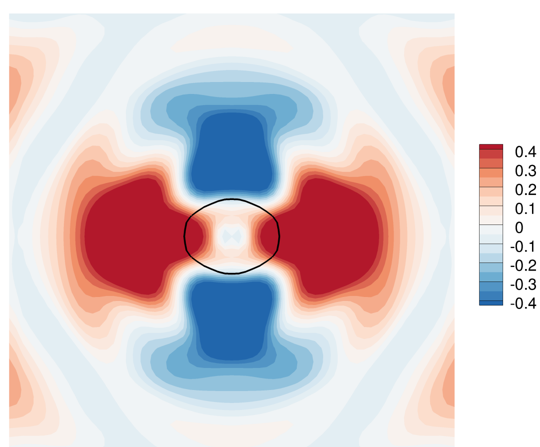


**Figure 9**: Acoustic pressure at maximum deflection in $x$-direction with concentration isoline for $\alpha = 0.5$

plots at approximately maximum drop deflections. In Figure 8 it can be seen that the drop emits positive acoustic waves in the $y$-direction while in Figure 9 positive acoustic waves travel in the opposite direction. The frequency of the drop oscillation is 60.1 Hz. In Figure 10 the Fourier analysis of the acoustic pressure is taken at point $Q = (0.25, 0.35)$. The maximum amplitude occurs at a frequency of 58.6 Hz. The difference between the frequencies of the drop oscillation and the acoustic pressure is within 2.5 % and therefore in a good agreement.

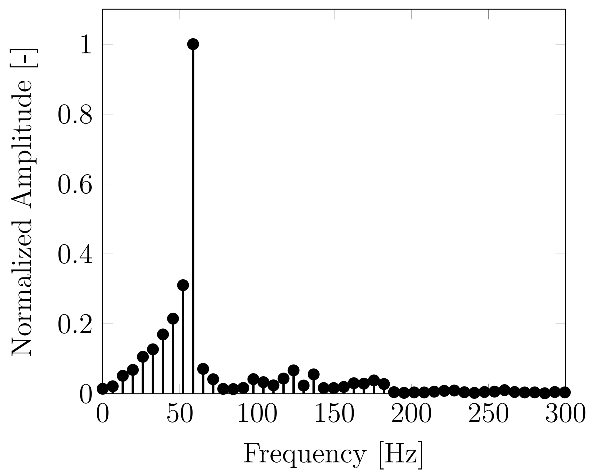


**Figure 10**: Fourier analysis of acoustic pressure at point $Q$

## 6 CONCLUSIONS AND OUTLOOK

An approach for the numerical simulation of acoustics in multiphase flows with moving interfaces has been presented. The linearized Euler equations are valid for each fluid in a multiphase phase system but not for the interface region in between. Within the one-directional finite-volume code consisting of an incompressible Navier-Stokes equations solver with integrated multiphase advection followed by the acoustic computation, a condition for the interface is introduced to suppress the unphysical acoustic sources.

Two different cases of moving interfaces have been investigated. In the first case the interfaces moves locally in the domain but has no relative movement to the background fluid and therefore should not produce any sound. A drop is sent through a duct with a constant velocity. Only with the proposed approach the acoustic pressure in the domain nearly vanishes. The second test case consists of a drop which oscillates due to surface tension. The background fluid is in rest so that the acoustic pressure emitted from the drop should have the same frequency. The frequencies are in a range of 2.5 % which is considered as a very good agreement.

In the next step this methodology will be applied to more complex problems, e.g. a water drop falling into a water pool.

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