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# STRUCTURAL DESIGN EMPLOYING A SEQUENTIAL HYBRID APPROXIMATE OPTIMIZATION

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**Key Words:** Sequential hybrid approximation optimization; Hybrid approximate model; adaptive sampling strategy; Structural design optimization.

**Abstract:** This paper presents a sequential hybrid approximate optimization (SHAO) algorithm suitable for structural design optimizations. A hybrid approximate model is introduced and further employed in predicting structural analyses more accurately while also requiring significantly fewer training samples. Furthermore, an adaptive sampling strategy is utilized to create a balance between its ability to locate the global optimum and computational efficiency within the optimization process. Consequently, the optimal searching efficiency of the SHAO algorithm is substantially enhanced. Efficiency and reliability of the proposed method are demonstrated through several benchmark structural design cases. Numerical results herein obtained reveal the proposed SHAO becomes more efficient when compared to conventional SAO and most existing meta-heuristic methods in terms of quality of solution, computational cost and convergence rate.

## 1 INTRODUCTION

Over the years, optimization has been recognized as an inseparable component of structural design [1]. Large number of design variables, large domain of search space and greater number of design constraints needed to be controlled have shown to be the major limiting factors towards performing optimum design within a reasonable time frame. Despite these shortcomings, the desire for optimal structures are continually on the increase. Within the past decades, a number of optimization algorithms were extensively utilized for structural optimization schedules, amongst these are design optimizations with meta-heuristics algorithms [2]. Meta-heuristics algorithms, such as genetic algorithms (GA) [3], simulated annealing (SA) [4], particle swarm optimization (PSO) [5] [6], ant colony algorithm (ACO) [7] and differential evolution (DE) [8] are typically nature inspired methods, where their working principles tend to mimic natural phenomena [9]. State-of-the-art reviews of these algorithms as well as their various applications towards structural optimization problems are clearly outlined in Refs.[2][10]. Highly different from gradient-based optimization algorithms, the evolutionary-based algorithms do not require gradient based search and they offer great adaptability to a large range of diverse problems. The stochastic nature of evolutionary-based algorithms enables such algorithms to be more likely and better at finding good and acceptable solutions for complicated optimization problems. Due to their robust and reliable search features, they efficiently proffer solutions for practical structural design optimization problems.

Engineering optimizations for large-scale systems are generally complex, sophisticated and highly time-consuming. Despite the advantageous characteristics of meta-heuristics algorithms, their slow rate of convergence towards the optimum is often conceived as a major downside of its technique and applicability for realistic structural optimization problems. A large number of function evaluations are usually required to find the global optimum via the meta-heuristics algorithms. In general, a good and acceptable structural optimization algorithm is required to possess reduced computational cost, generality, robustness, and high accuracy [11].

Recently, surrogate models are employed, thereby, further playing key roles towards efficiently solving design optimization problems [12]. Surrogate-based optimization (SBO) [13] is an effective approach for design of computationally expensive models such as those found in aerospace systems involving aerodynamics, structures and propulsion systems, etc. [14]. The sequential approximate optimization (SAO) [15]-[17] strategy has been recognized as one of the most attractive approach for engineering optimization. Unlike the classical surrogate -based optimization procedure summarized in [17], the SAO algorithm initially conducts a small-size design of experiment, using some surrogate model to further construct a surrogate model. The surrogate model global optimum is found by meta-heuristics methods, and the surrogate models are continuously updated through addition of new sampling points by an elaborate sampling strategy until a specified termination criterion is satisfied. SAO has been applied in numerous structural design optimization problems [17] and has proven to be an effective method.

The effectiveness of a successful SAO algorithm depends to a large extent on the construction and continuous updating of the surrogate model. In the implementation of a standard SAO, the link between design objectives and design variables are treated as a ‘black box’, where no prior knowledge about the process is assumed and subsequent manipulation is aimed at developing a surrogate model based only on observations of its input-output behavior. Hence, the SAO algorithm spends excess function evaluations in order to construct surrogate models accurate enough to“learn” the frequently complex dynamic behavior of the ‘black-box’ process, which inevitably reduces the optimal searching efficiency.

In this study, a hybrid approximate model is proposed, which focuses on imposing some prior knowledge of the structural analysis process on construction of the surrogate model in order to enhance the efficiency.The following sections of this paper are structured as: Section 2 provides the mathematical formulation of the considered structural design optimization problem and elaborates on the proposed sequential field approximate optimization (SHAO) specialized for structural design optimization. In Section 3, performance of the proposed algorithm is investigated by several test cases. Section 4 provides a brief conclusion.

## 2 SEQUENTIAL HYBRID APPROXIMATE OPTIMIZATION

### 2.1 Formulation of the structural design optimization problem

The structural optimization problem considered for this paper is in the following nonlinear programming form.

Find the design variables $\boldsymbol{X} = \{x_1, x_2, ..., x_n\}$, $x_{\min,i} \le x_i \le x_{\max,i}$ $i = 1, 2, ..., n$ to minimize:

$$M(\boldsymbol{X}) \tag{1}$$

satisfying

$$\boldsymbol{\delta}^L \le \boldsymbol{\delta} \le \boldsymbol{\delta}^U \quad \text{(displacement constraints)} \tag{2}$$

$$\boldsymbol{\sigma}^L \le \boldsymbol{\sigma} \le \boldsymbol{\sigma}^U \quad \text{(stress constraints)} \tag{3}$$

L and U are superscripts denoting lower and upper bounds, respectively. The lower and upper bounds are usually pre-assigned parameters.

## 2.2 Sequential approximate optimization (SAO).

SAO initially conducts a small-size design of experiment (DOE), using various approximate techniques to construct a surrogate model. The global optimum of the surrogate model is then found by optimal optimization methods such as meta-heuristics algorithms. The SAO algorithm terminates when a specified termination criterion is satisfied, e.g., when no further improvement of the surrogate model is observed or the maximum number of iterations are reached. Otherwise, the surrogate model is updated by repeating the procedure of adding points to the sample set adaptively and sequentially. The sampling strategy is structured to decide where to add new sampling points and how the approximation model is refined. This iterative process converges to a much higher accurate global optimum after far fewer function evaluations when compared to the meta-heuristics algorithms.

The SAO algorithm has been recognized as one of the most attractive approach for engineering optimization, however, it is worthwhile emphasizing once more that in the standard SAO, a black-box surrogate model of the entire structural analysis process is constructed. The goal is to develop a process model based only on observations of its input (design variables)-output (objective and constraints) behavior and no prior knowledge about the structural simulation process is utilized.

## 2.3 Description of proposed SHAO

### 2.3.1 Hybrid approximate model for structural analysis

In the standard SAO procedures, surrogate models are usually used to develop the black-box models. Carrying out a surrogate without prior knowledge of the process has often proved successful and oftentimes is the only possible approach especially when knowledge of the process is entirely unavailable. However, for the structural analysis model illustrated in section 2.1, its interior process structure is not completely unobservable. It is known that the structural simulation process consists of an FEA stage and the post processing stage, and both stages perform different tasks and possess different computational complexities. Therefore, some well-assessed phenomenon can be described by fundamental theoretical approach, while some others, being very difficult to interpret, can be modeled by means of rather simple "cause-effect" models. Based on this principle, we make a difference between the two stages of structural analysis process.

As shown in Fig.1, the FEA stage is performed in order to calculate the physical field of displacements and stresses, which is usually time-consuming and hard to interpret, thus surrogate models are needed to be imposed within this stage. Unlike the conventional 'black-box' approximate models, a field approximate model is established by constructing a surrogate model of the displacement for each node and the stress for each element. The field approximate model is used to surrogate the FEA stage and in predicting the physical field of displacements and stresses. The post processing stage contains manipulations such as linear superposition and max, which can be implemented analytically without computational costs. Therefore, a real model of the post processing stage instead of a surrogate model is implemented to obtain the values of the objective and constraints from the outcomes of the field approximate model. Finally, the field approximate model of the FEA and the real model of the post processing stage constitutes a hybrid approximate model used to minimize the uncertainty in existence by the use of solely surrogate models.

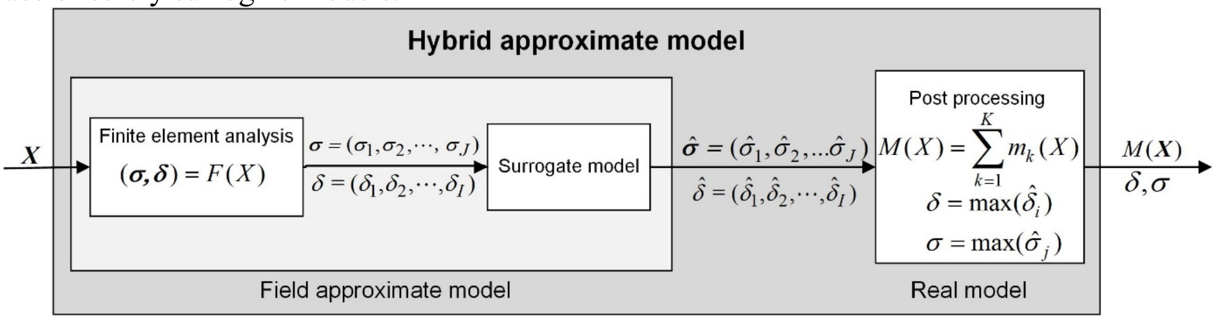


**Figure 1**: The hybrid approximate model

The hybrid approximate model combines a partial first principles model, which incorporates the available prior knowledge about the process being modeled, with the field approximate models which serve as estimator of unmeasured process parameters that are difficult to model from first principles. This produces combined models that are more reliable, while also generalizing and predicting more accurately compared to standard black-box surrogate models. The proposed field approximate model within the hybrid model are restricted to modeling terms for which a priori models are difficult to obtain. Of equal importance is the fact that significantly less data are required for training hybrid neural networks. Thus, the hybrid approximate model gives far better approximation efficiency for the same number of training data than the standard 'black-box' models.

2.3.2 The proposed SHAO algorithm

In this section, hybrid approximate models are introduced to enhance the standard SAO strategy, and a sequential hybrid approximate optimization (SHAO) strategy is proposed. The General framework of the SHAO method is shown in Fig.2:

**Step 1: Design of Experiments (DOE)**

Suppose the dimensionality of X is $n$, then the Optimal Latin Hypercube Design (OLHD) method is employed to sample $N$ ( $N \geq 2n$ ) points in the feasible domain of X. The points sampled are evaluated by invoking the FEA module to obtain the corresponding stress field and displacement field simultaneously, which are formulated as follows:

$$\begin{bmatrix} \boldsymbol{X}_1, \delta(\boldsymbol{X}_1), \sigma(\boldsymbol{X}_1) \\ \boldsymbol{X}_2, \delta(\boldsymbol{X}_2), \sigma(\boldsymbol{X}_2) \\ \dots \\ \boldsymbol{X}_N, \delta(\boldsymbol{X}_N), \sigma(\boldsymbol{X}_N) \end{bmatrix} \tag{4}$$

**Step 2: Hybrid surrogate model**

Based on the calculated vectors of delta and sigma in (4), surrogate models $\hat{\delta}_i(\boldsymbol{X})$ $(i = 1, 2, \cdots, I)$ for each node and $\hat{\sigma}_j(\boldsymbol{X})(j = 1, 2, \cdots, J)$ for each element are constructed using the RBF method. The field approximate model is presented as follows:

$$\begin{aligned} &[\hat{\delta}_1(\boldsymbol{X}), \hat{\delta}_2(\boldsymbol{X}), \cdots, \hat{\delta}_I(\boldsymbol{X})] \\ &[\hat{\sigma}_1(\boldsymbol{X}), \hat{\sigma}_2(\boldsymbol{X}), \cdots, \hat{\sigma}_J(\boldsymbol{X})] \end{aligned} \tag{5}$$

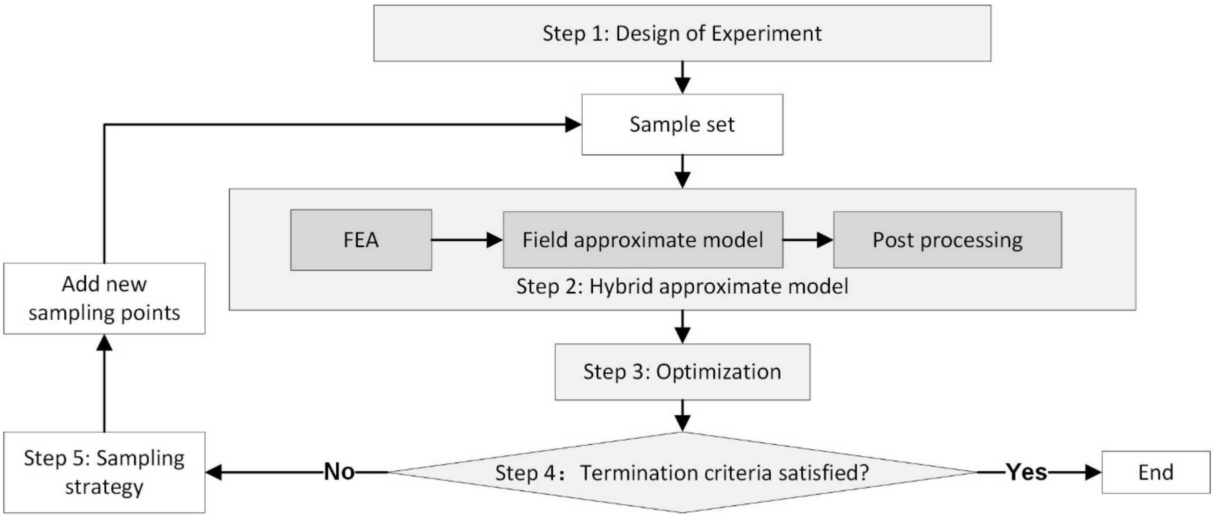


**Figure 2**: General framework of the SHAO method

After the field approximate model is constructed, the objective and constraints are obtained through the following theoretical equations:

$$\begin{aligned} &M(X) = \sum_{k=1}^{K} m_k(X) \\ &\delta = \max(\hat{\delta}_i)(i = 1, 2, \cdots, I) \\ &\sigma = \max(\hat{\sigma}_j)(j = 1, 2, \cdots, J) \end{aligned} \tag{6}$$

**Step 3: Optimization**

The objective and constraints are calculated through the field approximate model. Computational costs of the surrogate models in the field approximate model are rather low, hence, any meta-heuristics algorithm can be employed to solve the optimization problem, and the deferential evolution (DE) algorithm is chosen in this paper because of its outstanding performance. DE is originally credited to Storn and Price. Excellent surveys on the multi-faceted research aspects of DE can be found in journal articles. DE optimizes a problem by maintaining a population of candidate solutions and creating new candidate solutions by combining existing solutions according to its simple formulae, further keeping whichever candidate solution possess the best score or fitness on the optimization problem at hand. In this way the optimization problem is treated as a black box that merely provides a measure of quality given a candidate solution and the gradient is therefore not needed.

Step 4: Termination criteria

The SHAO is terminated under the following criteria:

(i) If the relative distance between the optimal solutions of two successive iterations is below 1%, then evaluate criterion (ii). Otherwise, advance SHAO to the Sampling stage;

(ii) If the relative error $e$, formulated as eq.(7), is less than 0.1%, then convergence is reached and the proposed SAO algorithm is terminated. Otherwise, advance SHAO to the Sampling stage.

$$e=\sqrt{\frac{1}{I}\sum_{i=1}^{I}\frac{\left(\hat{\delta}_i-\delta_i\right)^2}{\left(\delta_i-\bar{\delta}\right)^2}+\frac{1}{J}\sum_{j=1}^{J}\frac{\left(\hat{\sigma}_j-\sigma_j\right)^2}{\left(\sigma_j-\bar{\sigma}\right)^2}} \tag{7}$$

where $\hat{\delta}_i$ is displacement predicted by the surrogate model for the $i$th node and $\hat{\sigma}_j$ the stress predicted by the surrogate model for the $j$th element; $\bar{\delta}$ and $\bar{\sigma}$ are mean values of the true responses computed from the deterministic FEA module.

**Step 5: Sampling strategy**

Sampling strategy is highly critical for a successful SAO. A number of sampling strategies have been applied to the SAO algorithm. In the adaptive sampling strategy, the simplest way of exploitation sampling is to find the optimum of the surrogate model $s(\boldsymbol{x})$ and the pure exploration is to maximize the minimum Euler distance $d(\boldsymbol{x})$ between sampling points, which is given by

$$d(\boldsymbol{x})=\min(\sqrt{(\boldsymbol{x}-\boldsymbol{x}_i^{(n)})^T(\boldsymbol{x}-\boldsymbol{x}_i^{(n)})})\quad (i=1,2,\cdots,N^{(n)}) \tag{8}$$

where $N^{(n)}$ is the number of sampling points before the $n$th sequential sampling. The adaptive sampling method of solving the multi-objective optimization problem (9) is therefore proposed to balance the exploitation and exploration.

$$\begin{aligned} max:&\ \ s^{(n)}(\boldsymbol{x}),\ d(\boldsymbol{x}) \quad && \boldsymbol{x}_{\min}\le\boldsymbol{x}\le\boldsymbol{x}_{\max} \\ s.t.&\ \ g_i^{(n)}(\boldsymbol{x})\le 0 && i=1,2,\cdots,l \end{aligned} \tag{9}$$

where $l$ is the number of inequality constraints of the original optimization problem, $s^{(n)}(\boldsymbol{x})$ is the meta-model constructed before the $n$th sequential sampling. The optimal solution of (9) together with the real response evaluated by the original model will be regarded as a new sampling point and further used to update the surrogate model.

To simplify the solving process and obtain a preferable solution, the objective $d(\boldsymbol{x})$ is turned into a constraint and the multi-objective problem is thus converted to a single-objective optimization problem as follows:

$$\begin{aligned} max: &\ s^{(n)}(\boldsymbol{x}) \qquad \boldsymbol{x}_{\min} \le \boldsymbol{x} \le \boldsymbol{x}_{\max} \\ s.t. &\ g_i^{(n)}(\boldsymbol{x}) \le 0 \qquad i = 1, 2, \cdots, l \\ &\ d(\boldsymbol{x}) \ge d_0 \end{aligned} \tag{10}$$

In this paper, the determination of $d_0$ is given by:

$$d_0 = \min(\sqrt{(\boldsymbol{x}_i - \boldsymbol{x}_j)^T (\boldsymbol{x}_i - \boldsymbol{x}_j)}) \quad (i, j = 1, 2, \cdots, M, i \ne j) \tag{11}$$

where $M$ is the number of sampling points before the $n$th sequential sampling. It is concluded from Eq.(11) that when sampling points are sparse, the value of $d_0$ ought to be great. Knowledge of the response space is relatively inadequate. Hence, the sampling strategy will emphasize particularly on exploration. As more points are sampled, the value of $d_0$ will inevitably decrease. Meanwhile, knowledge of global response space accumulates and promising regions for global optimum will be located easily. Therefore, $d_0$ will need a relatively small value to bring exploitation over exploration. New sampling points are further added around such promising regions in order to precisely approximate for global optimum while details of non-promising regions are ignored. In this way, the number of true function calls are remarkably reduced, which is of great significance for computational-intensive structural design optimization tasks.

The adaptive sampling strategy is effective in balancing exploration and exploitation, allowing high-efficiency searching of the global optimum during the optimization process. The adaptive sampling strategy substantially reduces the number of evaluations of the true functions required to find the optimal solutions.

## 3 Benchmark case studies

In this section, two well-known truss structures are optimized by the proposed SHAO algorithm. The results obtained are subsequently compared to solutions from equally advanced and well documented optimization methods so as to demonstrate the efficiency of this proposed method.

### 3.1 Case study 1: 72-bar truss system

The second study is carried out using the proposed SHAO algorithm to execute an optimization problem for a 72-bar spatial truss structure as shown in Fig. 3. The material properties, as well as node and member numbering system are as shown in Fig.3. There are 72 truss elements, and these are divided into 16 groups as shown in Table I. This grouping reduces the number of design variables to 16 member groups and their areas vary from 0.1 to 2.5 $in^2$. The material density is 0.1 $lb/in^3$ and the modulus of elasticity is $10^4$ ksi. Stress limitations of the members are ±25,000 psi. All nodal displacements must be smaller than ±0.25 in. The structure is subjected to two loading conditions, as detailed in Table II.

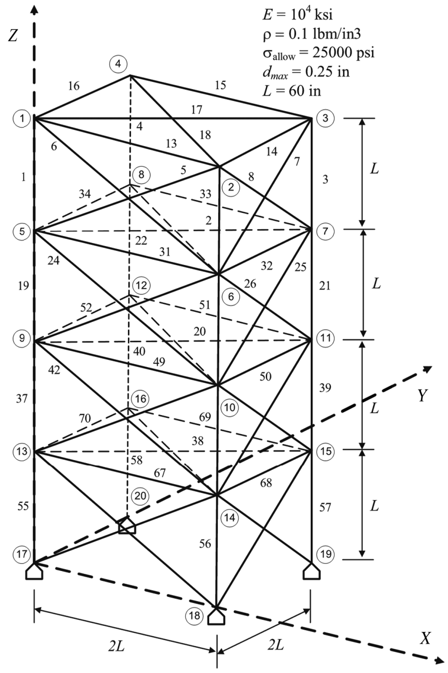


**Figure 3**: A 72-bar spatial truss structure.

**Table 1**: 72-bar truss member area groups.

| Area group | Truss members | Design variables |
|---|---|---|
| 1 | 1, 2, 3, 4 | x1 |
| 2 | 5, 6, 7, 8, 9, 10, 11, 12 | x2 |
| 3 | 13, 14, 15, 16 | x3 |
| 4 | 17, 18 | x4 |
| 5 | 19, 20, 21, 22 | x5 |
| 6 | 23, 24, 25, 26, 27, 28, | x6 |
| 7 | 31, 32, 33, 34 | x7 |
| 8 | 35, 36 | x8 |
| 9 | 37, 38, 39, 40 | x9 |
| 10 | 41, 42, 43, 44, 45, 46, | x10 |
| 11 | 49, 50, 51, 52 | x11 |
| 12 | 53, 54 | x12 |
| 13 | 55, 56, 57, 58 | x13 |
| 14 | 59, 60, 61, 62, 63, 64, | x14 |
| 15 | 67, 68, 69, 70 | x15 |
| 16 | 71, 72 | x16 |

**Table 2**: 72-bar truss loading cases.

| Load case | Node | Fx [kips] | Fy [kips] | Fz [kips] |
|---|---|---|---|---|
| 1 | 1 | 5.0 | 5.0 | -5.0 |
| 2 | 1 | 0.0 | 0.0 | -5.0 |
| | 2 | 0.0 | 0.0 | -5.0 |
| | 3 | 0.0 | 0.0 | -5.0 |
| | 4 | 0.0 | 0.0 | -5.0 |

**Table 3**: Optimization results for the 72-bar truss.

| **Design Variables** | **SHAO (The best)** | **SHAO (The worst)** | **SAO** | **ALPSO** | **PSO** | **HBB-BC** | **ACO** |
|---|---|---|---|---|---|---|---|
| **$x_1$ [in$^2$]** | 0.156 | 0.159 | 0.157 | 0.157 | 0.162 | 0.157 | 0.156 |
| **$x_2$ [in$^2$]** | 0.551 | 0.512 | 0.549 | 0.546 | 0.509 | 0.542 | 0.550 |
| **$x_3$ [in$^2$]** | 0.408 | 0.43 | 0.406 | 0.405 | 0.497 | 0.413 | 0.390 |
| **$x_4$ [in$^2$]** | 0.560 | 0.554 | 0.555 | 0.566 | 0.562 | 0.576 | 0.592 |
| **$x_5$ [in$^2$]** | 0.527 | 0.478 | 0.513 | 0.520 | 0.514 | 0.518 | 0.561 |
| **$x_6$ [in$^2$]** | 0.514 | 0.492 | 0.529 | 0.518 | 0.546 | 0.521 | 0.492 |
| **$x_7$ [in$^2$]** | 0.10 | 0.10 | 0.100 | 0.100 | 0.100 | 0.100 | 0.100 |
| **$x_8$ [in$^2$]** | 0.10 | 0.10 | 0.100 | 0.100 | 0.110 | 0.101 | 0.107 |
| **$x_9$ [in$^2$]** | 1.290 | 1.233 | 1.252 | 1.258 | 1.308 | 1.258 | 1.303 |

| Design Variables | SHAO (The best) | SHAO (The worst) | SAO | ALPSO | PSO | HBB-BC | ACO |
|---|---|---|---|---|---|---|---|
| $x_{10}$ [in$^2$] | 0.515 | 0.507 | 0.524 | 0.513 | 0.519 | 0.503 | 0.511 |
| $x_{11}$ [in$^2$] | 0.10 | 0.10 | 0.100 | 0.100 | 0.100 | 0.100 | 0.101 |
| $x_{12}$ [in$^2$] | 0.10 | 0.10 | 0.100 | 0.100 | 0.100 | 0.100 | 0.100 |
| $x_{13}$ [in$^2$] | 1.881 | 2.399 | 1.832 | 1.898 | 1.743 | 1.904 | 1.948 |
| $x_{14}$ [in$^2$] | 0.506 | 0.504 | 0.512 | 0.513 | 0.519 | 0.516 | 0.508 |
| $x_{15}$ [in$^2$] | 0.10 | 0.10 | 0.100 | 0.100 | 0.100 | 0.100 | 0.101 |
| $x_{16}$ [in$^2$] | 0.10 | 0.100 | 0.100 | 0.100 | 0.100 | 0.100 | 0.102 |
| Max. stress [psi] | 24958.61 | 24999.67 | 24943.87 | 24999.67 | 24485.67 | 24948.16 | 24939.59 |
| Max. disp.[in] | 0.24994 | 0.2500 | 0.24992 | 0.2500 | 0.2497 | 0.2501 | 0.2500 |
| Weight [lb] | 379.93 | 383.13 | 379.90 | 379.61 | 381.91 | 379.66 | 380.24 |
| No. of analyses | 138 | 153 | 252 | >$10^3$ | N/A | 13200 | 18500 |

The optimization problem is solved by the proposed SHAO algorithm with 50 initial sampling points. The termination criterion is satisfied after 88 iterations, as revealed by variation of relative error shown in Fig.4. Observing Fig.4, we notice that the approximate accuracy of the physical fields improves with the optimization iterations. Evolution of the objective function is displayed in Fig.5. Table III summarizes results for the 72-bar truss problem, and compares this with different optimization techniques. For proper comparison, the best and worst results from twenty independent SHAO trials are also listed.

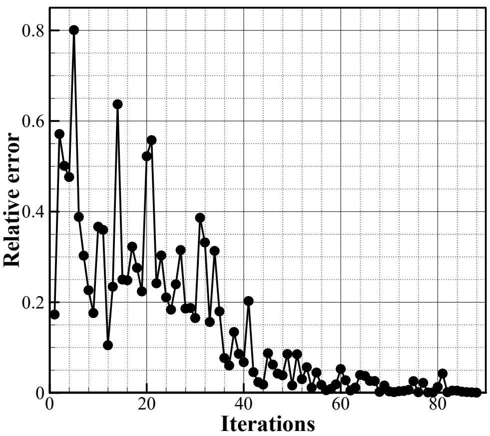


**Figure 4**: Convergence of SHAO for 72-bar truss optimization

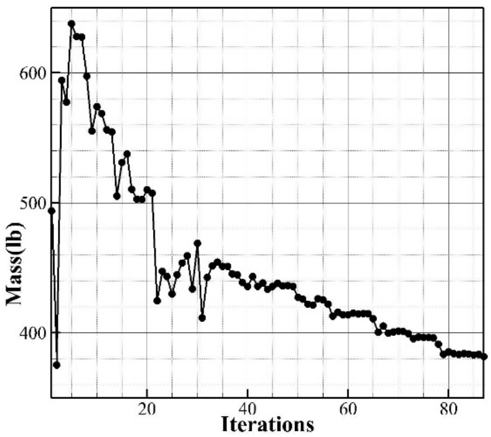


**Figure 5**: The objective function history.

### 3.2 Case study 2: 582-bar tower truss system

The final study carried out applies the SHAO algorithm to a 582-bar tower truss system as shown in Fig.6. The 582 structural members are categorized as 32 independent size variables. The lower and upper bounds on size variables are taken as 1.55 in$^2$ (10 cm$^2$) and 155.0 in$^2$ (1000 cm$^2$), respectively. A single load case is considered consisting of lateral loads of 5.0 kN (1.12 kips) applied in both x- and y-directions and a vertical load of 30 kN (6.74 kips) applied in the z-direction at all nodes of the tower. Modulus of elasticity is 29,000 ksi (203.89 GPa), allowable tensile and compressive stresses are 100MPa and the limitations of nodal displacements are no more than 8.0 cm in all directions.

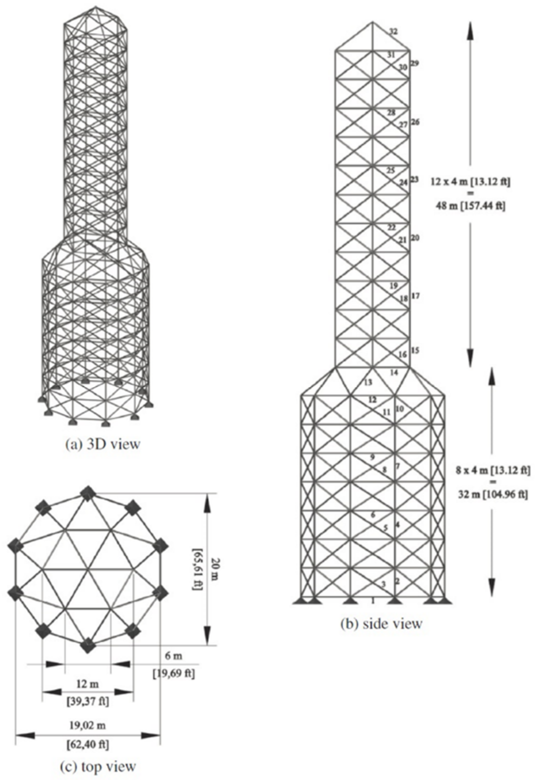


**Figure 6**: A 582-bar spatial truss structure.

**Table 4**: Optimization results for the 582-bar truss.

| Design Variables | SHAO (The best) | SHAO (The worst) | SAO | DE |
|---|---|---|---|---|
| $x_1$ [cm$^2$] | 10.01 | 14.22 | 10.13 | 16.47 |
| $x_2$ [cm$^2$] | 164.72 | 160.18 | 162.22 | 171.02 |
| $x_3$ [cm$^2$] | 21.98 | 21.83 | 23.33 | 20.40 |
| $x_4$ [cm$^2$] | 151.91 | 143.05 | 145.86 | 143.22 |
| $x_5$ [cm$^2$] | 20.68 | 22.93 | 18.51 | 23.24 |
| $x_6$ [cm$^2$] | 10.00 | 10.00 | 10.70 | 10.00 |
| $x_7$ [cm$^2$] | 116.84 | 113.86 | 130.25 | 120.42 |
| $x_8$ [cm$^2$] | 21.65 | 32.46 | 15.66 | 30.72 |
| $x_9$ [cm$^2$] | 10.00 | 10.00 | 10.01 | 10.00 |
| $x_{10}$ [cm$^2$] | 103.85 | 125.32 | 98.53 | 103.55 |
| $x_{11}$ [cm$^2$] | 16.07 | 31.55 | 15.67 | 20.38 |
| $x_{12}$ [cm$^2$] | 113.98 | 107.58 | 128.84 | 110.83 |
| $x_{13}$ [cm$^2$] | 183.58 | 173.43 | 178.87 | 183.14 |
| $x_{14}$ [cm$^2$] | 143.46 | 132.98 | 151.83 | 134.22 |
| $x_{15}$ [cm$^2$] | 186.02 | 161.16 | 193.86 | 170.82 |
| $x_{16}$ [cm$^2$] | 29.13 | 44.74 | 32.63 | 28.89 |
| $x_{17}$ [cm$^2$] | 152.25 | 152.83 | 153.33 | 153.65 |
| $x_{18}$ [cm$^2$] | 20.70 | 22.14 | 20.46 | 25.61 |
| $x_{19}$ [cm$^2$] | 10.00 | 10.00 | 10.13 | 10.00 |
| $x_{20}$ [cm$^2$] | 108.84 | 90.11 | 109.81 | 98.21 |
| $x_{21}$ [cm$^2$] | 19.30 | 18.40 | 19.22 | 19.72 |
| $x_{22}$ [cm$^2$] | 10.13 | 37.79 | 10.24 | 33.77 |
| $x_{23}$ [cm$^2$] | 55.60 | 62.41 | 61.77 | 59.48 |
| $x_{24}$ [cm$^2$] | 17.00 | 16.96 | 15.92 | 15.34 |
| $x_{25}$ [cm$^2$] | 10.00 | 10.00 | 10.07 | 10.00 |
| $x_{26}$ [cm$^2$] | 28.33 | 33.35 | 27.73 | 30.40 |
| $x_{27}$ [cm$^2$] | 16.35 | 18.42 | 10.07 | 16.34 |
| $x_{28}$ [cm$^2$] | 10.00 | 10.00 | 10.01 | 10.00 |
| $x_{29}$ [cm$^2$] | 10.00 | 10.00 | 10.15 | 10.00 |
| $x_{30}$ [cm$^2$] | 10.00 | 12.64 | 10.43 | 13.35 |
| $x_{31}$ [cm$^2$] | 10.00 | 10.00 | 10.65 | 10.01 |
| $x_{32}$ [cm$^2$] | 10.00 | 10.00 | 10.49 | 10.00 |
| Max. stress [Mpa] | 76.01 | 78.5 | 76.53 | 77.7 |
| Max. disp.[cm] | 8.00 | 7.96 | 8.00 | 7.96 |
| V(m$^3$) | 15.63 | 16.37 | 16.13 | 15.67 |
| No. ofanalyses | 419 | 433 | 586 | 30000 |

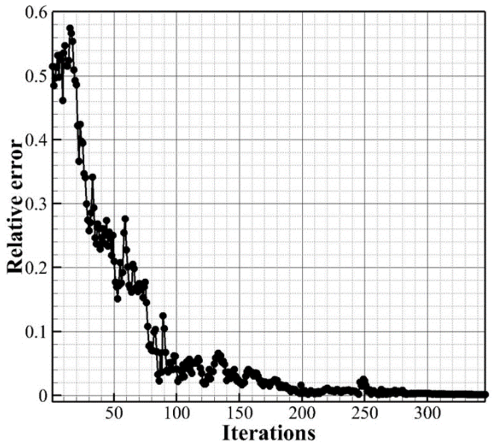


**Figure 7**: Convergence of SHAO for the 582-bar truss optimization

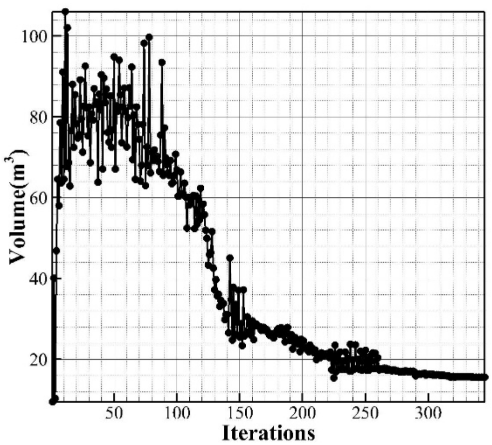


**Figure 8**: The objective function history.

This optimization problem is solved using the proposed SHAO algorithm with 64 initial sampling points. The termination criterion is satisfied after 355 iterations as revealed by the variation of relative error shown in Fig.7, and evolution of objective function as displayed in Fig. 8. To verify the effectiveness of this proposed method, SAO [17] and DE [18] are also utilized to solve the same problem. For SAO, 64 initial sampling points are used to initialize the optimization, while for the DE, population of 50 individuals are designated. Table IV presents the optimal results obtained by the proposed SHAO, SAO and DE, respectively.

## 4 CONCLUSIONS

This research paper presented a SHAO approach suitable for structural design optimization tasks. This approach significantly reduces the computational costs normally associated with structural design problems. With the existing trend of utilizing structural analysis processes as a seemingly ‘black box’ manner within the conventional SAO, this proposed SHAO breaks down the process and disintegrates it into differentiated stages: a time-consuming and hard-to-interpret FEA stage, and the well-assessed, easy-to-handle post processing stage. The former is surrogated by field surrogate models and the latter is handled in a fundamental theoretical manner. This manipulation produces a hybrid model that enables capturing of more complex dynamic behaviors of the true structural analysis process when compared to the ‘black-box’ prediction models. The hybrid model projects the error signals to a subspace that is easier to sufficiently explore with small number of training points and can further generalize and predict more accurately when compared to standard ‘black-box’ surrogate models. Thus, the efficacy and efficiency of the optimization process is improved substantially.

The proposed method was evaluated using two benchmark test cases; the 72-bar truss system and the 582-bar tower truss system. Compared with results of previously published studies, the SHAO algorithm yielded equivalent or much better objective values for the tested structural design optimization tasks. Furthermore, the number of true function evaluations required to find the same global optima was significantly reduced by multiple orders of magnitude, which further highlights the applicability of the proposed SHAO algorithm towards engineering structural design optimization problems.

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