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Shallow water flow estimation analysis using the ensemble Kalman filter FEM

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1 Introduction

The SUPG method is frequently used to solve the governing equation. It is necessary to carry out calculations based on several procedures that have been presented to obtain stabilization parameters[1]. However, even if a numerical simulation is carried out based on the stabilized FEM, the numerical solution is rarely in close agreement with real-world observed values. It has been confirmed that if the Kalman filter FEM is employed to compute the flow behavior while taking observed values into account, the computed value is in better agreement with the observed value than the computational results obtained using the conventional FEM[2]. However, the Kalman filter FEM can't be applyed to problem using the non-liniar system equation. Therefore, the ensemble Kaiman filter FEM is introduced in this study[3]. In the ensemble Kalman filter FEM, the stochastic distribution of the state variables is expressed by the ensemble approximation, and the special distribution of the state variables is obtained by the FEM. The shallow water equation is introduced as the governing equation, and the SUPG method is applied to discretize the governing equation. Some results of numerical example is shown in this study by using open channel model.

2 Computational flow of the ensemble Kalman filter FEM

In this study, the shallow water equation is applied to simulate the flow field. The momentum and the continuity equations are expressed as Eqs. (1) and (2), respectively.

$$\dot{u}_i + u_j u_{i,j} + g\eta_{,i} - \nu(u_{i,j} + u_{j,i})_{,j} = 0 \tag{1}$$

$$\dot{\eta} + ((h+\eta)u_i)_{,i} = 0 \tag{2}$$

where u_i, g, h, η and ν denote the velocity component for x and y directions, the water elevation, the gravitational acceleration, the standard water depth and the kinematic viscosity coefficient, respectively. The kinematic viscosity coefficient ν is expressed by Eq. (3).

$$\nu = \frac{\kappa_l}{6} u_* h \tag{3}$$

In Eq. (3), the Kalman constant κ_l is set as 0.41, and the friction velocity u_* is calculated by Eq. (4).

$$u_* = \frac{g n_*^2 \sqrt{u_k u_k}}{h^{1/3}} \tag{4}$$

Here, n_* indicates Manning's roughness coefficient. The SUPG and the backward Euler methods are employed to discretize the governing equation in space and time, respectively. The system equation in the ensemble Kalman filter FEM is obtained by adding the system noise term to the discretized governing equation.

The system and the observation equations shown in Eqs. (5) and (6) is introduced in the ensemble Kalman filter FEM:

$$[A_1(\phi)]\{\phi^{n+1(i)}\} = [A_2(\phi)]\{\phi^{n(i)}\} + [\Gamma]\{q^{n(i)}\} \quad i = 1, 2, \cdots, M \tag{5}$$

$$\{z^{n+1(i)}\} = [H]\{\phi^{n+1(i)}\} + \{r^{n+1(i)}\} \quad i = 1, 2, \cdots, M \tag{6}$$

where $[A_1(\phi)]$, $[A_2(\phi)]$, $\{\phi\}$, $[\Gamma]$, $\{q\}$, $\{z\}$, $[H]$ and $\{r\}$ respectively indicate the coefficient matrices of the finite element equation, the state variable vector, the driving matrix, the system noise vector, the observation value vector, the observation matrix and the observation noise. Here, n and (i) indicate the number of time steps and sample number, with M being the total sample number. The number of the initial conditions is M. The initial conditions are prepared by the Gaussian distribution. The computational flow of the ensemble Kalman filter is shown as follows. In the following computational flow, $\{\tilde{\phi}^{n(i)}\}$ and $\{\tilde{z}^{n(i)}\}$ represent $\{\phi^{n(i)}\} - \{\bar{\phi}^n\}$ and $\{z^{n(i)}\} - \{\bar{z}^n\}$, respectively. $\{\bar{\phi}^n\}$ and $\{\bar{z}^n\}$ represent the individual average values of all ensemble components.

1. Item input of the ensemble matrix for the state value at the initial step, $[\tilde{\Phi}^0_{(-)}] = [\{\tilde{\phi}^{0(1)}_{(-)}\}, \{\tilde{\phi}^{n(2)}_{(-)}\} \cdots \{\tilde{\phi}^{0(M)}_{(-)}\}]$, and n=0.
2. Calculation of the ensemble matrix for the observation value, $[\tilde{Z}^n_{(-)}] = [\{\tilde{z}^{n(1)}_{(-)}\}, \{\tilde{z}^{n(2)}_{(-)}\} \cdots \{\tilde{z}^{n(M)}_{(-)}\}]$.
3. Calculation of covariance matrices, $[U^{\bar{n}}_{(-)}] = \frac{1}{M-1}[\tilde{Z}^n_{(-)}][\tilde{Z}^n_{(-)}]^T$ and $[V^{\bar{n}}_{(-)}] = \frac{1}{M-1}[\tilde{\Phi}^n_{(-)}][\tilde{Z}^n_{(-)}]^T$.
4. Calculation of the Kalman gain matrix, $[K^n] = [\bar{U}^n_{(-)}][\bar{V}^n_{(-)}]$.
5. Calculation of the optimal estimation value, $\{\phi^{n(i)}_{(+)}\} = \{\phi^{n(i)}_{(-)}\} + [K^n](z^n - z^{n(i)}_{(-)}) \quad i = 1, 2, \cdots, M.$
6. Calculation of the averaged optimal estimation value, $\{\bar{\phi}^n_{(+)}\} = \frac{1}{M}\sum_{i=1}^{M}\{\phi^{n(i)}_{(+)}\}$.
7. Calculation of the system equation, $[A_1(\phi)]\{\phi^{n+1(i)}_{(+)}\} = [A_2(\phi)]\{\phi^{n(i)}_{(+)}\} + [\Gamma]\{q^{n(i)}\}$
8. Calculation of the ensemble matrix for the state value, $[\tilde{\Phi}^{n+1}_{(+)}] = [\{\tilde{\phi}^{n+1(1)}_{(+)}\}, \{\tilde{\phi}^{n+1(2)}_{(+)}\} \cdots \{\tilde{\phi}^{n+1(M)}_{(+)}\}]$.
9. Update of time step $n \Rightarrow n + 1$ and $[\tilde{\Phi}^{n+1}_{(+)}] \Rightarrow [\tilde{\Phi}^n_{(-)}]$ and return to step 2.

3 Numerical example

Some results of the numerical experiment using the ensemble Kalman filter using the SUPG FEM are shown in this section. The numerical model is shown in Fig. 1. The observation points are set at points A, B and C, and the estimation points are set at points D, E and F. The coordinates for each point are listed in Table 1. The definition of boundary conditions is also shown in Fig.2. The numerical conditions are listed in Table 2. As an example of observed water elevation, the time history of water elevation at point C is shown in Fig.3.

Fig.4 shows the time history of estimated water elevation in case of some sample numbers at point F. The sample numbers is set as 100, 125, 250, 1000. The red line indicate the true value of the water elevation, and the other lines indicate the estimated water elevation in case of each sample number. It is seen that appropriate estimated water elevation is obtained in case of the sample number is equal to 1000.

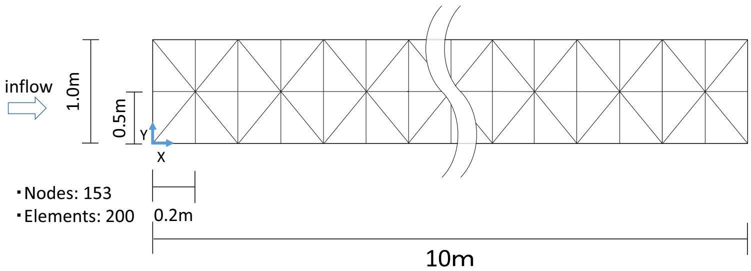

Figure 1: Computational model.

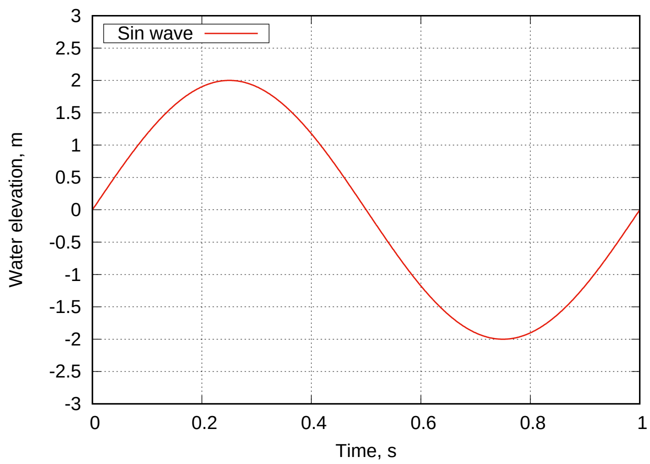

Figure 2: Boundary condition on the left hand side boundary.

4 Conclusions

Shallow water flow estimation analysis based on the Kalman filter FEM was shown in this paper. The shallow water equation was employed as the governing equation, and the SUPG and the backward Euler methods were applied to discretize the governing equation in space and time, respectively. The open channel model was employed as the computational model, and the estimation accuracy was investigated by changing the

Table 1: Position of observation points and estimation points.

A (Observation point)	$(x,y) = (0.0,0.5)$
B (Observation point)	$(x,y) = (1.0,0.5)$
C (Observation point)	$(x,y) = (2.0,0.5)$
D (Estimation point)	$(x,y) = (3.0,0.5)$
E (Estimation point)	$(x,y) = (4.0,0.5)$
F (Estimation point)	$(x,y) = (5.0,0.5)$

Table 2: Computational conditions.

Time increment, s	0.001
Number of time step	1000
Standard water depth, m	10.0
Manning's roughness coefficient n_*, $\mathrm{m}^{-1/3}\mathrm{s}$	0.05
Gravitational acceleration, $\mathrm{m/s}^2$	9.81
Initial condition (Average)	0.0
Initial condition (Variance)	0.001
System noise (Average)	0.0
System noise (Variance)	0.001
Observation noise (Average)	0.0
Observation noise (Variance)	0.1
Sample number M	100, 125, 250, 1000

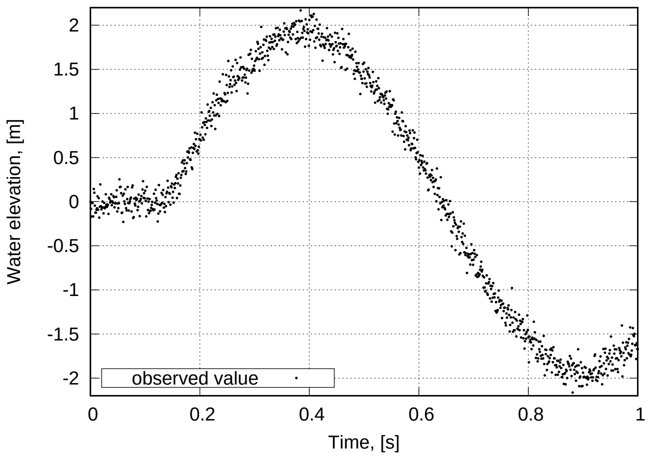

Figure 3: Time history of observed water elevation at point C.

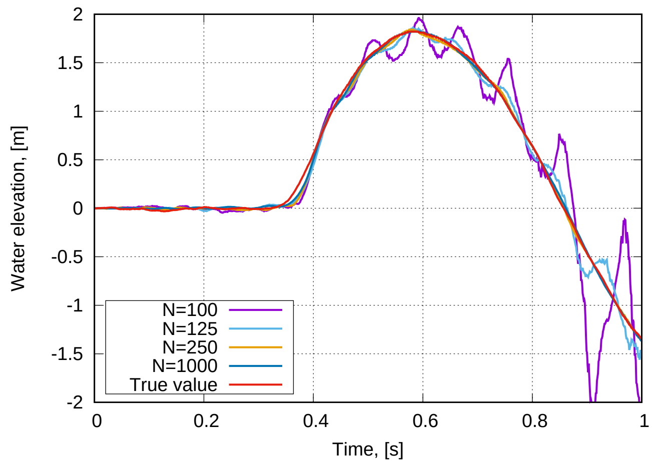

Figure 4: Time history of estimated water elevation in case of some sample numbers at point F.

sample number. In this study, we confirmed that the flow behaviour is appropriately obtained using the present method in case that enough sample number is given as the computational condition. In future studies, it will be necessary to apply the present method to the flow estimation problems using the practical observed data.

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