AN EFFICIENT HYBRID APPROACH TO GEAR CONTACT SIMULATION IN MULTIBODY SYSTEMS LEVERAGING REDUCED ORDER MODELS

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Key words: Multibody dynamics, Contact mechanics, Gear contact

Abstract. A novel method to solve complex gear contact problems is presented. The method is implemented in the Siemens PLM multibody solver *Motion* and employs an accurate and fast dedicated contact detection strategy using a hybrid numerical/analytical teeth contact stiffness representation. Experimental and numerical comparisons are proposed to validate the quality of the approach.

1 INTRODUCTION

Due to increasing awareness on environmental concerns, stringent targets are being set for energy efficiency and emissions. Industry is forced to improve their design under conflicting requirements between customers (increasing performance) and regulatory bodies (increasing efficiency). Mechanical transmissions are responsible for significant energy losses (6-8% of total) in automotive and wind energy applications. Recent research studies [1,2] document margins to significantly reduce transmission losses by 50%, showing potential for saving 9.3 million tons of CO_2 emissions in the automotive field. On the other hand, the complex interactions between efficiency optimization and other crucial performance attributes such as durability and noise are not well studied nor understood. One of the main reasons for this deficiency is the lack of proper simulation tools allowing to calculate the dynamic behavior of systems that include geared transmissions, with a sufficient level of speed and accuracy. In fact, gearboxes present significant engineering challenges in many application fields, ranging from automotive (cars, trucks, buses) to wind turbines and helicopters. The basic components of a transmission system are gears, bearings, shafts and the supporting structure. They are all mutually connected and the supporting structure substantially affects bearings contacts and gear meshing through e.g. several types of misalignment. The interactions between the different components, result oftentimes in poor dynamics and NVH performances. A key performance indicator (KPI) generally adopted to evaluate the NVH performances of transmissions is the transmission error (TE) [3].

In recent years, indeed substantial effort is being dedicated to analyze transmissions at a system level in order to achieve better performances in terms of efficiency, noise and reliability. The main challenge still remains the lack of methods to capture the non-linear system dynamics in a sufficiently detailed, yet computationally efficient way. Without claims of completeness, designers of transmissions can choose from three types of simulation platforms to support their design and simulation efforts. The first family is targeted at gear box design, providing a lot of gear box specific know-how that can be used in the design process, but lacking certain simulation capability such as e.g. dynamic system level simulations. A second family is represented by nonlinear finite element tools. These tools cover the highest end of the spectrum regarding solution accuracy, but are computationally too expensive for lengthy time-domain simulations and system level analysis. As a result, users are practically limited to a series of static simulations or short dynamic simulation of sub-systems such as a single gear pair. The third family is represented by general purpose multi-body simulation tools which are ideal for dynamic system-level simulations but usually lack specificity and/or accuracy regarding advanced drivetrain simulations.

The methodology presented in this work belongs to the latter family and proposes to extend the applicability range of multibody approaches towards dynamics drivetrain simulations. This is achieved thanks to a dedicated efficient contact detection and an advanced and scalable contact force computation method. In particular geometrical effects such as misalignment, microgeometry modifications and wedging, are taken into account thanks to a tailored contact detection scheme. In order to efficiently solve the contact problem, a non-linear complementarity problem is regularized and solved with a stable Newton scheme. This problem accounts for local non-linear Hertzian phenomena and linear flexibility of the gear bodies including coupling between multiple contacting teeth and lightweight gear bodies. This is efficiently attained thanks to notions related to the field of (discontinuous) model order reduction [4, 5, 6]. The implementation is available as a standard product in the Siemens multibody solver *Motion* (Siemens Virtual.Lab Motion [7], Siemens Simcenter 3D Motion [8]).

The solver is designed to be modular and includes the possibility to simulate stiffness, damping and friction contact forces for cylindrical internal and external gears. For what concerns stiffness contributions to contact forces, the user is able to select between three levels of complexity: Standard, Analytical and Advanced [9]. These three options also present some sub-options in order to satisfy advanced users and provide a wide range of simulation opportunities. Figure 1 highlights the main differences and characteristics of the available approaches.

This work analyzes in more details the Advanced - FE preprocessor option which represents the highest level of fidelity implemented and is a significant step forward towards efficient and accurate gear simulation. The Advanced - FE preprocessor allows to account a wide range of flexibility-induced phenomena (lightweight gears, convective coupling effects, etc.). The method and the preprocessing steps have been developed in cooperation with the University of Leuven (KU Leuven - Belgium) and the University of Calabria (Italy).

	Standard	Analytical	Empirical	FE Preprocessor
Input stiffness	ISO	ISO+Cai	Bulk stiffness curves	FE-based tooth stiffness data
	ISO+TE			
	UserDef Const			
	UserDef Const +TE			
	UserDef Var			
Friction	х	1	1	1
Possibility of slicing	х	✓	✓	1
Possibility of microgeometry	x	1	✓	1
Contact stiffness (linear or nonlinear)	Linear	Linear	Nonlinear	Nonlinear
Accounts for varying width of the contact	1	1	✓	1
Internal gears	✓ Approximately	✓ Approximately	✓ Approximately	1
Light weight	x	х	х	1
Blank flexibility	х	х	✓ Approximately	✓
Convective coupling between slices and teeth	х	х	х	✔ Both
Misalignments, tip contact, wedging	x	✓	✓	1
Usage recommendation	When data for stiffness and TE is available. Design studies on system dynamics	Bulky gears, parametric studies on microgeometry, system dynamics, misalignement, friction influence. Preliminary NVH	Cylindrical gear: spur and helical external. When body flexibility plays a role but not for light- weight or highly flexible.	Cylindrical gear: spur and helical, internal and external, includ- ing lightweight and highly deformable bodies. Possibilit to extend to Bev and hypoid gear.

Figure 1: Gear contact methods available in Virtual.Lab Motion and Simcenter 3D Motion

2 METHODOLOGY

The *Advanced – FE preprocessor* method belongs to the larger class of penalty based contact problems [10]. These methods are generally defined by two computational steps: (1) a contact detection step and (2) a force computation. Penalty methods allow the bodies in contact to unphysically penetrate between each other. By means of this penetration and a penalty stiffness, contact forces are produced. These forces counter the penetration between the bodies in contact, so that only a limited amount of penetration is allowed until the force equilibrium is achieved. Within (non-linear) FE formulations, the penalty factors used are - generally speaking- set to values that are several orders of magnitude larger than the FE contact stiffness. This results in a negligible penetration depth, but in an increased numerical stiffness of the problem to solve. This often requires very small time-steps for integration stability and leads to unacceptable computational performances.

In the field of contact in multibody systems, penalty formulations are generally adopted to compute contact interactions between very stiff bodies. In this context the overall body deformation is negligible with respect to the local contact deformation. The latter can oftentimes be approximated thanks to the usage of several analytical expressions, mostly including Hertz-based non-linear analytical formulas and coefficients of restitutions to simulate dissipative effects [11]. These methods are generally efficient and accurate for contact between stiff bodies, but become inaccurate when the global body deformations start to play an important role. Contact between gears lies in the difficult realm of contact problems for which the linear global deformation of the bodies is small but significant with respect to the non-linear contact deformations. For this reason the *Advanced - FE preprocessor* combines the best of two words by performing an efficient rigid contact detection and merging numerical and analytical methods for the computation of the contact stiffness.

2.1 Numerical aspects of contact detection

The presented method makes use of a tailored efficient contact detection strategy. The main assumptions are listed as follows:

- The method assumes contact between cylindrical spur/helical internal/external gears and profile shapes that are involute or close to involute;
- The tangent and normal vectors to the contact surfaces are computed assuming perfectly involute profiles;
- The contact points along the gear flank remain located along the line of action defined by the involute profiles.

Thanks to the above assumptions, it is possible to exploit well know analytical formulas [12, 13] that allow to predict the locations of potential contact between the gears teeth. These assumptions are usually acceptable in the field of gear contact especially when metallic gears are considered. In particular, the shapes of the majority of commercial cylindrical gears is very close to an involute and the surface normals are only slightly affected by profile modifications or tooth deformation. The contact point migration along the flank profile due to microgeometry and deformation is mostly negligible. Moreover, the contact stiffness variation caused by this migration is negligible. These assumptions might lose some of their effectiveness when extremely deformable gears are considered (e.g. rubber-like materials).

Within the presented approach several effects are taken in to account:

- **Microgeometry modifications:** The penetration between the teeth flank profiles is corrected for microgeometry induced modifications along the normal to the contact;
- Axial overlap: The contact detection strategy determines the axial overlap between the contacting gears based on geometrical considerations on-line during simulation;
- **Misalignment:** One of the most important characteristics of the implemented contact method lies in the possibility to simplify translational and rotational misalignments between gears thanks to dedicated geometrical computations performed on-line during simulation. Angular misalignments are complex to resolve in an exact manner but have a large impact on the static and dynamic behavior of geared transmissions. The proposed method introduces an approach to deal with these conditions thanks to the concept of *'average rotational axis'*. First a coarse contact detection step is performed to simplify the contact detection by virtually re-aligning the gears such that their rotation axes become aligned along the *'average rotational axis'*. This coarse contact detection effectively reduces the problem to the solution of contact detection of aligned gears. In order to compute a reliable approximation of the penetration occurring between points in the original misaligned configuration. This two-step approach is accurate for small angular misalignments as found in most engineering applications;
- Slicing: In order to increase the accuracy of a contact simulation, the contact detection has the possibility to divide the axial overlap in equidistant segments (slices). This is done by taking into account various effects, such as flank modifications along the axial direction, misalignment and stiffness variations along the tooth width. The penetrations and velocities are computed for each slice center and are considered constant along each slice width.

The implemented contact detection includes several important features that allow its usage

for a wide variety of applications in the geared transmissions world. Thanks to the assumption of contact between rigid bodies that is typical of multibody applications, the method remains computationally superior to FE-based contact detection methods.

2.2 Numerical of contact force computation

After the contact detection, a second computational step is performed to translate the penetrations into contact forces. Here, the discussion is focused towards the computation of contact forces acting along the teeth normals and caused by stiffness-related effects. Friction forces and damping forces are not detailed in this work for sake of conciseness.

The *Advanced - FE preprocessor* method is inspired by several works [4-6, 14-17] and is a hybrid analytical/numerical penalty approach. The overall idea behind the method is detailed in the next paragraphs. By reference to Figure 2, the overall contact deformation is decomposed in two components:



Figure 2: Decomposition in global and local deformation [5]

- A local non-linear deformation caused by locally non-linear Hertzian-like effects;
- A global linear deformation caused by the tooth bending and shear as well as by complex effects due to the gear body geometry and coupling between different teeth (e.g. when one tooth is loaded, also adjacent teeth will deform).

In order to capture the first contribution, it is possible to rely on analytical formulas available in the literature. In the method presented in this work, a formula derived by Weber and Banaschek [16, 17] based on Hertzian theory is used;

The second contribution includes global effects that despite being linear with respect to the applied loads, is generally not representable by any closed-form analytical expression. In-fact, the types of combinations of gear body geometries and teeth shapes cannot be captured properly by a parametric analytical formula. These types of effects are typically well captured by gears modelled with the finite element (FE) method. To this end, an FE model of each gear involved in the contact is created. To help the user in this task, a parametric the FE preprocessor is accessible [9]. Once the mesh is available, a numerical strategy is adopted to decouple the part of the deformation captured by the underlying FE model and the part captured by the non-linear analytical formulas. It is of paramount importance to make sure that this decomposition is performed accurately, so that local deformation effects are removed from the contribution included in the FE representation. The adopted technique is described in [5, 17]. This approach is schematically represented in Figure 3 where the first two images on the right-side represent FE-based calculations. In the same figure, the rightmost image represents the analytical non-linear contact deformation. The FE simulations result in a compliance matrix file computed for each gear to be considered. The method allows the user a large range of flexibility.



Figure 3: Computational steps to represent gear total deformation

A special attention is given to the amount and types of *"coupling"* convective terms to be included in the problem solution. With coupling terms we indicate the following effects:

- Slice coupling effects: When a single axial slice is loaded on a tooth flank, the other slices on the same tooth flank will experience a deformation, effectively contributing to the compliance matrix of the tooth. These terms are always included in the computations;
- **Teeth coupling effects**: When a single axial slice is loaded on a tooth flank, slices belonging to all of the adjacent teeth will experience a deformation that effectively contributes to the compliance matrix of the gear pair. In common applications, only a limited amount of teeth is in contact at a certain instant in time, such that the amount of teeth coupling terms can be limited, consequently reducing storage. The user can chose to increase the complexity and accuracy of the simulation by including these effects.

The coupling terms between slices and – more importantly – between teeth are a truly unique feature of this method for multibody systems and is an important contribution to the high accuracy of the method. The above method description can be summarized in mathematical terms in a non-linear complementarity problem (NLCP) [18] of the following form:

$$\Delta_{l}^{12} - f(\overline{F_{l}^{12} \cdot n_{l}^{12}}, geom_{1,2}, E_{1,2}, \nu_{1,2}) + [C^{1}(geom_{1}, E_{1}, \nu_{1}) + C^{2}(geom_{2}, E_{2}, \nu_{2})] \cdot \overline{[F_{l}^{12} \cdot n_{l}^{12}]} = (1)$$

= $q(\overline{F_{l}^{12} \cdot n_{l}^{12}}, geom_{1,2}, E_{1,2}, \nu_{1,2}) = 0$

$$g(F_l^{12} \cdot n_l^{12}, geom_{1,2}, E_{1,2}, \nu_{1,2}) \ge \mathbf{0}$$
(2)

$$\overline{[F_l^{12} \cdot n_l^{12}]} \ge 0 \tag{3}$$

$$g(\overline{F_{l}^{12}} \cdot n_{l}^{12}, geom_{1,2}, E_{1,2}, v_{1,2})^{T} \cdot \overline{[F_{l}^{12} \cdot n_{l}^{12}]} = 0$$
(4)

Where:

- Δ_l^{12} is the vector of instantaneous penetrations on each active slice *l* for each of the teeth that are simultaneously in contact at a certain time instant. The penetration vector is corrected for the contribution due to the microgeometry in the normal direction;
- $[F_l^{12} \cdot n_l^{12}]$ is the vector of unknown contact forces in the normal direction n_l^{12} with respect to the involute profile;
- f(F_l¹² · n_l¹², geom_{1,2}, E_{1,2}, v_{1,2}) is a vector of non-linear functions of the normal contact forces that accounts for the non-linear Hertzian contact compliance effects as according to [8]. geom_{1,2}, E_{1,2}, and v_{1,2} represent known input parameters that relate to the gears geometry and material properties;
- $C^{i}(geom_{i}, E_{i}, v_{i})$ is a full matrix that includes the FE based compliance for gear *i*;
- $g(\overline{F_l^{12}} \cdot n_l^{12}, geom_{1,2}, E_{1,2}, v_{1,2})$ is the residual function.

It can be noticed that due to the full nature of the C^i matrices, the amount of penetration on each slice is highly depended on the penetration (and contact forces) acting on all the potentially active slices. In practice, the amount and the locations of the active slices are not known before the solution of (1)-(4). In physical terms, this can be explained thinking that some slices might exit or enter contact due to the influence of loads acting on other loaded slices. Several strategies exist to solve the above problem. When a solution exists, the system of equations can be solved exactly using an NLCP solver or approximately using a regularization scheme and a modified Newton solver. Given the difficulty in guaranteeing the existence and uniqueness of the solution of the NLCP, the second alternative was selected. An algorithm has been tailored to guarantee convergence of the solution by exploiting the particularly well defined structure of the nonlinear contribution f(-) of the system of equations. The following considerations make the suggested approach computationally attractive and scalable from a user point of view:

- The separation between global and contact compliance allows to adopt a relatively coarse mesh (usually 6-12 elements on the profile and 4-12 elements on the width are enough for cylindrical gears), as compared to typical contact mechanics FE meshes where the contact areas have to be highly discretized;
- The slicing approach allows the user to select a number of slices in which the instantaneous axial overlap is divided. A number of slices that is similar to the amount of element along the teeth axes is usually suggested;
- Since contact forces are computed including a non-linear local compliance term, the contact stiffness presents the typical stiffening behavior with increasing contact loads.

2.3 Stiffness map creation thanks to model order reduction

As mentioned in previous sections, the creation of the gear compliance matrix involves the computation of a reduction space that is used to condense the information contained in the underlying FE stiffness matrix of the gears. The used space reduction is obtained based on a procedure similar to the one detailed in [5], which leverages a series of static FE solutions for each potential contact node. The reduction space contains only the global deformations the gears. In order to further limit the amount of preprocessing and the CPU time during the multibody simulation, the user is provided with a parameter that allows to limit the amount of calculations. In fact, in many cases, it is not necessary to compute global deformation patterns for each of the nodes on a gear flank, since global shapes obtained from applying loads to adjacent nodes are very similar and they add only little amount of information to the compliance database. For the majority of the cases, it is sufficient to extract deformation patterns for a limited set of FE nodes (e.g. 50-70% of the nodes on the teeth flanks).

The obtained reduction space Ψ^i is used to reduce the stiffness matrix of the underlying FE model, as shown in Eq. (5):

$$\mathbf{K}_{\rm red}^i = {\mathbf{\Psi}^i}^T \mathbf{K}_{\rm FE}^i {\mathbf{\Psi}^i} \tag{5}$$

The matrix K_{FE}^{i} is the FE stiffness matrix of gear and K_{red}^{i} is the reduced stiffness matrix. The inverse of the reduced stiffness matrix is used internally in the multibody solver to create the compliance matrix C^{i} mapped onto the gear axial slices.

It is well known that the amount of vectors that need to be included in the reduction space can become significant. As a consequence, the reduced stiffness matrix K_{red}^i becomes prohibitively large [4] (especially in the case of non-axisymmetric gears with a large number

of teeth). In addition, it is important to notice that due to the rotation of the gears, only a few teeth are actively involved in the contact at a certain moment in time. For this reason, the assembly of the reduced stiffness matrix is performed dynamically during the simulation, using only the segment of the database related to the part of K_{red}^i that is involved in the contact (at a certain moment in time). This procedure is closely related to the static modes switching method proposed in [4]. Since the contact forces are computed in a quasi-static manner, the numerical procedure remains smooth and convergence is obtained at each time-step (without spurious oscillations). The update of the compliance database takes place every time a new tooth pair enters/exits contact, or when the amount of teeth potentially in contact changes (e.g. due to a center distance variation). In this way, an optimally small database is retained and the inverse of the reduced stiffness matrix is updated only when necessary. The coupling terms between the teeth are also assembled at the same time, depending on how many coupling terms the user decided to select. Symmetry is also exploited when possible. The application of the abovementioned features results in a big benefit for the user in terms of computational effort.

3 VALIDATION

The Advanced - FE preprocessor methodology has been validated using multiple experimental results. The main goal of the validation process is to evaluate the accuracy and robustness of the modeling techniques (simulation data) using test measurements (experimental data). The validation process has been carried out thanks to the usage of an in-house precision gear test-rig [19] – see Figure 4 and Figure 5– jointly developed by Siemens PLM Software, KU Leuven and the University of Calabria.



Figure 4: Picture of the test rig (left), and the corresponding CAD design (right). Image credit: SISW.

The test rig has been designed and manufactured to assess typical gear-related physical quantities in static and dynamic conditions, under imposed conditions of misalignment and shaft compliances. The setup is composed of two main parts: (1) the test side and (2) the reaction side. These parts are dynamically separated from each other by using flexible couplings. Designed in a power-recirculating arrangement, it allows a small electric motor to spin the shafts. This simplifies the application of smooth speed and constant torque preloads and facilitates the measurement of rotational and lateral vibrations in the system. The raw data obtained from the experiment is then post-processed to obtain frequency and order domain results.

The gears used in the test campaign were manufactured with tight tolerances and measured tooth by tooth in the profile and lead directions. Tooth surfaces have been hardened and

precision ground to ISO quality 3 (equivalent to AGMA 15). The gears are designed in such a way that the signal to noise ratio of the TE is emphasized, due to higher tooth compliance. In this paper, two gear sets are tested for validation purposes: (A) a set of gears with no micro-modifications (3U-4U); and (B) a set of gears including micro-modifications (P1-P2). The geometrical specifications and micro-modifications of the gears are summarized in Table 1.



Figure 5: Cylindrical gears mounted on the test side of the setup (left) and test rig instrumentation (right).

Parameters	Units	Gear set 3U-4U	Gear set P1-P2
Driving gear tooth number	[-]	57	57
Driven gear tooth number	[-]	57	57
Helix angle	[deg]	0	0
Pressure angle	[deg]	20	20
Module	[mm]	2.6	2.6
Face width	[mm]	23	23
Center distance	[mm]	150	150
Micro-modifications	[µm]	-	P1: 5 (profile crowning)
		-	P2: 10 (profile crowning)

Table 1: Gear specifications.

Gear set A: 3U-4U: In this test configuration, the TE is measured while the gears are rotating at 10 rpm. The applied torque values are ranging from 50 to 350 Nm. Figure 6 presents the results obtained after the post-processing of raw TE data. The left plot shows the comparison of the computed and measured transmission error, corresponding to a selected set of torque values. In the right plot, the amplitudes of the 1st and 2nd meshing orders (orders 57 and 114) of the gears are presented for every value of applied torque. Given the low speed of rotation, a high Coulomb friction coefficient of 0.4 has been applied to the gear contact element during simulation. The matching between measurements and simulations is very good both in angle and order domain for all torque levels. This shows the accuracy of the proposed method to capture the stiffening behavior proper to gear contact and the stability of the method with respect to the discontinuous number of teeth in contact.

Gear set B: P1-P2: The gears in set B include tooth micro-modifications, see Table 1. The rotation speed is 10 rpm and different values of torque are applied, similar to the gear set A. The measured TE results are presented in Figure 7 (left). The computation of TE is performed by considering different values of friction coefficient ($\mu \in [0.30, 0.35, 0.40]$). The numerical results (TE) corresponding to each friction coefficient are presented together with the measured results. To better illustrate the effect of friction on the results, the amplitudes of the 1st and 2nd

meshing orders are presented in Figure 8. Similar to the previous case, the low speed of rotation imposes large values of Coulomb friction coefficients.



Figure 6: TE at different torque levels in angle (left), and order domain (right)



Figure 7: TE at different torque levels (left) and zoom on friction effect at higher torque levels: i.e. 350 Nm.



Figure 8: The amplitudes of simulated TE vary in function of the friction coefficient.

At first, it can be observed how the matching of the TE for low values of torques follows the typical parabolic shape dictated by the microgeometry. This shows the accuracy of the selected contact detection method. It is interesting to see how both measurements and simulation show the same discontinuous behavior in TE. At each angular pitch, the friction reversal effect happens due to the sign variation of the relative speed between the contact points (see Figure 7 – curve at 150 and 250 Nm). The solver can cope well with this highly discontinuous behavior as well. The effect of friction become prominent at higher loads (see Figure 7 – curve at 350

Nm and zoom at the right). Qualitatively, it is important to notice that the behavior shown in Figure 7 is particularly complex to capture. In particular, gears with profile crowing are designed in order to obtain a minimal transmission error at a nominal loading (torque). Due to the combined effect of microgeometry, friction and tooth flexibility, the TE shows instead a monotonically increasing trend. This effect cannot be captured by uncoupled analytical methods. Finally, Figure 8 shows the influence of the friction coefficient with respect to the first two orders of the TE, further highlighting the change in qualitative and quantitative behavior of this gear pair with respect to friction.

4 CONCLUSIONS

This paper proposes a novel method to solve gear contact problem for detailed NVH and system level multibody analysis of complex transmissions. The method is implemented in Siemens Virtual.Lab Motion and Siemens Simcenter 3D Motion multibody solvers. The *Advanced FE-preprocessor* method allows to retrieve quantitative results with an accuracy similar to non-linear FE methods with a much lower computational cost. It can be used for system level dynamics assessments, NVH analyses and design studies including microgeometry, dynamic center distance variations, and misalignment. Its hybrid FE formulation offers the possibility to analyze complex systems including ring gears and lightweight gears with a great level of accuracy. A first validation against experimental results shows a very good match of the results with respect to transmission error. The results show a great potential of the method for dynamic system level analysis as well. Future works will validate the method in more complex systems under dynamic conditions.

AKNOWLEDGMENTS

The research leading to these results has received funding from VLAIO for the project 150394 "**ECO-Powertrain**" and from the People Programme of the European Union's 7th Framework Programme FP7/2007-2013/ under REA grant agreement no. 324336 DEMETRA.

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