6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11 – 15 June 2018, Glasgow, UK

THE TEORETICAL AND EXPERIMENTAL EXPLORATION OF SUPERSONIC FLOW

A. NASTASE[1],

[1]RWTH, Aachen University
Templergraben 55, 52062 Aachen, GERMANY
nastase@lafaero.rwth-aachen.de

Key words: Supersonic Flow, Non-Classical Three-Dimensional Hyperbolic Potential Solutions, Enlarged Variational Problems with Free Boundaries, Surrogate Models, Hybrid Navier-Stokes Solutions.

Abstract. *The theoretical and experimental exploration of supersonic flow over flying configurations (FCs) was performed with the help of eight models of FCs. The lift, pitching moment and pressure coefficients on the surfaces of these models, were tested in the trisonic wind tunnel of the DLR Cologne in the frame of some research projects of the author, sponsored by the DFG. A very good agreements between the experimental- correlated lift and pitching moment coefficients and the theoretical predicted values of these coefficients by using the non-classical three-dimensional hyperbolic potential solutions of the author are obtained for all these models. These three-dimensional hyperbolic solutions are used:for the proposed hybrid numerical solutions of full Navier-Stokes PDEs and as start solutions for the determination of the inviscid global optimized (GO) of the shapes of three FC models namely, of the delta wing alone Adela and of the fully-integrated wing-fuselage models Fadet I and Fadet II. The deternination of the GO shape of a FC leads to an enlarged variational problem with free boundaries. The here proposed optimum-optimorum strategy can determine the GO shape of a FC among of a class of elitary FC defined by their chosen common properties. These GO models are of minimum inviscid drag at cruising Mach numbers 2, 2.2 and, respectively, 3 These inviscid GO shapes are used for the new proposed GO shape of aerospace models and as surrogate models, the first step of an more recent developed iterative optimum-optimorum theory*

1 INTRODUCTION

The theoretical and experimental exploration of supersonic flow over FCs was performed by using eight FC models, presented in the (Fig.1) and (Fig. 2) namely: five non-optimized models of the wedged and the double wedged wings, the wedged delta wing fitted with a conical fuselage, the wedged and the cambered rectangular wings and three models with inviscid GO shapes, it is, the delta wing alone Adela and the integrated wing-fuselage models Fadet I and Fadet II, which use the own three-dimensional hyperbolic analytical potential solutions (HASs), as start solutions for the design of their GO shapes. The capabilities of these rapid, non-classical HASs, written in integrated forms , as in [1]- [4], to predict the real world, were cheked in the trisonic wind tunnel of the DLR Cologne, in the frame of some research contracts of the author, sponsored by the DFG. A very good agreement between the theoretical predicted and the experimental measured lift and pitching moment coefficients for the range of angles of attack $\alpha = -20^{\circ},\ 20^{\circ}$, which is greater as in subsonic flow, was found. A good agreement was also obtained for the pressure coefficients for the range of angles of attack $\alpha = -10^{\circ},\ 10^{\circ}$. For flattened FC at moderate angle of attack, very important conclusions for this exploration are found, namely, the more economic flight with characteritic surface (instead of shock surface) is possible, the flow is laminar, as supposed here, the influence of the friction over the lift, pitching moment and pressure coefficients is neglectable, the developed software for the computation of these coefficients are confirmed and the solutions for the Navier-Stokes layer (NSL) must have the HASs, as asymptotes for small perturbation. The high performant surrogate models have inviscid GO shapes and are used in the first step of her iterative optimum-optimorum (OO) theory .

Figures. 1 The global optimized delta wing model Adela and others five non optimized FC models

Figures 2a,b: The views of GO shapes of fully- integrated wing-fuselage modelsls Fadet I and Fadet II

A refinement of the start solutions was further made. These three-dimensional HASs were replaced with hybrid numerical solutions for the NSL PDEs. They use the HASs twice: as outer flow at the NSL's edge and to reinforce the NSL's solutions, which are expressed as products of HAS with polynomes with free coefficients, which are determined by fulfilling the NSL's PDEs in a number of points, as in [1]-[4]. These NSLs solutions have also analytical properties due to HAS as: the correct last behaviors, have correct jumps over the singular lines, fulfill all the boundary conditions, the derivatives are exact computed, are split and speed up the computation time and are used up the second step of her iterative OO theory .

2 NON- CLASSICAL, THREE-DIMENSIONAL HYPERBOLICAL POTENERAL SOLUTIONS

Here is supposed that the downwashes on the surface of FCs are written or can be picewise approximated in form of superpositions of homogeneous polynomes in two variables. Dimensionless variables are here introduced as follows:

$$\tilde{x}_1 = \frac{x_1}{h_1} \quad , \qquad \tilde{x}_2 = \frac{x_2}{\ell_1} \quad , \qquad \tilde{x}_3 = \frac{x_3}{h_1} \quad , \tag{1}$$

$$\left(\ \tilde{y} = \frac{y}{\ell} \quad , \quad \ell = \frac{\ell_1}{h_1} \quad , \ c = \frac{c_l}{h_l} \quad , \ \nu = B\ell \ \ , \ \ \bar{\nu} = Bc \quad , \quad B = \sqrt{M_\infty^2 - 1} \ \right) \quad ,$$

$$w = \tilde{w} \quad , \qquad w^* = \tilde{w}^* \quad , \quad w'^* = \overline{w}^* \quad , \qquad u = \ell \ \tilde{u} \quad , \qquad u^* = \ell \ \tilde{u}^* \quad , \tag{2a-e}$$

$$C_\ell = \ell \ \tilde{C}_\ell \quad , \qquad C_m = \ell \ \tilde{C}_m \quad , \qquad C_d = \ell \ \tilde{C}_d \quad , \qquad C_d^* = \ell \ \tilde{C}_d^* \quad , \tag{.3a-d}$$

$$C_d^{(i)} \equiv \ell \ \tilde{C}_d^{(i)} = C_d + C_d^* = \ell \ \left(\ \tilde{C}_d + \tilde{C}_d^* \right) \quad , \tag{4}$$

Hereby u and $u^\bullet$ and w and $w^\bullet$ are the axial disturbance velocities and the downwashes on the thin and thick-symmetrical wing-fuselage components ℓ_1, h_1 and ℓ are the half-span, the maximal depth and the dimensionless span on the entire FC, ν and $\bar{\nu}$ are the similarity parameters of the planforms of the entire FC and of its fuselage $\tilde{u}$ and $\tilde{u}^\bullet$ and $\tilde{w}$ and $\tilde{w}^\bullet$ are the axial disturbance velocities and the downwashes on the thin and thick-symmetrical FC components of the transformed FC, C_ℓ , C_m and $C_d^{(i)}$ are the lift, the pitching moment and the inviscid drag coefficients of the FC. The downwashes on the thin and thick-symmetrical components of an integrated thick, lifting delta wing fitted with a central fuselage are : for the thin FC component, supposed continous

$$w \equiv \tilde{w} = \sum_{m=1}^{N} \ \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \ \tilde{w}_{m-k-1,k} \ \left| \tilde{y} \right|^k \tag{5}$$

and for the thick-symmetrical FC, on the wing and on the fuselage

$$w^{*} \equiv \widetilde{w}^{\bullet} = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \widetilde{w}^{\bullet}_{m-k-1,k} \left|\widetilde{y}\right|^{k} \quad , \qquad w'^{*} \equiv \overline{w}^{\bullet} = \sum_{m=1}^{N} \widetilde{x}_1^{m-1} \sum_{k=0}^{m-1} \overline{w}^{\bullet}_{m-k-1,k} \left|\widetilde{y}\right|^{k} \qquad \text{(6a,b)}$$

The non-classical, three-dimensional hyperbolical potential solutions for the computation of the axial disturbance velocities over the thin and thick-symmetrical components of an integrated flattened FCs are obtained by the author in integrated form, by using the compatibility conditions of P. Germain, the prtnciple of minimal singularities, (which fulfill the jumps) of M. van Dyke and the hydrodinamic analogy of E. Carafoli, as in [1]-[4]

$$u \equiv \ell\ \widetilde{u} = \ell \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left\{ \sum_{q=0}^{E\left(\frac{n}{2}\right)} \frac{\widetilde{A}_{n,2q}\widetilde{y}^{2q}}{\sqrt{1-\widetilde{y}^2}} + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{C}_{n,2q}\widetilde{y}^{2q} \cosh^{-1}\sqrt{\frac{1}{\widetilde{y}^2}} \right\} \qquad (7)$$

and

$$u^{*} \equiv \ell\ \widetilde{u}^{*} = \ell \sum_{n=1}^{N} \widetilde{x}_1^{n-1} \left\{ \sum_{q=0}^{n-1} \widetilde{H}^{*}_{nq}\widetilde{y}^{q} \left(\cosh^{-1} M_1 + (-1)^q \cosh^{-1} M_2 \right) \right.$$

$$+ \sum_{q=0}^{n-1} \widetilde{G}^{*}_{nq}\ \widetilde{y}^{q} \left(\cosh^{-1} S_1 + (-1)^q \cosh^{-1} S_2 \right)$$

$$\left. + \sum_{q=1}^{E\left(\frac{n-1}{2}\right)} \widetilde{C}^{*}_{n,2q}\ \widetilde{y}^{2q} \cosh^{-1}\sqrt{\frac{1}{\nu^2\widetilde{y}^2}} + \sum_{q=0}^{E\left(\frac{n-2}{2}\right)} \widetilde{D}^{*}_{n,2q}\ \widetilde{y}^{2q}\sqrt{1-\nu^2\widetilde{y}^2} \right\} . \qquad (8)$$

$$\left(M_{1,2} = \sqrt{\frac{(1+\nu)(1\mp\nu\ \widetilde{y})}{2\nu(1\mp\widetilde{y})}} \quad , \quad S_{1,2} = \sqrt{\frac{\left(1+\overline{\nu}\right)\left(1\mp\nu\ \widetilde{y}\right)}{2\left(\overline{\nu}\mp\nu\ \widetilde{y}\right)}} \right)$$

The coefficients of these axial disturbance velocities, namely, for the thin and thick-symmetrical components of the integrated wing-fuselage FC

$$\widetilde{A}_{n,2q} = \sum_{j=0}^{n-1} \widetilde{a}^{(n)}_{2q,j}\ \widetilde{w}_{n-j-1,j} \quad , \qquad \widetilde{C}_{n,2q} = \sum_{j=0}^{n-1} \widetilde{c}^{(n)}_{2q,j}\ \widetilde{w}_{n-j-1,j} \qquad \text{(9a,b)}$$

$$\widetilde{H}^{*}_{nq} = \sum_{j=0}^{n-1} \widetilde{h}^{*}_{n,q,j}\widetilde{w}^{*}_{n-j-1,j} \quad , \quad \widetilde{D}^{*}_{n,2q} = \sum_{j=0}^{n-1} \left(\widetilde{d}^{*}_{n,2q,j}\widetilde{w}^{*}_{n-j-1,j} + \overline{d}^{*}_{n,2q,j}\overline{w}^{*}_{n-j-1,j} \right) ,$$

$$\widetilde{C}^{*}_{n,2q} = \sum_{j=0}^{n-1} \overline{c}^{*}_{n,2q,j}\overline{w}^{*}_{n-j-1,j} \quad , \quad \widetilde{G}^{*}_{nq} = \sum_{j=0}^{n-1} \left(\widetilde{g}^{*}_{n,q,j}\widetilde{w}^{*}_{n-j-1,j} + \overline{g}^{*}_{n,q,j}\overline{w}^{*}_{n-j-1,j} \right) , \qquad \text{(10a-d)}$$

are linear and homogeneous functions of the coefficients of downwashes,which depand on the shape of the surface of FC and are multiplied with coefficients, which depand only on the similarity parameters of the planform of FC. The axial disturbance velocities on delta wing and on the delta wing-fuselage FC models are obtained as particular cases of the formulas (7) and (8), as given in [1] . The axial disturrbance velocities on rectangular wing models are deduced also in [1]-

3. THE OPTIMUM- OPTIMORUM THEORY

The determination of a global optimized (GO) shape of a flying configuration (FC) (namely the simultaneous optimization of its distributions of camber, twist and thickness and also of the similarity parameters of its planform) leads to an extended variational problem with free boundaries The discontinous surface of an elitary FC is picewise approximed in form of two different superpositions of homogeneous polynoms in two variables, one on the wing and the other on the fuselage. The coefficients of these polynoms, together with the similarity parameters of their planforms, are the free parameters of optimization. The classical optimization strategy allows the determination of the shape of the surface of a FC with fixed shape of its planform and is here called ellitary FC. The optimum-optimorum (OO) theory is an optimization strategy, which allows the determination of the GO shape of the surface of a FC and also of the similarity parameters of its planform , inside a class of elitary FCs defined by their common properties, as in [1]-[4]. A lower-limit hypersurface of the inviscid drag functional, as function of the similarity parameters of the planforms of elitary FCs of the class is defined and the elitary FC, which corresponds to the minimum of this hypersurface is, in the same time, the GO shape of the FC of the class. The free parameters of the optimization are the coefficients of the downwashes w , w^* and w'^* and also the similarity parameters of the planforms of the wing and of the fuselage. Further, the quotient of these similarity parameters, which is determined for the purpose of the FC, is considered constant.

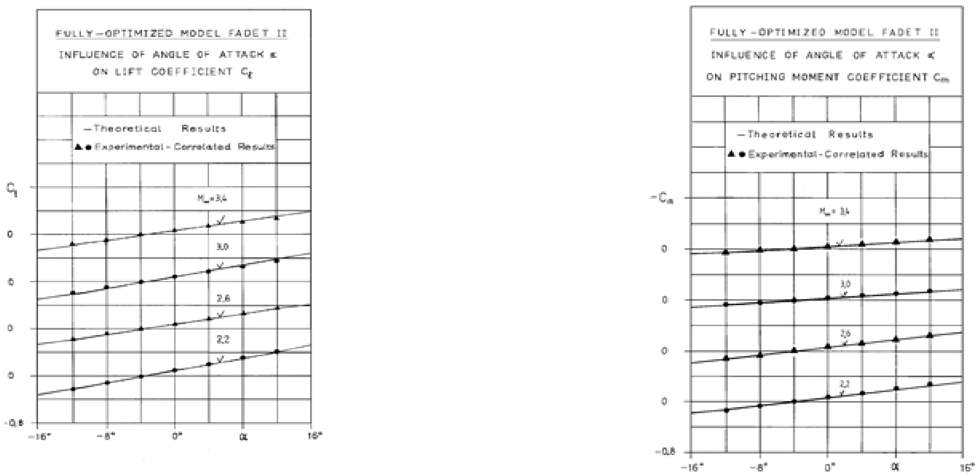
This OO strategy was used for the determination of the surfaces of the inviscid GO shapes of the delta wing alone Adela, presented in the middle of the (Fig. 1) and of the fully-integrated wing-fuselage models Fadet I and Fadet II, presented in the (Fig. 2a,b). These models are of minimum inviscid drag at cruising Mach numbers 2, 2.2 and, respectively, 3.

4 THE COMPARISON BETWEEN THE THEORETICAL AND THE EXPERIMENTAL RESULTS

The theoretical predicted pressure distribution and the aerodynamic characteristics of these models were checked in the trisonic wind tunnel of DLR Collogne in the frame of research projects of the author, sponsored by the DFG. The results of this checking are here given for the fully-integrated wing-fuselage model Fadet II, GO with respect of minimum inviscid drag at cruising Mach number $M_\infty = 3$ presented in the (Fig. 2b) and (Fig. 3), is choosen here for exemplification. A very good agreement between the theoretical predicted and the experimental- correlated lift and pitching moment coefficients are presented in the (Fig.4a,b).

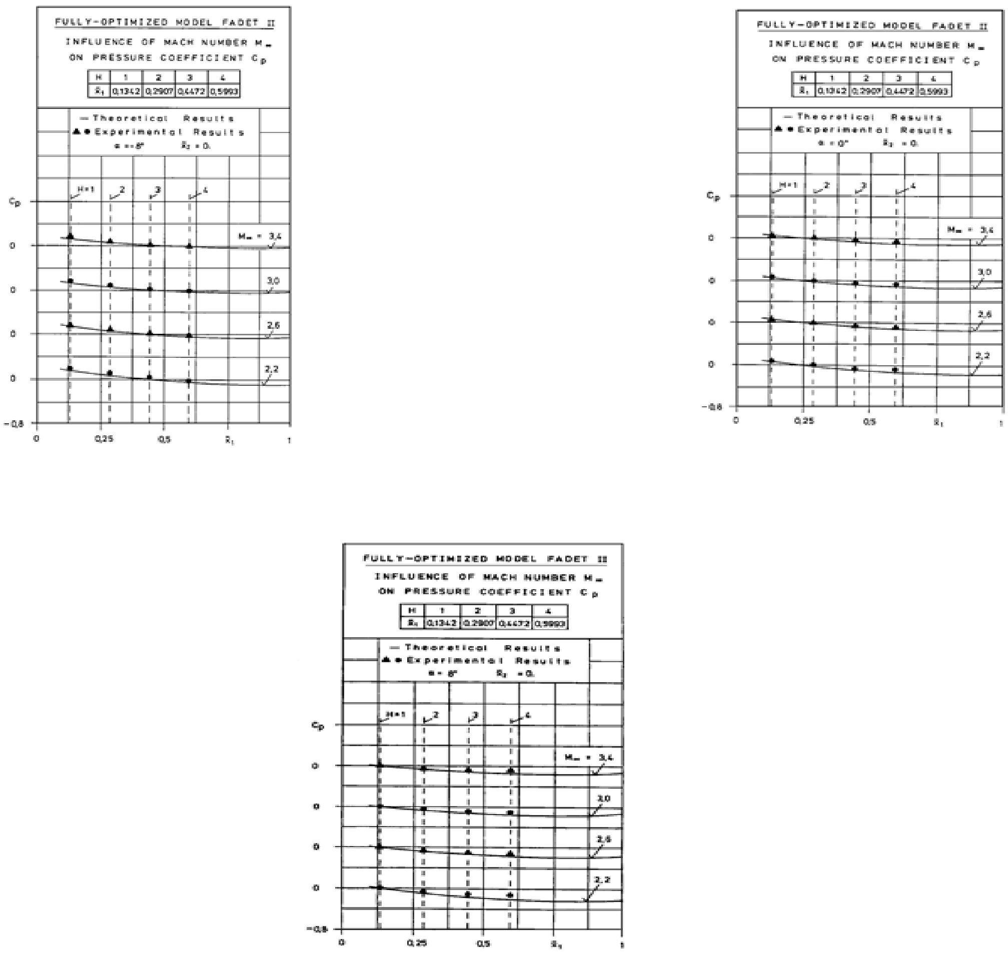
A good agreement between the theoretical predicted and the local interpolated values of the measured pressure coefficients on the upper side of the model in its central longitudinal section are presented in the (Fig. 5a-c) .

Fig. 3 The view of the fully- integrated and GO model Fadet II



Figures 4a,b The agreement between the seoretrical and experimental-correlated lift and pitching moment coefficients of the fully- integrated and GO model Fadet II

A very good agreement between the theoretical and the experimental-correlated lift and pitching moment coefficients are obtained also for the others seven models.



Figures. 5a-c The agreements between the theretical and the experimental determinated pressure coefficients along the longitudinal central section of the fully-integrated GO model Fadet II, at the angles of attack

$$\alpha = -8^\circ,\ 0^\circ,\ 8^\circ .$$

4 NEW GO SHAPES FOR AEROSPACE MODELS

The high performant inviscid GO shapes of models Fadet I and Fadet II are here used as source of inspiration of new GO shapes of aerospace catamaran models of GEO and LEO with twin fuselages, which are only almost integrated in the wing thickness in order to have windows on their both sides. Two different locations of twin fuselages are here proposed. The first variant for the GO shape of LEO with more stiffness has two parallel twin fuselages located in the central zone of the wing and almost embedded in its thickness is presented in the (Fig. 6) and a new variant with twin fuselages located in Λ . form, as in (Fig. 7), which has windows on both sides, for a better view. The both variants have rescue bars, located closed to the cabine of pilots. These bars are the meeting point of twin fuselages and have two doors for entrance and exit from the space vehicle

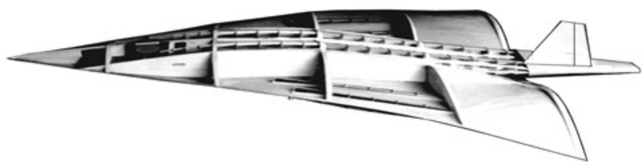


Figure 6: The catamaran space vehicle model with twin central fuselages

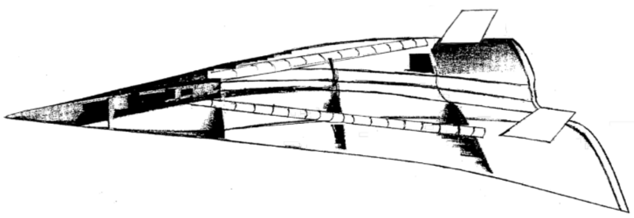


Figure 7 The catamaran space vehicle model with twin fuselages in Λ . form .

5 THE PROPOSED HYBRID SOLUTIONS FOR THE NAVIER-STOKES PDS

Let us firstly introduce the coordinate η inside the NSL, it is

$$\eta = \frac{x_3 - Z(x_1,\ x_2)}{\delta(x_1,\ x_2)} \qquad (0 \le \eta \le 1) \tag{11}$$

The hyperbolic potential solutions are here used twice, it is: as outer flow at the NSL's edge and to reinforce the numerical NSL's solutions of the full PDEs of the NSL, as in [1] - [4}. The proposed hybrid solutions of the velocity components, the here introduced density function $R = \ln \rho$ and the absolute temperature T are the following:

$$u_\delta = u_e \sum_{i=1}^{N} u_i\,\eta^i \quad , \qquad v_\delta = v_e \sum_{i=1}^{N} v_i\,\eta^i \quad , \qquad w_\delta = w_e \sum_{i=1}^{N} w_i\,\eta^i \qquad \text{(12a-c)}$$

$$R = R_w + (R_e - R_w) \sum_{i=1}^{N} r_i\,\eta^i \qquad\qquad T = T_w + (T_e - T_w) \sum_{i=1}^{N} t_i\,\eta^i \qquad \text{(13a,b)}$$

Hereby are: δ the thickness of the NSL, u_e , v_e , w_e the velocity components of the hyperbolic potential flow at the NSL's edge, R_e and T_e the density function and the absolute temperature of the outer flow at the NSL's edge and R_w and T_w their values at the wall of the FC. The free coefficients r_i , t_i , u_i , v_i , w_i are used to satisfy the NSL's PDEs and the boundary conditions at the NSL's edge in some points. The viscosity μ fu lfills an exponential low and the pressure p is obtained from the physical equation of the ideal gas, it is

$$\mu = \mu_\infty \left(\frac{T}{T_\infty} \right)^{n_1} \qquad , \qquad p = R_g\;\rho\;T = R_g\;e^R\;T \qquad \text{(14a,b)}$$

All the physical entitis are expressed as functions of the velocity's coefficients by using the continuity and the temperature PDEs. The velocity's coefficients are determined by iterative solving of a quadratic algebraic system. More details are given in [1]- [4] .
The friction , the friction drag and the total drag coefficients of an integrated wing-fuselage FC are the following:

$$\tau_{x_1}^{(w)} \equiv \tau_{x_1} \Big|_{\eta=0} = \mu_f \frac{\partial\, u_\delta}{\partial\, \eta} \Big|_{\eta=0} = \mu_f u_1 u_e \quad , \qquad \text{(15)}$$

$$C_d^{(f)} = 8 \int_{\tilde{O}\tilde{A}_1\tilde{C}} v_f\; u_1\; u_e\; \tilde{x}_1\, d\tilde{x}_1\, d\tilde{y} \quad , \qquad C_d^{(t)} = C_d^{(f)} + C_d^{(i)} \qquad \text{(16a,b)}$$

6 THE ITERATIVE OPTIMUM-OPTIMORUM THEORY

The viscous iterative OO theory uses the inviscid hyperbolic potential solutions as start solutions and the inviscid GO shape of these FCs as surrogate models, **only** in its first step of iteration. An intermediate computational checking of this inviscid GO shape of FC is made with own hybrid solvers, for the three-dimensional compressible NSL. The friction drag coefficient $C_d^{(f)}$ of the FC is computed and the inviscid GO shape is checked also for the structure point of view. A weak interaction aerodynamics-structure is proposed. Additional or modified constraints, introduced in order to control the camber, twist and thickness distributions of the GO shape, for structure reasons, are here proposed. In the second step of optimization, the predicted inviscid GO shape of the FC is corrected by including these additional constraints in the variational problem and of the friction drag coefficient in the drag functional.

7 CONCLUSIONS

The agreement between the experimental and the theoretical analytical hyperbolic solutions for the pressure, lift and pitching moment of flattened FCs flying at moderate angles of attack, leads to the following important conclusions:
- the flow is laminar, as supposed here, and it remains attached in supersonic flow, for larger range of angles of attack than by subsonic flow;
- the flight with characteristic surface, which is more economic, instead of the flight with shock wave surface is confirmed;
- the validity of the three-dimensional hyperbolic analytic potential solutions for the axial disturbance velocity, with the chosen balanced minimal singularities and the corresponding software for the computation of the above coefficients are confirmed;
- the influence of friction upon these coefficients is neglectable;
- these analytic solutions are very usefull for the computation of proposed hybrid solutions for the NSL's PDEs, which are abble to compute the total drag, including friction ;
- these analytic solutions were used by the author as start solutions for the inviscid global optimization of the shapes of the high performant models Adela , Fadet I and Fadet II ;
- these inviscid global optimized models can also be considered as surrogate models, which are used in the first step of a proposed iterative optimum-optimorum strategy of the determi--nation of the GO shapes of FCs, in supersonic flow, as in [1]-[4];
- these hybrid solutions do not need interface at the NSL's edge.

REFERENCES

[1] Nastase, A. C*omputation of supersonic flow over flying configurations*. Elsevier, Oxford, UK (2008).
[2] Nastase, A. Hybrid Navier-Stokes solutions for aerodynamical, global optimal shape's design, Proc. of International Conference EngOpt, Paper 750, Rio de Janeiro, Brazil, (2008).
[3] Nastase, A. Use of surrogate models for the global optimization of flying configurations, in supersonic flow, Extended Abs. of EUROGEN 2015, Univ.of Strathclyde, Glasgow, UK, p. 52-57, (2015).
[4] Nastase, A. Evolutionary, iterative optimum-optimorum theory, INCAS Bulletin, Bucharest, Romania, Vol. 2, Number 4, p. 153-161, (2010).
.