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ERROR INVESTIGATION FOR COUPLED SIMULATIONS USING DISCONTINUOUS GALERKIN METHOD FOR DISCRETISATION

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Abstract. The simulation of fluid-structure-acoustics interaction is still a challenge in the engineering field. Simulating the overall problem with a monolithic approach is not feasible, since different phenomenons appear on different scales, which have to be resolved up to the smallest scale. In order to be able to run such compute intensive simulations we consider partitioned coupling, where we split the overall problem into smaller domains, which than can be treated with the best-suited scheme order, spatial discretisation and equations. This allows the realisation of such complex simulations with affordable computational time. Due to the decomposition of the domain, the communication and the data-exchange has to be maintained, for that we compare two different coupling approaches, which connect the subdomains with each other at the boundary interfaces. The data exchange is done at coupling points for each domain. The first coupling approach is based on black-box, where just the coupling points at the interface are known, thus for the data-exchange from one domain to the other an interpolation method has to be used. The second approach is integrated into our framework APES, thus taking advantage of the direct evaluation of the underlying polynomial, which results in very accurate simulation results, while the first approach is more generally applicable. For the coupled simulation we have to ensure, that the simulation error is as small as possible, when compared to the monolithic simulation result. Therefore we investigate the accuracy of the coupled simulation, considering the two approaches and the different methods they use for the data-exchange. Thus we study different scenarios for our simulations, where we compare the L2error for the methods.

1 INTRODUCTION

In the recent years the simulation of fluid-structure-acoustics interactions became more and more important in the engineering field. Due to increasing computational power, these kind of complex simulations can be recently realised. The opportunity to simulate more complex scenarios is coupled with higher computational cost. Considering monolithic approaches for the numerical simulation results in not feasible computation. Since we still want to realise simulations with such interactions, in order to allow improvements in the early stage of the design process, we make use of partitioned coupling. This leads to the subdivision of the whole problem into smaller parts, hence solving each subdomain according to the requested physics. That enables the application of different numerical schemes, spatial discretisation and equations for each subdomain. The decomposition of the domain and the different treatment of each subdomain results in the reduction of computational time, thus a decrease in computational cost. For the simulations we consider for the discretisation the Discontinuous Galerkin method (DG), using high order scheme, that enables low dispersion and dissipation considering e.g. acoustic wave propagation.

Through the decomposition of the overall problem size the need of a tool, which connects the subdomains with each other and allows the communication between them is necessary. Therefore we consider an external coupling approach called preCICE [4] and an in-house approach named APESmate [9]. preCICE is a black-box approach, which allows the surface coupling of different solvers. For the data-exchange between the domains it requires point values from each domain, at which the communication occurs. In order to exchange the values from one domain to the other, this tool makes use of different interpolation methods. Three well known methods are the Nearest-Neighbour, Nearest-Projection and the Radial-Basis-Function interpolation. On the other hand our in-house approach is integrated in our framework, which allows the direct evaluation of the underlying polynomials, thus resulting in very high precision simulation results with small error.

In this paper we investigate the accuracy of the coupled simulations, when compared to the monolithic simulation, where we solve the whole domain with the same equations. We start our investigation with a convergence study, considering different scheme orders and element size to identify how the L2error converges. After determining an error limit, we decompose our domain and couple two different domains, using different scheme orders and element size in respect to the previous findings regarding the error convergences. For the coupling we consider the named interpolation methods provided by preCICE and the evaluation method used by APESmate. Considering the error, we also have a look at the computational cost for each method respectively.

2 Interpolation/ Evaluation Method

This section is devoted to the different interpolation methods and the evaluation of the state variables for the data-exchange at the coupling surface. First a short description about the interpolation methods in preCICE is presented, afterwards the evaluation

method used in our integrated approach APESmate is shown.

2.1 Mapping by interpolation in preCICE

The multi-solver preCICE allows the coupling of different solvers considering them as a black-box. Hence it has no information about the discretisation method in each coupling domain. The data-exchange at the coupling surfaces happens using coupling points. In order to exchange requested point values from one domain to the other, preCICE provides different interpolation methods, while three of them are mostly used, the Nearest-Neighbour, Nearest Projection and the Radial-Basis-Function interpolation. Since the involved coupling domains request point values located at arbitrary locations on the coupling surface, the information has to be provided at those points from one domain to the other. This makes interpolation methods necessary to compute values at those requested points. More information regarding preCICE can be found in [12] [5], [3] and [4].

In [6] we already gave an overview over the different interpolation methods as well as some findings when using these methods. This paper will extend those findings further. The interpolation method Nearest-Neighbour (NN (see Fig. 1a) [4]) is the easiest applicable method provided by preCICE. It is a first order accurate method, since point values are copied from one domain to the other. Hence this method is just reasonable when having a matching coupling interface, i.e. same number and distribution of the coupling points. Since NN is not qualified for non-matching coupling surfaces and we already showed in [6], that the error increases considerably with increasing non-matching coupling surfaces, we will not consider this method further for our investigation.

The second method which we want to investigate further, is the Nearest-Projection method (see Fig. 1b), which leads to second order accuracy. For this method beside the coupling points, further information regarding the neighbourhood is needed [4].

The third method considered for our study is the second order accurate Radial-Basis-Function, where no neighbourhood information is needed for the coupling. This method provides different basis functions, which can be used according to the users requirements. For our studies we apply the Gaussian function as basis function, where we have to provide a shape-parameter to preCICE, which defines the width of the Gaussian function. It is important to mention here, that the distribution of the coupling points plays a major role on the convergence of the linear equation system **??**, which has to be solved. Different scenarios can be thought of, where the coupling points on the domain interface are non-equidistant. One is the use of high order Discontinuous Galerkin (DG) [7] schemes, where the coupling points are the Gaussian integration points, which are not equally distributed. At the corners of the elements the points are tightly distributed, while in the middle area of the element the distance between the points is much larger. For non-equidistant point distribution the shape-parameter has to be calculated considering the smallest gap between the points, in order to cover all points accordingly (see [6]).

(a) (b)

Figure 1: a) Nearest-Neighbor, b) Nearest-Projection Interpolation [4]

2.2 Mapping by evaluation using APESmate

The integrated coupling approach APESmate [9] is implemented in our APES framework [8],[11] and [13], thus it has access to solver specific data. Therefore exchanging data at arbitrary exchange points with our DG solver Ateles at the coupling interface can be realised by the direct evaluation of the polynomial representations at the requested points. Hence no additional interpolation is necessary. One of the main beneficials of our coupling approach is, that with a higher order in the scheme a higher accuracy in the context of the interpolation can be obtained [9].

3 Results

In this section we compare the simulation results of the previous mentioned coupling approaches, when using the interpolation/evaluation method for the data-mapping. Our study contains a test case, where a Gaussian pressure pulse spreads over the domain during the simulation time. We start our investigation with a convergence study, which allows us to find out, how the domain has to be spatially discretised in order to get the same L2error for different scheme orders (O). This allows us to decompose the overall problem size (inner and outer domain) and use the coupling approaches to couple them with each other, while keeping the error for both domains the same, by using the knowledge from the previous convergence study. As a result, we are able to make a clear statement about the accuracy of each method. The study involves four test cases, where we increase the non-matching behaviour of the coupling domains, while keeping the inner domain the same (scheme order of three) and changing the outer domain by using different scheme orders (up to a scheme order of 14) and spatial discretisation for each of them. The L2error will be kept the same according to the convergence study.

3.1 Error convergence for higher scheme order

Before starting the investigation of the coupled simulation, we first consider a convergence study. Our test case is a 5x5x5 cube, where a Gaussian pressure pulse is located at the exact center (see Fig. 2). The pulse spreads over the domain in all spatial directions. The amplitude is set to 1.0 and the halfwidth of the pulse to 0.25. The ambient pres-

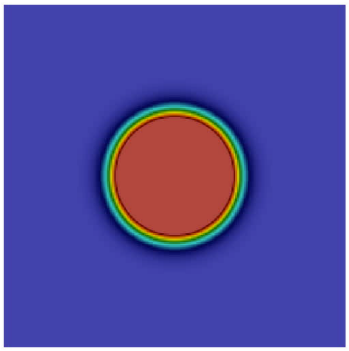


Figure 2: Gaussian pressure pulse centered inside the domain

sure is predefined by 100,000, while the density is set to 1.0. The velocity is defined as $\vec{v} = [0.0, 0.0, 0.0]$. The domain is solved using the Euler equations. This test case is used to study how the L2error converges to a small error, when using different scheme orders and spatial discretisations. Our study concerns different spatial discretisation for different scheme orders to obtain the same error (see red line in Fig. 3) for the simulation. The figure clearly illustrates, that the error drops much faster, when we increase the scheme order, while decreasing the spatial resolution (increasing element size). For the lower

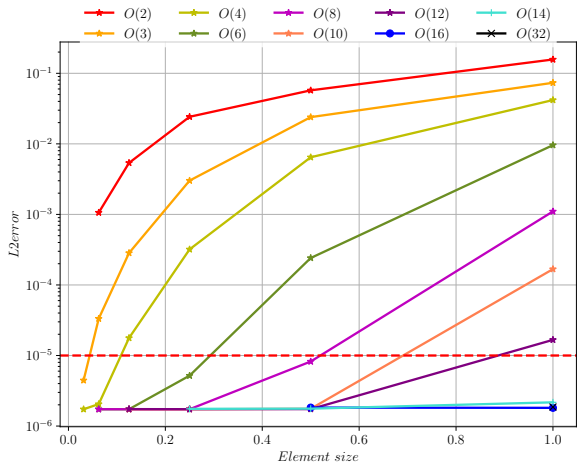


Figure 3: Convergence of the L2error with increasing spatial discretisation for different scheme orders using DG

scheme orders e.g. 2^{nd} or 4^{th} order we have to increase the spatial resolution (decreasing the element size) to obtain the same error as for the higher orders for e.g. the 14^{th} or 16^{th}order, where we can use a coarse mesh, thus increasing the element size to reach the aimed error of 10^{-5}. Hence for using higher scheme orders allows us to maintain a very small L2error, while using a very coarse mesh with few elements.

3.2 Coupled simulation - Test case setup

For the coupled simulation we use our findings from the convergence study to decompose our test case into inner and outer domain, while coupling them via the coupling approaches. Both domains have the scheme order and spatial discretisation as obtained from the convergence study respectively. The inner domain covers a 1x1x1 area, while the outer domain is 4x4x4 large (see Fig. 5). The Gaussian pressure pulse is located in the center of the inner domain, which than spreads over to the outer domain during the simulation. Our study involves four test cases, where we change the outer domain us-

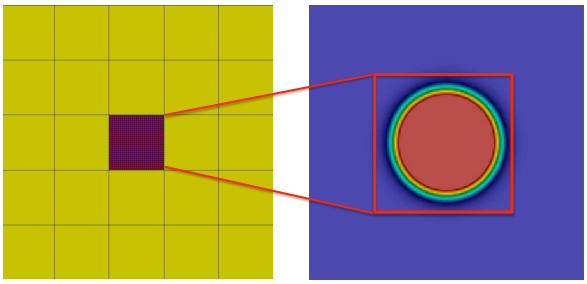


Figure 4: Decomposition of the domain into inner and outer domain as an example for test case d)

ing different scheme orders and spatial discretisation according to the convergence study, whereas the inner domain is kept always the same, using the same scheme order and spatial discretisation. With the change of the outer domain, also the number of coupling points changes. In Tab. 1 all test cases including the used scheme orders, total number of elements in the each domain and the coupling points are listed.

Table 1: Investigation of the interpolation/ evaluation method using four test cases

	Test case a		Test case b		Test case c		Test case d	
	inner	outer	inner	outer	inner	outer	inner	outer
nElements	32768	124000	32768	7936	32768	992	32768	124
nCoupling points	55296	9600	55296	3456	55296	1536	55296	1176
nScheme order	3	4	3	6	3	8	3	14

3.3 Error estimation for different data-mapping methods

In order to estimate the error for the different coupling methods we ensure, that the pressure pulse reaches the outer domain, but is still away from the boundaries, which could influence our error estimation. For the error computation the pulse reaches the same position as for the convergence study. In Fig. 5 first results for the coupled simulation are shown. As can be recognised the integrated approach shows as expected overall the same constant error, which is also the lowest. This is due to the direct evaluation of the

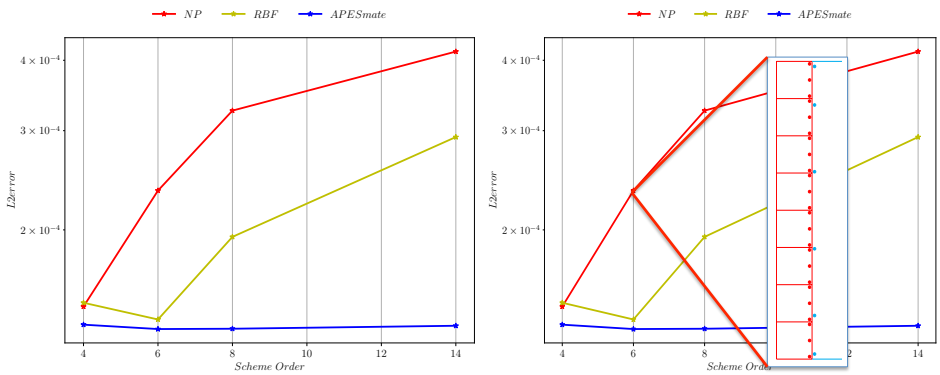


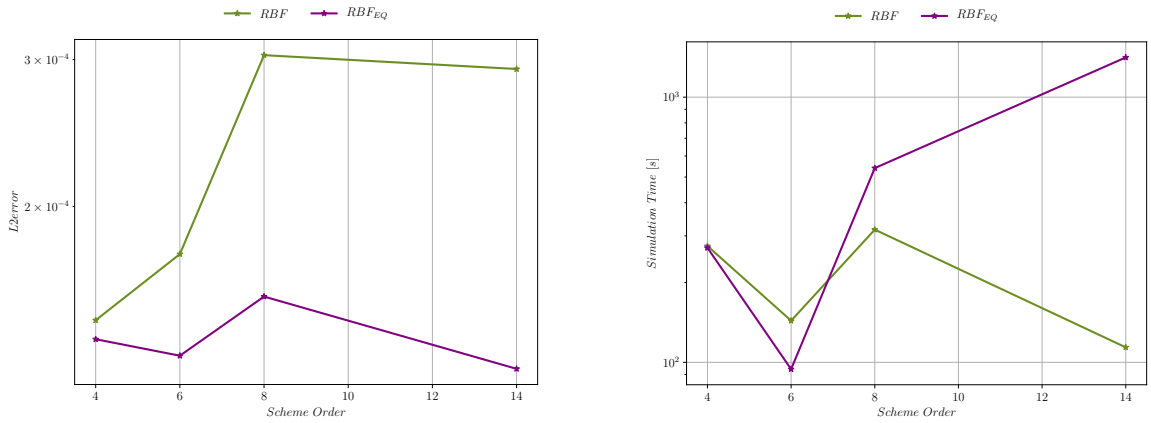
Figure 5: Comparison of the L2error for different test cases and exemplary illustration of the coupling point distribution when using DG for test case b)

polynomials, which allows a very high accuracy for the coupled simulation. Considering the NP and RBF method provided by preCICE, the error increases with increasing non-matching coupling interfaces. As shown in Fig. 5 (right image), the Gaussian integration points are non-equidistantly distributed per element. Both NP and RBF reconstruct a polynomial out of these points and evaluates the polynomial at the requested points for the other domain. As the error of the reconstruction depends on the distance of the points, the error increases with the maximum distance. As both schemes, NP and RBF, are 2^{nd} order, the error decreases only slowly with the number of total points in the middle of the elements. In addition, the stability of the reconstruction becomes sensitive to the non-uniformness of the point distribution. To counter this, we can instead sample our solution at equidistant points. While we still use the non-equidistant points for the Gaussian integration, the intermediate data on equidistant points provides a regularisation for the low order interpolation with a more uniform error distribution, and thus decreases the maximal error. We implemented equidistant points (see Fig. 6) in our solver, which we provide to preCICE to use for the interpolation, but still request non-equidistant point values from the approach. Then we run our simulations again, using the RBF method with the new implementation.

Figure 6: Providing equidistant coupling points (blue) for the interpolation but requesting point values for non-equidistant points (red) for preCICE

As can be seen in Fig. 7a the error drops considerably, when using equidistant points for the interpolation instead of the previous non-equidistant points. But having a look at the computational time it becomes clear, that the change does not only decrease the error, but also increased the computational time, since the coupling points have to computed twice, once as equidistant and once as non-equidistant e.g. test case d). Hence it can be concluded, that the RBFs show a good quality, but are overly expensive. Thus the ques-

tion is, whether we could improve the quality of the NP method with less computational effort. After investigating the RBF method in the previous paragraph, we now look at



(a) (b)

Figure 7: L2error behaviour (a) and computational time (b) for the RBF interpolation, when using equidistant ($\mathrm{RBF_{EQ}}$) and non-equidistant (RBF) point distribution for the interpolation

the NP method. We also compare non-equidistant and equidistant point delivery. But as NP is known to be less accurate, when using the same number of projection points as requested points, we also consider the use of an increasing number of sampling points (called oversampling, see Fig. 8). This allows us, to increase the accuracy of the simulation result clearly, when compared to the solution with non-equidistant points. Fig. 9 illustrates, how the L2error is higher when using equidistant points in the first step. The error increases here, due to the non-equidistant point distribution requested by the solver. Using as many points as the scheme order results in bad interpolation of the coupling points, which are located in the element corners, since there the points are more tight together. But increasing the number of equidistant coupling points also increases the accuracy, since even points located in the corners can better interpolated (see Fig. 9).

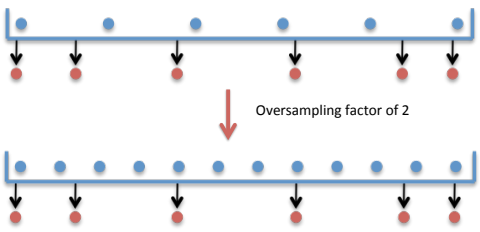


Figure 8: Providing equidistant and more (oversampling) coupling points for the NP interpolation method to preCICE exemplary shown for test case b)

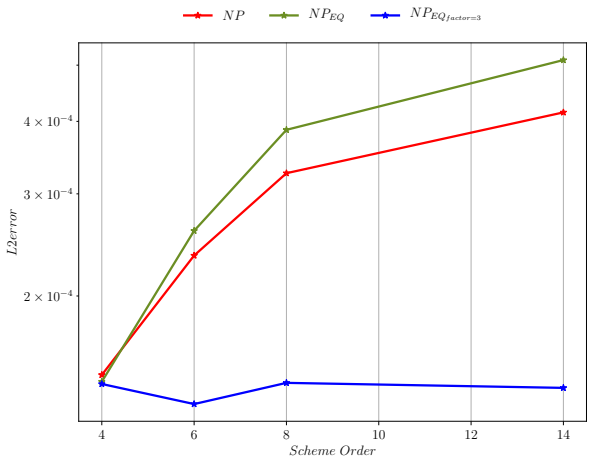
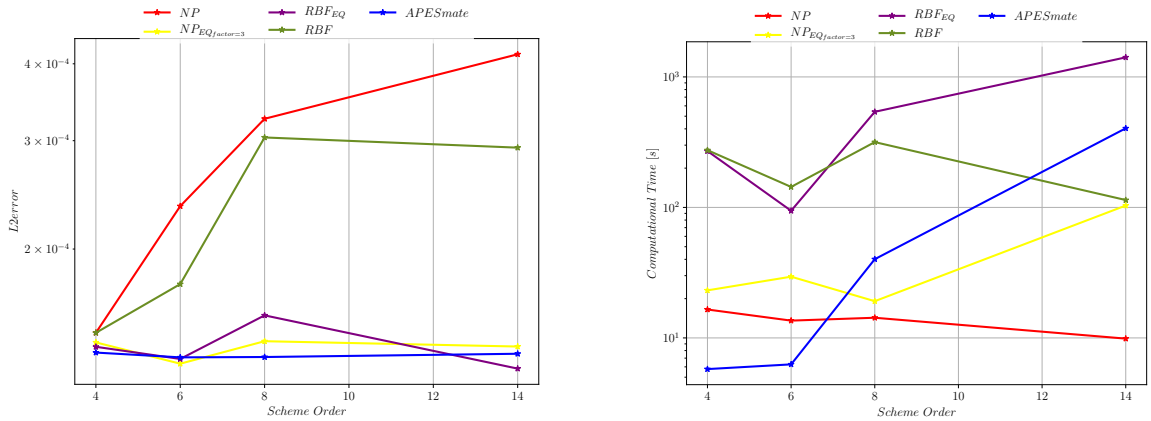


Figure 9: L2error over the scheme order for NP interpolation method, when using non-equidistant points, equidistant points $\mathrm{NP_{EQ}}$ and equidstant points with an oversamppling factor of three $\mathrm{NP_{EQ_{factor=3}}}$

To summarise the findings on the different interpolation methods, Fig. 10 shows the error plots as well as computational time for all methods. We plot one line for each of the 3 methods - NP, RBF and APESmate - with the points as they are (i.e. non-uniformly distributed Gaussian points) plus the best of the improved version, i.e. equidistant for RBF, equidisant and oversampled for the NP. Obviously, only these improved versions of the 2$^{\mathrm{nd}}$ order methods NP and RBF give comparable error to the APESmate version. Since the decomposition of the domain should help to keep the computational cost as low



(a) (b)

Figure 10: L2error and computational time for all methods

as possible, but still provide small error for the simulation results. As in Fig. 10a depicted, using the coupling approach APESmate results in very accurate simulation results,

independent how hight we go with the scheme order. For preCICE we have to provide equidistant coupling points for the interpolation when using RBF, to reduce the error. For NP just providing equidistant points is not sufficient, hence we have to increase the number of coupling points, to decrease the error of the simulation results (see Fig.7a). Considering also the computational cost, we notice, that the RBF with equidstant points are too expensive for our simulations hence not reasonable for us to use. The NP method provides very fast computation even when using higher number of points for the interpolation (see Fig. 10b). Our integrated approach APESmate shows increasing computational cost with increasing scheme order, this is due to the evaluation of the polynomials, which is costly for higher scheme order.

4 Conclusion

The numerical simulation of fluid-structure-acoustics interactions is still challenging when solving the problem with one solver (monolithic). Considering these kind of interactions monolithically is not feasible and expensive, since the occurring physics have to be resolved up to the smallest scales. In order to simulate such problems, we consider partitioned coupling, where we split the domain and solve each subdomain with the best-suited physics. In this paper we presented how the two coupling approaches use different methods to exchange point values at the coupling surface. All test cases were solved using our Discontinuous Galerkin (DG) solver, where the coupling points (Gaussian integration points) are non-equidistantly distributed on the element surface. For our investigation we considered the coupling approach preCICE, a black-box approach, which has no knowledge about solver specific information, hence considering just coupling points at the coupling surface. For the data-exchange between the coupling domains, this approach has to use an interpolation method. On the other hand we used our integrated approach APESmate. Hence this approach has knowledge about the underlying scheme and can take advantages of that, in order to evaluate the high order polynomial for the data-exchange, thus resulting in accurate simulation results.

We showed how to use preCICE when using non-equidistant points for the coupling and how to improve he accuracy and decrease the computational cost by providing equidistant points for the interpolation. For the interpolation we used the Nearest-Projection (NP) and the Radial-Basis-Function (RBF) method. Using non-equidistant points for the interpolation results in higher error, with increasing non-matching coupling surfaces. This is due to the Gaussian integration points, which are tightly located at the element corner, while in the middle of the element the gap between the points is much higher. The error is dominated by the huge gap in the middle area of the element. In order to overcome that, we provided equidistant points to preCICE for the interpolation. This increased the accuracy of the RBF method but also increased the computational cost for that. The basic problem with the RBF method is the fact, that the quality of the solution and the convergence of the system depends on how the coupling points are distributed. It is challenging to find a good agreement, when using the Gaussian function as basis function, how to chose a right shape-parameter for the simulations. In our previous study

[6] we were able to show, that depending on how the shape-parameter is chosen, the error of the solution was considerably low, when using equidistant points for the simulation. But this finding was just possible after trying certain shape-parameters for each setup. When using non-equidistant points the convergence rate of the linear system is even lower and finding a suitable shape-parameter even more challenging. Furthermore we showed, that providing equidistant points and an additional amount (oversampling) of coupling points increases the accuracy of the simulation and holds the computational cost in a reasonable range when using NP. This method requires additional neighbourhood information from the solver, it is able to provide accurate results, when using equidistant and a higher amount of equidistant points. Our in-house approach APESmate showed overall the lowest error, since it allows the direct evaluation of the polynomials for the requested point values. Even when using the most non-matching test case, our approach was able to keep an outstanding behaviour, by having an almost constant L2error for all simulations. Considering the computational time for the used methods, we can conclude that NP, when using equidistant points and an oversampling factor of three, shows not only a very low error but also a fast computation for the simulation, when compared to the RBF. Since APESmate evaluates the high order polynomials, thus keeping the coupling error as low as possible, the computational time increases with increasing scheme order.

Our future work is devoted to decrease the computational time for our coupling approach APESmate in the high order area, to reduce the computational cost for the coupled simulation further.

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