

NUMERICAL SIMULATIONS OF LAMB WAVES FOR OPTIMIZATION OF SENSOR PLACEMENT IN SHM SYSTEM

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Key words: Lamb Waves, Time-Domain Spectral Element Method, Structural Health Monitoring, Sensor Placement Optimization.

Abstract. Lamb waves are a type of waves propagating in thin-walled structures which have found many applications in aerospace and automotive industry. Due to the distribution of displacements in the whole cross-section and high sensitivity to material discontinuity, Lamb waves are often utilized for Structural Health Monitoring (SHM). In such systems, Lamb waves are usually excited and registered by a network of piezoelectric transducers. The algorithms for damage detection and localization are based on Lamb wave signals corresponding to all combinations of actuator-sensor pairs. It is very important to place piezoelectric transducer so that it is possible to inspect largest possible area without blind spots. This is an enormous optimization problem which requires a large database of Lamb wave signals. Hence, in the proposed method analytical solutions were utilized along with genetic algorithms for finding optimal sensor configuration. However, the analytical solution has many limitations as it does not take into account mass of piezoelectric transducers as well as the bonding layer. This research work aims to provide a database of full wavefield data of propagating Lamb waves for the final configuration of piezoelectric transducers in order to verify the methodology based on the analytic solution. The simulations consider realistic 3D plate model with or without physically bonded piezoelectric transducers. The in-house code of time domain spectral element method is used for the simulations. In order to speed up computation, parallel implementation on Graphics Processing Unit was applied.

1 INTRODUCTION

Lamb wave modelling is a challenging task due to high-frequency components of propagating waves. From the point of view of Structural Health Monitoring (SHM), the useful frequency range is from about 10 kHz up to about 350 kHz. Simulation of guided wave propagation (including Lamb wave propagation) can be performed by many methods including finite element method [1], spectral element method in frequency [2] or time domain [3], finite difference method [4], LISA [5] and many more. An excellent review of guided wave based structural health monitoring methods can be found in ref. [6].

Analytic and spectral element method in the frequency domain [2] are very efficient. However, these methods can be applied only to simple structures such as beams and rods, or for wave propagation in plates propagating along a line, i.e., between actuator and sensor without taking into account defects. Also, edge reflections are difficult to consider. Combination of analytic-based approaches with finite element method gives more flexibility [7]. In order to model appropriately SHM system including piezoelectric transducers and bonding layer as well as structural features such as stiffeners, complex meshes have to be used. It leads to long computation time, even in case of robust methods such as LISA [5] and spectral element method [9]. Parallel implementation on Graphics Processing Unit (GPU) is used for reducing computation time [9, 10].

Recently, other interesting approaches emerged. It includes spectral cell method utilizing octree mesh generation [11, 12]. Other methods are wavelet finite element method [13] and wavelet spectral finite element method [14] which are also implemented on GPU [15]. Also, the problem of crack and surface waves propagation modelling was addressed by using peridynamic approach [16]. Nevertheless, neither of these methods seem to be good enough for application for model-based SHM. Model accuracy must be further improved as well as computation time reduced. On the other hand, direct damage detection methods utilize signals registered by sensors. In such case sensor placement optimization is a vital problem. It is a complex problem, and it is difficult to use high-fidelity models due to computation power constraints.

In this paper comparison study of various numerical models based on the time domain, spectral element method [3] is performed in order to find their usefulness and limitations for piezoelectric transducer placement optimization. The concept of optimization is the same as presented by Soman et al. [17] except that instead of electromechanical impedance signals, Lamb wave signals are used. Optimization is performed based on analytical signals which do not take into account piezoelectric mass, coupling effect, a bonding layer, etc. A genetic algorithm is employed for optimization. The aim is to find out how much the numerical model can be reduced to provide satisfactory optimization results and if the analytic approach is feasible. Models based on Mindlin plate theory and 3D elasticity theory for a various number of nodes across the thickness are compared in terms of dispersion curves. Lamb wave propagation patterns are calculated for the final configuration of transducers, and the signals are compared for the case of the fully modelled piezoelectric transducer with the case of transverse force actuation.

2 NUMERICAL MODELS

Numerical models used for simulations are based on the time domain spectral element method. The advantage of the method is flexibility for modelling of complex structures, a diagonal mass matrix which leads to fast explicit time integration and high granularity of processes in parallel implementation [10].

Various spectral elements were tested which are derived by using Mindlin-Reissner first order shear deformation theory and 3D elasticity theory wherein 3D elements had 2, 3 and 4 nodes across the thickness of investigated aluminium plate. These elements are schematically shown in Figs. 1(a)-(d), respectively in the local coordinate system (ξ, η, ζ) . Mindlin-Reissner theory leads to the element which has 5 degrees of freedom per node: in-plane displacements u, v , transverse displacements w and two rotations φ_1 and φ_2 . 3D elements have 3 degrees of freedom per node: in-plane displacements u, v and transverse displacements w . It should be noted that 3D elements can also possess an electrical degree of freedom denoted in Figs. 1(b)-(d) by symbol ϕ . Therefore implementation of the piezoelectric transducer for Lamb wave actuation and sensing is straightforward [8]. On the other hand implementation of piezoelectric transducers in 2D elements is less realistic. An alternative method is a coupling of 2D host structure (e.g., plate) with 3D piezoelectric transducer by using Lagrange multipliers [18].

It can be seen in Fig. 1 that the distribution of nodes is non-uniform and corresponds to Gauss-Lobatto-Legendre points. Neutral plane is marked by dashed lines. Six nodes along ξ axis as well as η axis were used.

Plate of the size 1 m by 1 m and thickness 1 mm was investigated. The material properties of aluminium alloy were assumed as follows: Young modulus $E = 72$ GPa, Poisson ratio $\nu = 0.33$, mass density $\rho = 2660$ kg/m³. The material damping was not incorporated into the model.

2.1 Dispersion curves

Characteristic Lamb equations are nonlinear and can be solved numerically by using the methodology presented by Rose [19] combined with bisection method. For comparison, approximate solutions corresponding to models shown in Fig. 1 were derived. It means that for each theory equation of motion is derived, then exponential wave motion terms are substituted to the equation of motion, and traction-free boundary conditions are applied which leads to determinant containing wavenumber-frequency relations. Determinant must be zero which defines a set of branches approximating dispersion curves of particular Lamb wave modes.

The results in terms of group velocity dispersion curves are shown in Figs. 2-5. Lamb wave analytic approach is indicated by solid lines whereas approximate solutions are marked by diamond symbols. Approximate solutions also contain dispersion curves of shear horizontal waves. Analytic solutions for shear wave dispersion curves are omitted for better clarity. Approximate solutions are valid only for fundamental symmetric and antisymmetric modes (S0 and A0) in certain frequency range. Mindlin-Reissner model (Fig. 2) underestimate A0 mode group velocity around frequency 1 MHz whereas the 3D model with 2 and 3 nodes across the thickness overestimate it (Figs. 3-4). The best approximation of A0 mode dispersion curve is for a 3D model with 4 nodes across the thickness (Fig. 5). However, for the 100 kHz frequency even lower order model can lead to better estimation of A0 mode group velocity. It is presented in Table 1, where the lower error is for the 3D model with 2 nodes across the thickness. At the same frequency, all models estimate S0 mode velocity with very good accuracy (error

below 0.2%).

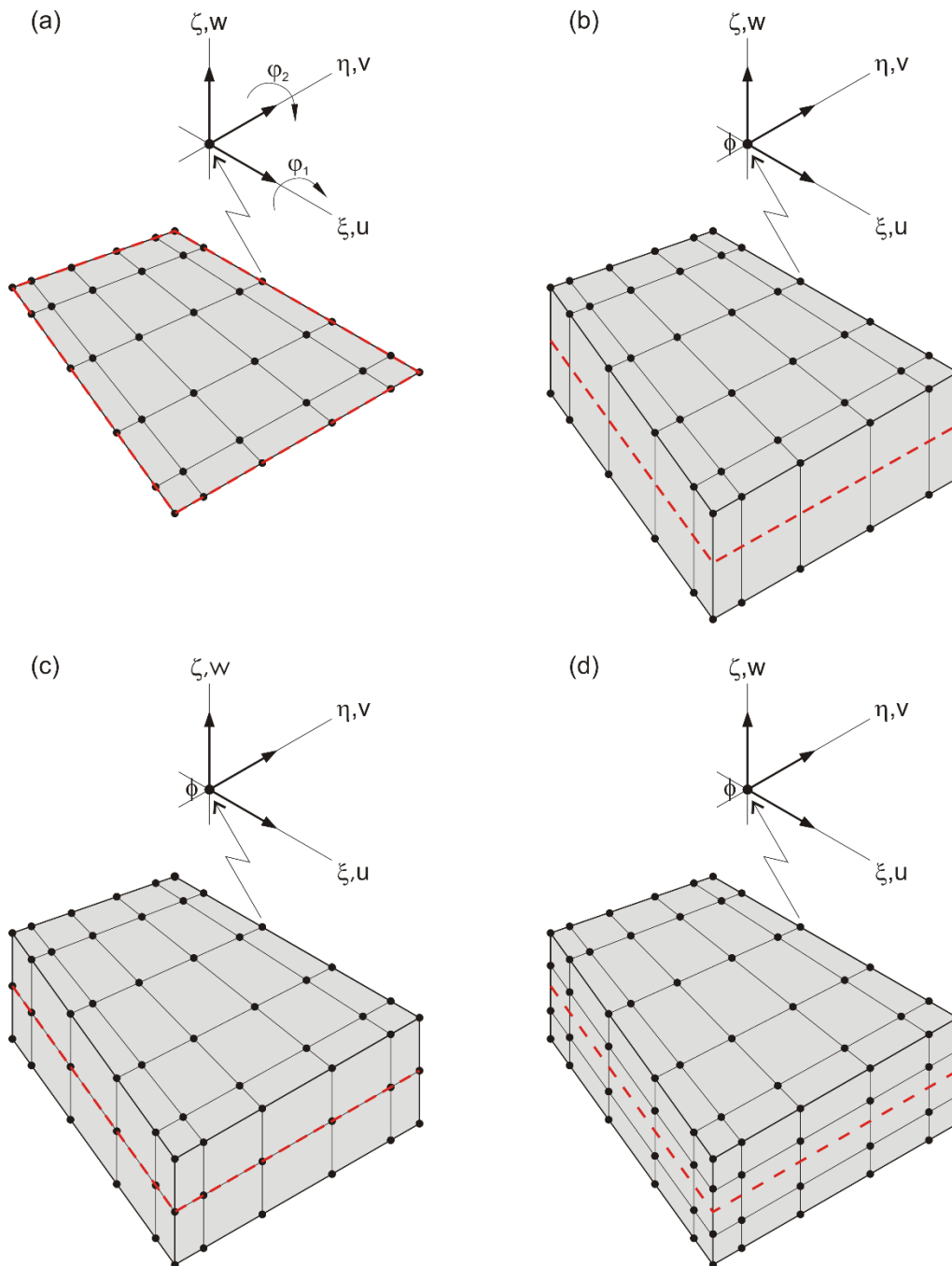


Figure 1: Modelling approaches: (a) plate spectral element based on Mindlin-Reissner first order shear deformation theory, (b) 3D spectral element with 2 nodes across the thickness, (c) 3D spectral element with 3 nodes across the thickness, and (d) 3D spectral element with 4 nodes across the thickness; red dashed lines indicate neutral plane

Table 1: Group velocity comparison for 100 kHz

Model	Lamb	Mindlin	3D: 2 nodes	3D: 3 nodes	3D: 4 nodes
A0 mode [m/s]	1921.09	1798.92	1931.15	1815.91	1799.73
Error	-	-6.4%	0.5%	-5.4%	-6.3%
S0 mode [m/s]	5509.00	5511.40	5509.99	5509.27	5509.27
Error	-	0.04%	0.18%	0.005%	0.005%

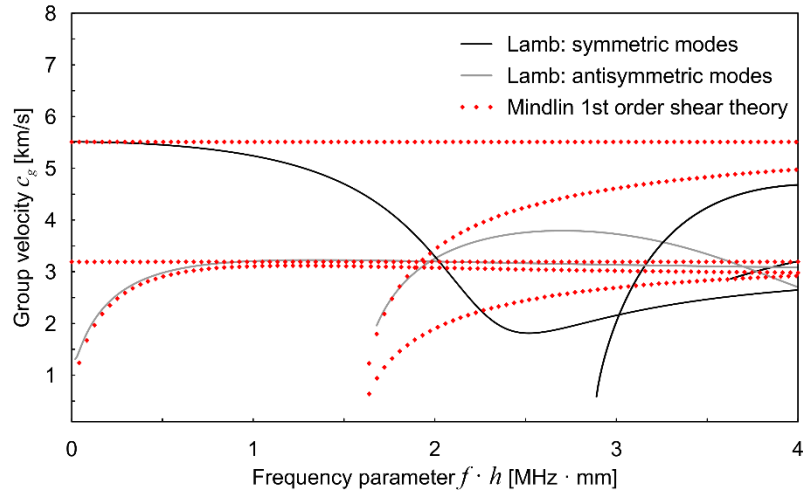


Figure 2: Comparison of group velocity dispersion curves obtained by using analytic Lamb equations and approximation based on Mindlin-Reissner first order shear deformation theory ($E = 72 \text{ GPa}$; $\nu = 0.33$; $\rho = 2660 \text{ kg/m}^3$)

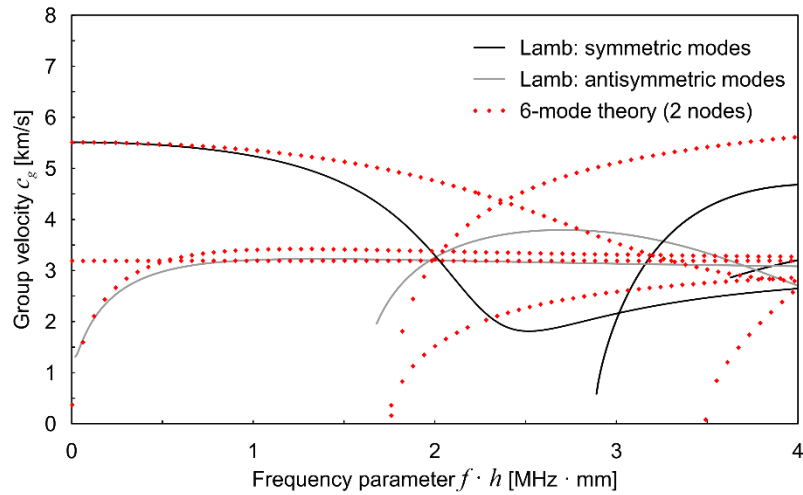


Figure 3: Comparison of group velocity dispersion curves obtained by using analytic Lamb equations and approximation based on 3D elasticity theory corresponding to 3D spectral element containing 2 nodes across the thickness of the plate ($E = 72 \text{ GPa}$; $\nu = 0.33$; $\rho = 2660 \text{ kg/m}^3$)

It should be noted that proper A0 mode approximation is challenging and accuracy of presented models is still too low for model-based SHM methods in which group velocity is key factor. Even 1% change of wave velocity, for example due to temperature change, leads to significant loss in the signal-to-noise ratio of subtracted signals. However, for estimation of optimal sensor coverage, edge reflection, traveled distance, etc., where required accuracy is below 10%, all models are good enough.

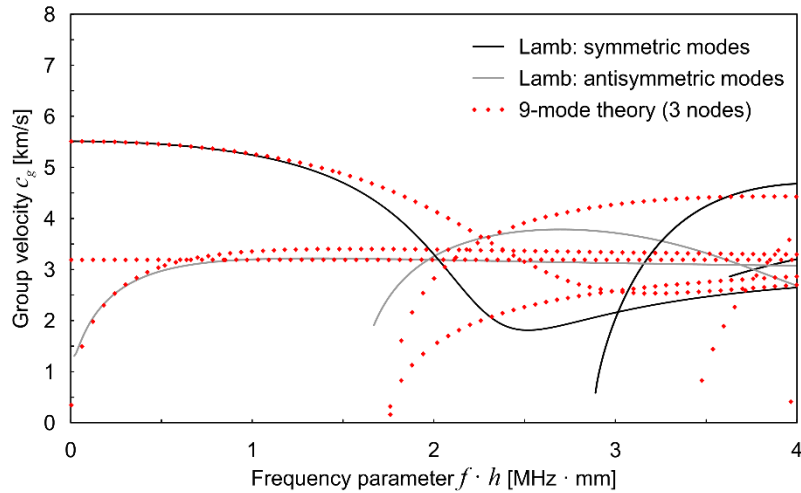


Figure 4: Comparison of group velocity dispersion curves obtained by using analytic Lamb equations and approximation based on 3D elasticity theory corresponding to the 3D spectral element containing 3 nodes across the thickness of the plate ($E = 72 \text{ GPa}$; $\nu = 0.33$; $\rho = 2660 \text{ kg/m}^3$)

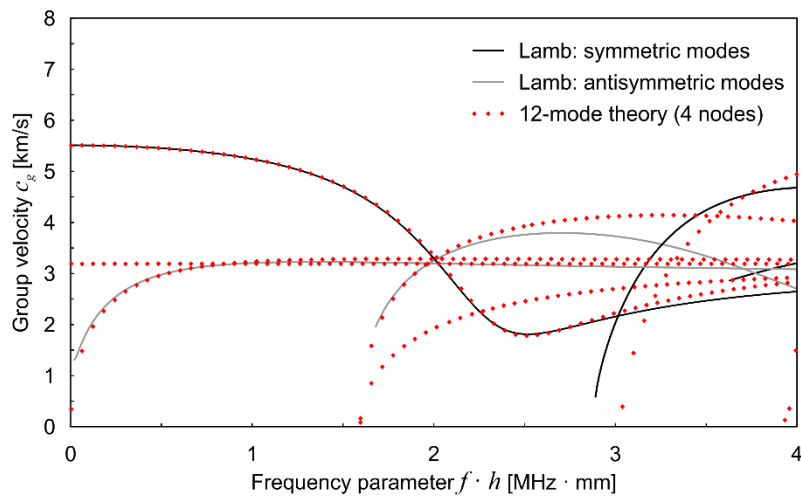


Figure 5: Comparison of group velocity dispersion curves obtained by using analytic Lamb equations and approximation based on 3D elasticity theory corresponding to the 3D spectral element containing 4 nodes across the thickness of the plate ($E = 72 \text{ GPa}$; $\nu = 0.33$; $\rho = 2660 \text{ kg/m}^3$)

2.2 Piezoelectric transducer model versus transverse force excitation

A piezoelectric transducer in the form of a disc of diameter 10 mm was modelled by 3D spectral elements (3 nodes across the thickness). Classic electromechanical coupling was assumed [8]. The actuation by the piezoelectric transducer was compared to the case in which transverse force was applied at the central point corresponding to PZT location. The analysis was performed for the excitation signal in the form of a wave packet of carrier frequency $f_c = 100$ kHz and modulation frequency $f_m = 20$ kHz. The results of transverse displacements are shown in Figs. 6-7. It can be seen that excited displacements in Fig. 6 are little bit shifted in time (see first dashed line) due to the size of PZT transducer in comparison to the force applied to the single node. Nevertheless, edge reflections (second dashed line) are incoming at the same time. It is obvious from Fig. 7 that for the isotropic material, transverse force excitation is not able to reproduce S0 mode of Lamb waves. Apart from that, signal shapes match very well signals excited by the PZT actuator.

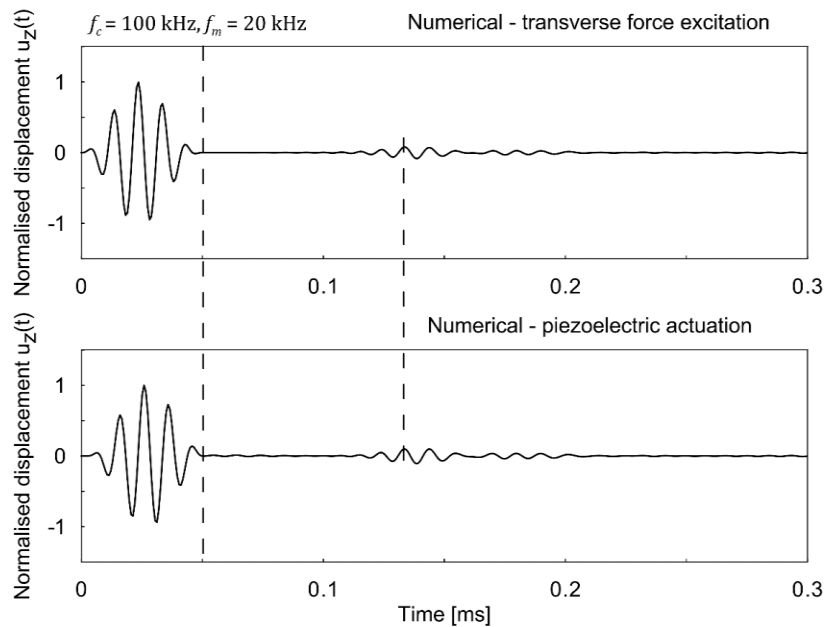


Figure 6: Comparison of normalized transverse displacements at the excited node for the case of transverse force excitation and fully modelled PZT, respectively

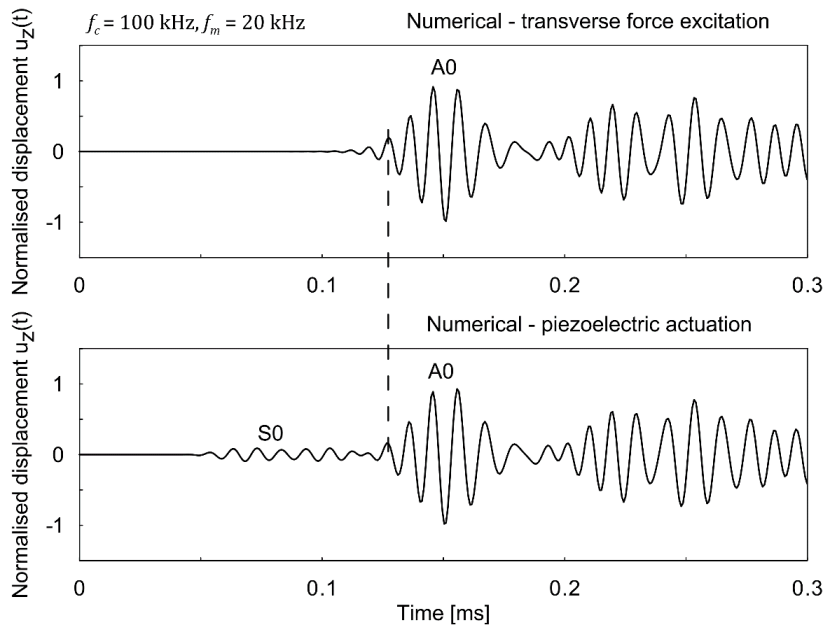


Figure 7: Comparison of normalized transverse displacements at the node distant about 22 cm from actuation point for the case of transverse force excitation and fully modelled PZT, respectively

2.3 Computation time

Computation time is a critical factor limiting the application of presented models. Hence, comparison of computation time of typical wave propagation problem for models with various complexities was performed. The total propagation time was assumed as 0.3 ms. The same planar mesh of spectral elements was used for 2D case based on Mindlin-Reissner theory and 3D models. 3 nodes across the thickness were used in the 3D model. For most complex mesh 3 layers of 3D spectral elements were used. The results are presented in Table 2. It is evident that a number of degrees of freedom (DOF) directly influences the computation time. Computation on Central Processing Unit (CPU) by using a mesh consisting of 3D elements with 3 nodes across the thickness takes more than twice the time needed for computation using plate elements based on Mindlin-Reissner theory. The computation time can be significantly reduced by using GPU (almost 10 times for this particular case). A layered approach, which seems to be suitable for modelling composite laminates, leads to very long computation time even with the application of parallel implementation on GPU. Therefore, the most practical approach is a homogenization of material properties and averaging over the thickness so that only one layer of 3D spectral elements can be used in the mesh.

Table 2: Comparison of computation time (CPU: Intel I5 4460 3.2GHZ, RAM 16 GB, GPU: Nvidia Tesla K20X, RAM 6GB)

Analysis	Mindlin 2.1 mln DOF	3D – 3 nodes 3.8 mln DOF		3D – 3 layers 8.8 mln DOF	
Per time step	CPU	CPU	GPU	CPU	GPU
	1.1 s	2.316 s	0.258 s	6.453 s	1.123 s
4 actuations	36h40 min	77h 12 min	8h 36 min	215h 4 min	36h 26 min

3 OPTIMIZATION OF SENSOR PLACEMENT

The methodology is similar to the concept presented by Soman et al. [17]. The aim was to develop a metric which ensures coverage of a maximum portion of the structure with at least 3 sensors used in triangulation methods for damage detection while using the minimum sensors possible. It is assumed that the sensors can be placed on a regular grid of points covering the investigated plate. The cost function c is defined as:

$$c = \alpha \times \frac{-\text{cov}_3}{S} + \beta \times \text{pen}_0 \quad (1)$$

where cov_3 is the number of points of the grid which lie within the sensing range of 3 or more sensors, pen_0 is the number of points which do not lie in the sensing range of a single sensor. α and β are weighting values to determine the relative merit for each of the parameters and S is the number of sensors. The cost function was employed in the genetic algorithm. The determination of the cov_3 and the pen_0 for each sensor placement was carried out analytically as the problem size is too big to be tackled through experiments or numerical methods. The final optimized coordinates of sensors are compiled in Table 3. The origin of the coordinate system is in the lower left corner of the plate.

Table 3: Coordinates of optimized configuration of transducers

Transducer no	1	2	3	4	5	6	7	8	9
Coordinate x [cm]	10	90	90	20	50	90	40	20	90
Coordinate y [cm]	10	10	20	30	40	60	70	80	90

The wave propagation patterns in the form of transverse displacements at three selected time instances for transducers 1-3 working as actuators are presented in Fig. 8. Two important remarks can be made. Dominating A_0 mode amplitude is quite dispersive at 100 kHz, and it causes stretching of the wave packet with the time and traveling distance. At the time $t=250 \mu\text{s}$ wave packet is quite wide, reducing the precision of damage localization. The actuator placement close to the edge causes immediate edge reflections which follows directly transmitted waves. It might hinder wave packet reflected from damage.

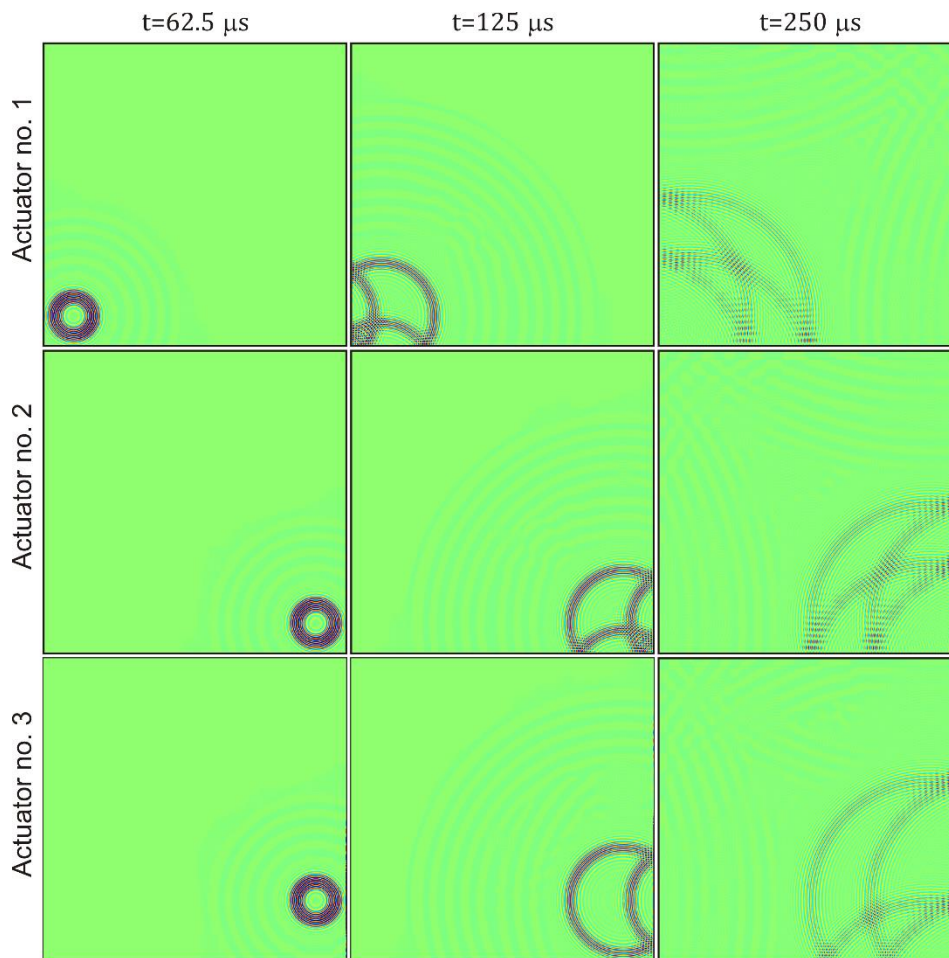


Figure 8: Wave propagation patterns at selected times propagating from actuators no 1, 2 and 3, respectively

4 CONCLUSIONS

- Literature review suggests that the time domain spectral element method is popular tool for modelling of Lamb wave propagation phenomenon.
- 2D spectral elements based on Mindlin-Reissner plate theory and 3D spectral elements seems to be accurate enough for sensor coverage optimization.
- Simplification of piezoelectric modelling by using transverse force excitation is reasonable for the case when A0 mode amplitude dominate.
- Computation time can be significantly reduced by using parallel implementation on GPU, but it is still too long to use signals calculated numerically for sensor optimization problem. Analytic solutions are still the best option for this purpose.
- Wave propagation patterns obtained numerically for optimized transducer location match well with analytic predictions.

ACKNOWLEDGEMENTS

The authors would like to thank National Science Centre, Poland for the financial support in the frame of the project no 2016/22/E/ST8/00068.

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