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APPLICATION OF FOURIER AND WAVELET METHODS TO ANALYSIS OF POSITRON EMISSION PARTICLE TRACKING DATA

A. Halidou[1], D.V.V. Kallon[1], A. Nel[1], I. Govender[2] AND A. N. Mainza[3]

[1] Department of Mechanical and Industrial Engineering Technology, University of Johannesburg
dkallon@uj.ac.za

[2] Department of Chemical Engineering, University of KwaZulu Natal

[3] Department of Chemical Engineering, University of Cape Town

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Abstract. This paper presents Fourier and wavelet methods applied to data derived from Positron Emission Particle Tracking (PEPT) experiments conducted at the iThemba LABS outside Cape Town in South Africa. The iThemba LABS runs a specialized cyclotron which produces positrons for use in medical PET and industrial-related PEPT research. The methods are applied as an aid in developing a computational procedure for determining the value of the circulation rates of the charge found in typical industrial tumbling mills. The circulation rate values are obtained as fundamental harmonics from power spectral plots using Fourier analysis. The wavelet method is applied to identify and remove noise from the data in order to improve on the computationally determined value of the circulation rate. For the analysis presented herein, parameters are obtained directly from the flow dynamics of the PEPT tracer particle.

1 INTRODUCTION

The particles in a tumbling mill can undergo various kinds of motion including spin, rotational and translational. Translational motion, also known as material transport defines motion along the axis of the mill between entry and discharge. Due to the mill rotational motion, however, most charge motion is in a plane at right angles to this axis and this plane is important in the description of grinding kinetics. For motion in this plane the charge is said to circulate at a rate known as its circulation rate. In a previous paper we established a model for the experimental determination of the circulation rate of the charge in these mills which was tested using data derived from Positron Emission Particle Tracking (PEPT) experiments [1]. In this paper we discuss a computational methodology for determining circulation rate using Fourier and Wavelet techniques.
According to Powell and Nurick [2], the motion of a tumbling mill is translated into

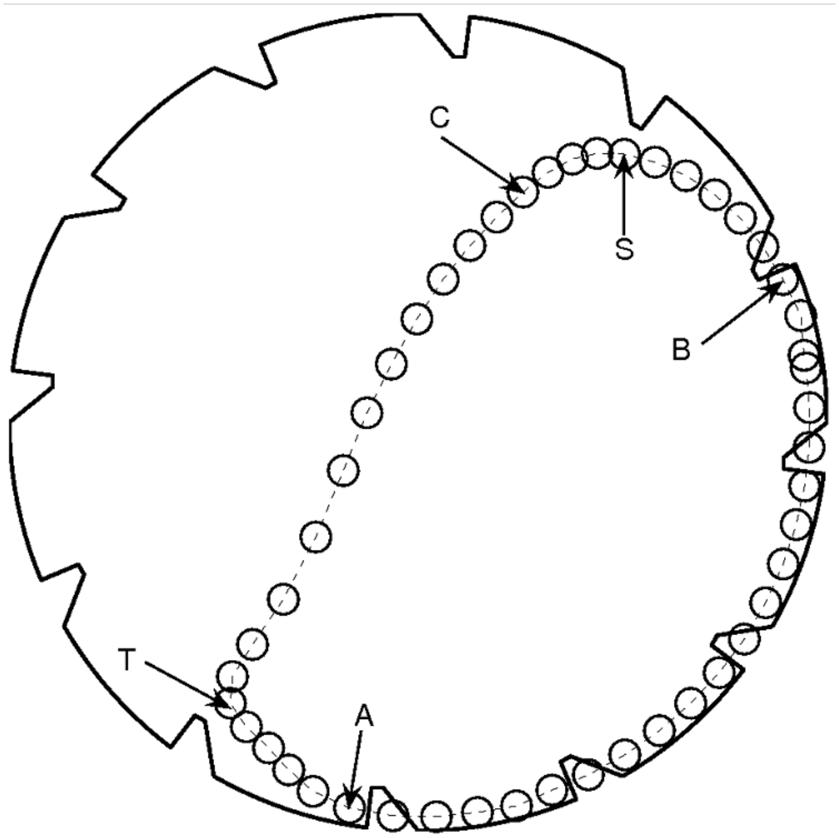


Figure 1: A typical circulation path followed by the ball, showing the various distinct zones

mechanical energy of the content of the mill, known as the charge, in the form of rotational motion. This charge motion arises from the mill internal design and mill speed and thus determines the power draft, stressing intensity and stressing probability, mixing kinetics inside the charge, material transport and the performance of a mill [3]. The subsequent work done on the charge can increase its potential energy, linear kinetic energy, rotational kinetic energy and temperature.
In a previous paper [1], it was established a model for determining the circulation rate of the charge in a tumbling mill relating it to both physical and geometric parameters of the mill operation. Physical parameters of typical tumbling mill operations include the mill speed, mill load fraction and internal friction while geometric parameters may include those of the center of circulation (CoC), the center of mass (CoM), the angle of repose, and the toe and shoulder angles. These features of tumbling mill operations have been well investigated [4] [5] [6] [7] [8] [9] [10] [1]. In order to fully appreciate the relationship between circulation rates and tumbling mill operating parameters we may need to divide the circulation plane of the charge into a number of distinct zones as proposed by Govender and Powell [11] [10], (see figure 1).

1. The circular path in the en-masse region, where the ball is lifted up by the rotary motion of the mill - from A to B. This zone is used to calculate the slip between

layers of charge.

2. The shoulder zone - B to C - where the ball falls away from the mill circular path, passes through the maximum height S, and then begins to fall down towards the toe region.

3. Cascading/cataracting zone - from C to T - where the ball tumbles down the other descending balls (cascading) or falls freely (cataracting), until it impacts in the toe region at T.

4. The toe region - T to A - where the ball impacts on the charge and is drawn into the rotary motion of the mill. This is an active region of the charge, and is where the interactive forces are greatest.

These key zones define the trajectory of a circulating particle which is influenced mostly by the mill operating physical and geometric parameters.

1.1 Review of Fourier and Wavelet Methods

Fourier and wavelet methods are powerful and fascinating branches of mathematics. In the analysis of complex functions one may sometimes need to make an educated guess about the identity of a function or the meaning of a result [12] [13]. Until recently these branches of mathematics had very little practical application. With the continued application of computers to mathematics both methods have emerged as central to many parts of science and technology. These methods have found applications in areas such as predicting ocean tides, analysis of noisy signals and mostly in the analysis of periodic data. We found their ability to analyse periodic data very useful in the discussion of data obtained from Positron Emission Particle Tracking (PEPT) experiments. The methods were therefore applied to our experimental data with reasonable degree of success. But before we develop our models we take a look at the basic concepts governing these methods.

1.1.1 Forward Transforms

Transforming a digital signal from time to frequency representation is always required in signal processing. The Fourier transform is used to convert signal from time to frequency domain by integrating over its time axis (see equation (1))[12]. It is an efficient method when the frequency bands of signals and noises are distinctives, the low-pass filters will reduce the noises, however it produces an affected signal output.

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \tag{1}$$

This last decade, wavelet has become a powerful denoising tool. It overcomes the shortages of Fourier transform by processing simultaneaously the time domain (of finite length) and frequency domain (of finite bandwidth) [13]. From equation (2) as dilation and translation of the mother wavelet occurs, very low frequency components can be observed at large s values, while very high frequency components can be located precisely at small s.

$$Wf(s,u) = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{s}}\psi^*(\frac{t-u}{s})dt \tag{2}$$

1.1.2 Inverse Transforms

The fourier transform F(ω) produces the fequency domain function with real and imaginary parts while the inverse Fourier Transform f(t) is used to recover a function from its Fourier transform. Knowing the frequencies and phases one can reconstruct the original signal. In many situations the Fourier transform is apply in one end and the Inverse Fourier transform to the other end. In this subsection, the inverse Fourier Transform f(t) is illustrated in equation (3).

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{i\omega t}dw \tag{3}$$

The wavelet transform is a useful and well known signal processing technique, once we get the wavelet transform we can reconstruct the original signal by computing the inverse wavelet transform as shown in equation (4).

$$f(t) = \frac{1}{C_\psi}\int_{0}^{\infty}\int_{-\infty}^{\infty} wf(s,u)\frac{1}{\sqrt{s}}\psi(\frac{t-u}{s})du\frac{ds}{s^2} \tag{4}$$

Where:

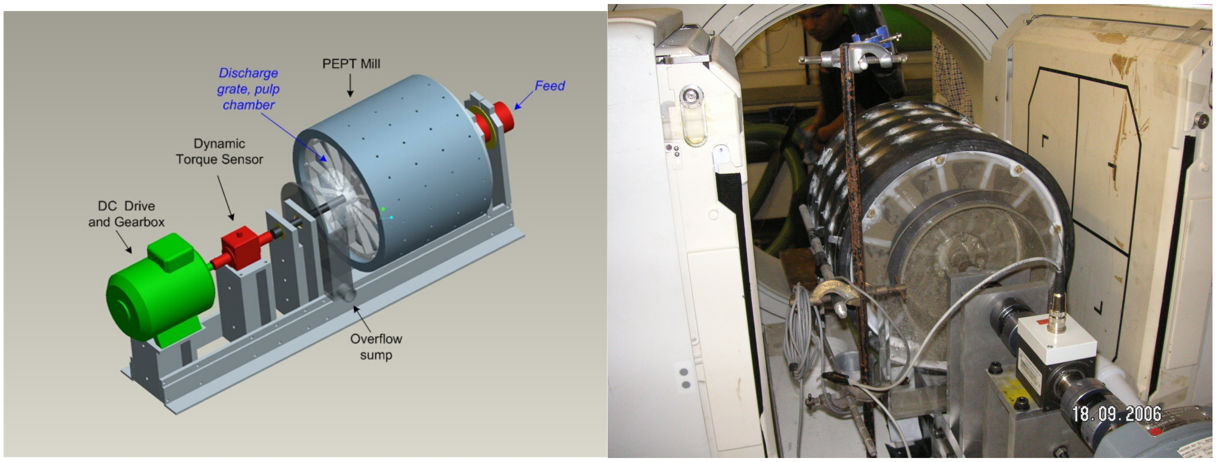
$$C_\psi = \int_{0}^{\infty} \frac{|\Psi(\omega)|^2}{\omega}d\omega < \infty \tag{5}$$

1.1.3 Fast Discrete Transforms

In the analysis of tumbling mill data obtained from PEPT experiments, sampling is necessary, meaning that a discrete formulation of both the Fourier and Wavelet transforms were necessary. In both cases the discrete formulation can be done in a more efficient manner by the use of the Fast Transform Algorithm (FTA). Hence, in the place of the Discrete Fourier Transforms (DFT) the Fast Fourier Transform (FFT) can be used and in the case of the Discrete Wavelet Transform (DWT) the Fast Wavelet Transform (FWT) can be employed. We shall show later how we successfully applied these two methods to our tumbling mill scenario to compute a free noise circulation rate.

2 Methodology

Todo............



(a) (b)

Figure 2: .....

3 Application of Fourier and Wavelet Methods to PEPT Data

We developed FFT and FWT and used them to analyze tumbling mill data obtained from our PEPT experiment conducted the iThemba LABS. From this we have developed a computational scheme for determining circulation rate of the charge in tumbling mills. The computational methodology presented herein compares well with experimental results and further validates our circulation rate model established in a previous paper [1].

3.1 Applying the Fourier Methods

Using the MATLAB FFT function, the DFT of the radial positions R(n) could be found. The power spectrum at various mill speeds is obtained from equation (8) and plotted in figures (3) and (4).

Some important inferences that can be drawn from the plots in figures (3) and (4) are:

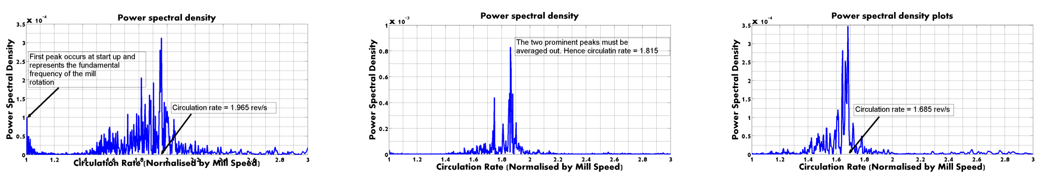
- In figure (3a), which shows the mill running at 20% of the mill critical speed (the lowest speed used in the experiments), an initial peak appears at start-up. Since the plots are normalized by mill speed this peak represents the fundamental frequency of the mill. The charge and mill are seen to move at the same velocity indicating that this fundamental frequency must be equal to unity $(\omega_0 = 1.0)$.

- Once the charge starts to circulate a peak or cluster of peaks appear to the right of this fundamental frequency. This peak is the fundamental harmonic from which the circulation rate of that experimental run can be read. For instance at a mill rotational speed of 35% of the critical speed, the circulation rate read off from the fundamental harmonic in figure (3b) is 1.815 rev/s.

- From figures (3), and (4) it can be seen that there is a gradual shift in the location of the peak (or cluster of peaks) towards the fundamental frequency as mill speed is increased. This can be explained by the fact that the trajectory path travelled by a particle in one circulation, as illustrated in figure 1, is considerably shorter than the mill circumference at lower speeds but this ratio reduces at higher speeds [2] [10] [1]. Hence at lower speeds the circulation rate of the charge in a tumbling mill is high. At higher speeds, when the charge trajectories are longer, the circulation rate approaches the fundamental frequency of $(\omega_0 = 1.0)$.

- These harmonics (circulation rates) are plotted as a function of mill speed in figure 5. An empirical line is fitted to the data with a high coefficient of determination ($R^2 = 0.98326$). From this plot a linear relationship between the fundamental harmonic (circulation rate) and mill speed is observed where the former decreases near linearly with increasing the later.

- In figure (4b), it can be observed that higher order harmonics may sometimes occur in the spectral plots. These may represent higher order charge circulation rates in the mill and could be as a result of unsteady flows nearer to the end of the experimental run. However there may be better/more physical interpretations of this occurrence and its consequence for mill performance needs to be further investigated. For this reason we developed and applied the wavelet methods to tumbling mills using our PEPT data.



a b c

Figure 3: Power spectral plots showing circulation rates obtained from the first peak or an average of closer peaks. (a) Circulation rate at 20% mill speedand 40% mill load; (b) Circulation rate at 35% mill speed and 40% mill load; (c)Circulation rate at 50% mill speed and 40% mill load

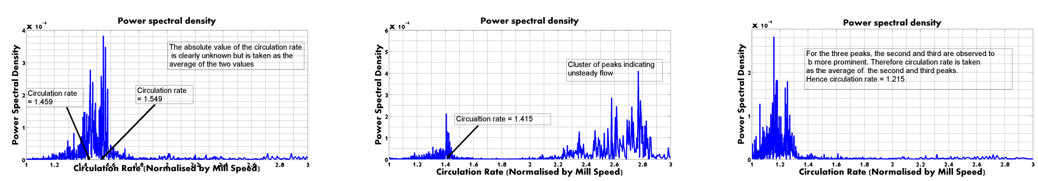


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Figure 4: Power spectral plots showing circulation rates obtained from the first peak or an average of closer peaks. (a) Circulation rate at 65% mill speedand 40% mill load; (b) Circulation rate at 75% mill speed and 40% mill load; (c)Circulation rate at 90% mill speed and 40% mill load

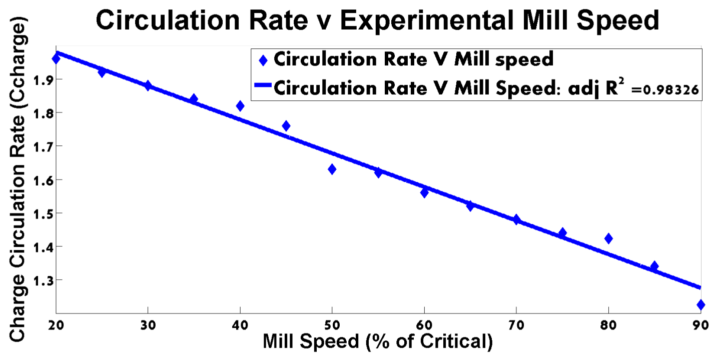


Figure 5: Plot of power spectral peaks (harmonics or circulation rates) as a function of mill speed.

3.2 Applying the Wavelet Methods to PEPT Data

We used the Daubechies wavelet family to obtain the approximation signal and details of the PEPT tracer particle kinematic distribution within the opaque grinding environment of the tumbling mill. The Daubechies wavelets are a family of orthogonal wavelets defining a discrete wavelet transform and for some given support they are characterized by a maximal number of vanishing moments [13]. Suppose the scaling and wavelet functions are given as Daubechies, lets say the basis is known. We approximate a discrete signal in $I^2(Z)$ by equation (9):

$$f[n] = \frac{1}{\sqrt{M}} \sum_k W_\phi[j_0, k]\phi_{j_0,k}[n] + \frac{1}{\sqrt{M}} \sum_{j=j_0}^{\infty} \sum_k W_\psi[j, k]\psi_{j,k}[n] \tag{6}$$

Here

$$f[n], \phi_{j_0,k}[n] \& \psi_{j,k}[n] \tag{7}$$

These are discrete functions defined in $[0, M-1]$, totaling M points. We note that the sets in equations (11) and (12) are orthogonal to each other. We take the inner product to obtain the wavelet coefficient as in equation (13). Hence equation (13) is the approximation coefficient and equation (14) is the wavelet coefficient and from equation (15) we obtain the detailed coefficient shown in equation (16).

$$\{\phi_{j_0,k}[n]\}_{k\in Z} \tag{8}$$

$$\{\psi_{j,k}[n]\}_{(j,k)\in Z^2, j\geq j_0} \tag{9}$$

$$W_\phi[j_0, k] = \frac{1}{\sqrt{M}} \sum_n f[n]\phi_{j_0,k}[n] \tag{10}$$

$$W_\psi[j, k] = \frac{1}{\sqrt{M}} \sum_n f[n]\psi_{j,k}[n] j \geq j_0 \tag{11}$$

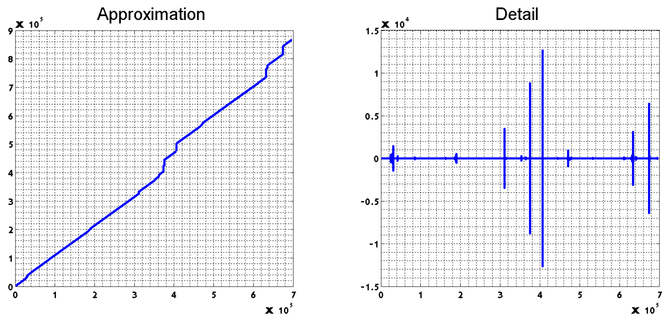


Figure 6: Approximation and details of PEPT data using Daubechies wavelet(Left: Approximation of PEPT data, Right: The details of the PEPT data)

$$\phi_{j,k}[n] = 2^{j/2}\phi[2^j n - k] = \sum_{n'} h_\phi[n']\sqrt{2}\phi[2(2^j n - k) - n'] \tag{12}$$

Let $n' = m - 2K$, then:

$$\phi_{j,k}[n] = \sum_{m} h_\phi[m - 2k]\sqrt{2}\phi[2^{j+1}n - m] \tag{13}$$

On applying this approximation model to our PEPT data we obtained the graph to the left of figure (6). In theory the approximation graph should give a straight proportional line, however, from the plot we note significant instability in the signal. This instability arises from noise in the data which affects the performance of the computerized analysis of tumbling mill circulation rate: poor experimental design and errors in the experimental procedure; noise during data acquisition; external noises from the environment all add to the noise in the signal. This noise can be observed in the discrete shifts in the plots towards higher regions giving a poor approximation of the signal (left of figure 6). The noise contributes to reducing the performance of visual and computerised analysis of the PEPT data.

In the detail plot (shown to the right of figure 6) the frequency components are obtained with their energy magnitudes plotted on the vertical axis. This plot reveals that there exist high frequency components in the data obtained from a typical PEPT experiment. It can be further noted that the positions of these high frequency components correspond to the locations of the discrete shifts in the approximation plots to the left of figure 6. These high frequency components are, therefore, responsible for the instability in the signal. For lengthy periods the signal appears stable, but the high energy frequencies create noise in the signal. This noise exists in only small segments of the signal around the region of the discrete shifts in the approximation plot. The noise needs to be removed by applying denoising using wavelet methods. Denoising is a process of removing the noisy segments (high frequency components) of a signal while retaining the quality of the original signal.

An algorithm was developed for processing the PEPT data in matlab and was successfully applied to denoise the PEPT data. But before applying denoising we examine whether

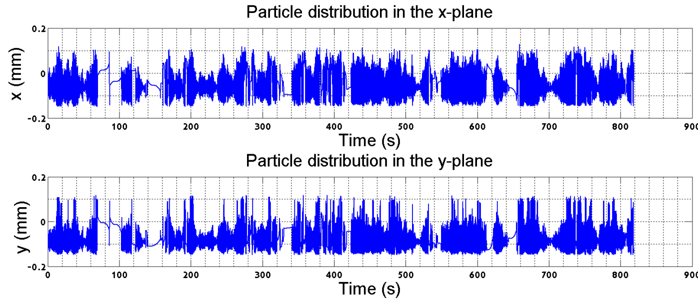


Figure 7: The X, Y plots of kinematic distribution of the tracer from PEPT experiments

thresholding is needed or not. Applying thresholding at some point in the data discards a portion of the details that exceeds a certain predetermined limit. In figure (7) we examine the X, Y plots of kinematic flow of the PEPT tracer particle from the tumbling mill experiments. The plots reveal that there exist discrete gaps in the particle flow patterns in the grinding environment of the tumbling mill. The occurrence of these gaps indicates erratic motions of the tracer particle giving rise to high frequency components in the signal.

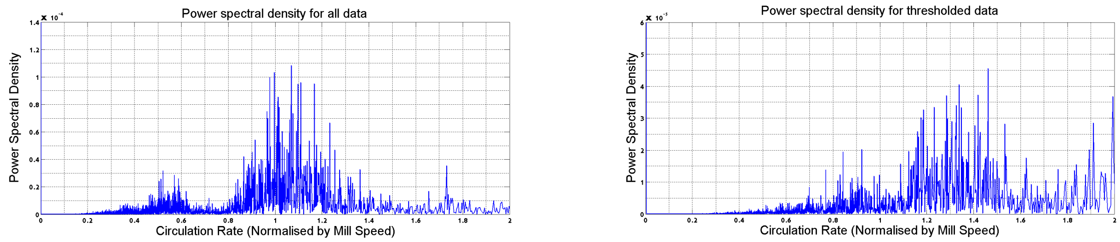
In the case of PEPT data the noise attributed to the data is found to be non-stationary so an assumption has to be made to fix the noise as stationary in order to apply a reasonable denoising model. The general wavelet denoising model is given in equation (17). In this model, the noisy signal $y\left(i\right)$ depends on the noise free signal $x\left(i\right)$ and some independent normal random variables $\varepsilon\left(i\right)$. In equation (17), represents the intensity of the noise in $y\left(i\right)$. Thus the approach models noise as high frequency signals superimposed on the original signal.

$$y(i) = x(i) + \sigma\varepsilon(i), \mathrm{i} = 1, 2, ..., \mathrm{n} - 1 \tag{14}$$

Applying denoising to the PEPT Data using the Daubechies wavelet method we obtained the plot in figure (8). Denoising based on the Daubechies wavelet is a technique that preserves some part of the high frequencies that are nonetheless relevant to the overall frequency representation of the signal and provide important information about the signal. Using the inverse wavelet transform we obtain a denoised signal to which the Fourier scheme earlier developed is reapplied to produce the denoised plot of the circulation rate of the charge in the tumbling mill as shown at the bottom of figure (8). A close comparison of the plots before denoising (figure 8a) and after denoising (figure 8b) shows that after removing some of the high frequency components through denoising the position of the harmonic shifts thereby increasing the value of the circulation rate. Thus the denoising process improves the circulation rate determination procedure and shifts its value towards an acceptable and reasonable magnitude.

4 CONCLUSIONS

An investigation was conducted into the causes of the failure of the Fourier scheme developed for the computational determination of the value of the circulation rate of



(a) before denoissing (b) after denoising

Figure 8: Denoising

tumbling charge using data derived from PEPT experiments. It was observed that the main cause was the occurrence of high frequency components in the signal generated from this data. These high frequency components manifest themself as noise during data acquisition or pre-processing of the data. The problem may also be caused by a raft of other factors which may include but not limited to the following:

- Poorly designed experiments,
- Experimental errors,
- Parameterization of sinogram data,
- Data sampling method,
- Triangulation methods,
- Etc.

Because the sources of noise in the data are not fully known we conclude that the problem must be fixed at the pre-experimental and experimental stages. Noise in most signals may be insignificant and negligible; however, when the noise is seen as corrupting the signal in significant ways it is best that it be removed before further meaningful analysis can be carried out. In this work we observed that the noise in the PEPT data has no frequency selectivity, is nonlinear and relatively dispersed and as such cannot lend itself to Additive White Gaussian Noise controls. And until the source of the noise in the data is well placed one cannot assume a Poison/Laplace noise either. As a consequence a tractable mathematical model that could provide insight into the underlying behavior of the data could not be developed at this stage.

It may be possible to implement a Signal-to-Noise Improvement Ratio at the instrumentation stage based on discrete wavelet transforms. A Signal-to-Noise Ratio (SNR) is a ratio of the power of a signal to that of the power of the noise in the signal. The higher the ratio the more useful information one can obtain from the signal. We recommend that further experiments be conducted with a view to reducing experimental errors, removal of noise in the data and an overall signal-to-noise ratio improvement model should be implemented.

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REFERENCES

[1] D. Kallon, I. Govender, A. Mainza, Circulation rate modelling of mill charge using position emission particle tracking, Minerals Engineering 24 (3-4) (2011) 282–289.

[2] M. Powell, G. Nurick, A study of charge motion in rotary mills part 1extension of the theory, Minerals Engineering 9 (2) (1996) 259–268.

[3] T. J. Napier-Munn, S. Morrell, R. D. Morrison, T. Kojovic, Mineral comminution circuits: their operation and optimisation, Vol. 2, Julius Kruttschnitt Mineral Research Centre, University of Queensland, 1996.

[4] H. White, The theory of the tube mill, The Journal of the Chemical, Metallurgical and Mining Society of South Africa (1905) 290–305.

[5] E. Davis, Fine crushing in ball mills, Trans. AIME 61 (1919) 250.

[6] H. E. Rose, R. M. E. Sullivan, A treatise on the internal mechanics of ball, tube, and rod mills, Constable, 1958.

[7] H. Watanabe, Critical rotation speed for ball-milling, Powder technology 104 (1) (1999) 95–99.

[8] J. Mellmann, The transverse motion of solids in rotating cylindersforms of motion and transition behavior, Powder Technology 118 (3) (2001) 251–270.

[9] M. Powell, A. McBride, A three-dimensional analysis of media motion and grinding regions in mills, Minerals Engineering 17 (11-12) (2004) 1099–1109.

[10] I. Govender, M. Powell, Investigating sag model assumptions using laboratory 3d trajectory data, in: International autogenous and semiautogenous grinding technology, University of British Columbia, Dept of Mining and Mineral Process Engineering, 2006, pp. 436–450.

[11] M. S. Powell, A study of charge motion in rotary mills, with particular reference to the grinding action, Ph.D. thesis, University of Cape Town (1993).

[12] E. M. Stein, G. Weiss, Introduction to Fourier analysis on Euclidean spaces (PMS-32), Vol. 32, Princeton university press, 2016.

[13] M. W. Frazier, An introduction to wavelets through linear algebra, Springer Science & Business Media, 2006.

Biography

Aminou Halidou is a Cameroonian PhD holder since 2014 from Hauzhong University of Science and Technology (HUST) in Wuhan, China. His PhD research concerned on-road accident prevention based on pedestrian detection technology and he is thus an expert on 3D image reconstruction. He holds three years experience as a Lecturer at the University of Yaound in Cameroon. He is attached with the Department of Computer Science where he has been teaching at Bachelors and Masters Levels. Dr, Halidous primary research interests are 3D image processing and reconstruction, pattern recognition, Machine Learning and Computer Vision.

B. D.V.V. Kallon is a Sierra Leonean holder of a PhD degree obtained from the University of Cape Town (UCT) in 2013. He holds a year-long experience as a Postdoctoral researcher and four months of employment as a Scientific Officer at UCT. In May 2014 he is transferred to the University of Johannesburg as a full-time Lecturer in the Department of Mechanical and Industrial Engineering Technology (MIET). He has been teaching at the BTech level of this department since. At the start of October 2015 he was promoted to a Senior Lecturer position. His primary research areas are Acoustics Technologies, Mathematical Analysis and Optimization, Vibration analysis, Pollution Control and Water Research.

Andre Nel has BSc (Eng) MEng D Eng and BA (Class Lang) degrees. He is currently head of the School of Mechanical and Industrial Engineering (SOMIE) at the University of Johannesburg and is interested in most fields of Mechanical Engineering. He has worked at UJ and its predecessor for more than 25 years and also has 5 years of Industrial experience. His main interests are in image processing, CFD and separation of multiphase flows and control applied to visual feedback in robotics.

Indresan Govender is a Professor of Particle Technology & Mineral Processing, and Research Director of the Positron Imaging Laboratory at iThemba LABS the only industrial positron imaging laboratory in the southern hemisphere. He currently serves as an executive member of the Global Comminution Collaborative (GCC), International Comminution Research Association (ICRA), Minerals to Metals Signature Theme (MtM), Centre for Minerals Research (CMR), and the Centre for Research in Computational and Applied Mechanics (CERECAM). He has produced over 50 publications in international journals and refereed conference proceedings, given as many talks at conferences and workshops, and raised more than 15 million Rand in research funding from industry and the NRF. He has 16 years of lecturing experience and has successfully supervised 28 postgraduate students, with a current cohort of 8 PhDs, 2 MScs, 2 postdoctoral fellows and a Research Officer.

Aubrey Mainza is a Deputy Director and Head of Comminution Research for the Centre for Minerals Research. He teaches core Chemical Engineering, and participates extensively in collaborative research with international universities, mining companies, and comminution and classification equipment manufacturers. He holds a C1 NRF rating and has produced more than 70 publications and is regularly invited to give keynote addresses at conferences. He has successfully supervised more than 30 PhD and MSc students to completion. Aubrey is a founder member of the Global Comminution Collaborative and the African Chapter Chairperson for the International Communition Researchers Association (ICRA) and sits on the Executive Committee for the Minerals to Metal Signature Theme and the Centre for Research in Computational and Applied Mechanics (CERECAM). Chairs the faculty Equipment Committee and represents senate on the Universities Institutional Forum.