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MODELLING DAMAGE AND PROGRESSIVE COLLAPSE OF FRAMES USING A GAUSSIAN SPRINGS BASED APPLIED ELEMENT METHOD

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Abstract. The Gaussian based AEM is implemented in damage and progressive collapse of frames of subjected to dynamic loading. A new return mapping scheme is also developed. This is achieved by introducing a separation strain criteria that describes the complete failure of the material. If the strain at a spring exceeds the separation strain, that spring is removed from the element. Once all springs of the element fail, the element will be separated from its adjacent element. This leads to a load redistribution, and the progressive collapse analysis of the structure proceeds. The results showed that modelling the collapse of the structure was straight forward, with a low computational cost and high accuracy. The framework developed allows to perform AEM analysis on a Finite Element model.

1 INTRODUCTION

The Applied Element Method (AEM) [1] was developed in 1997 by Meguro and Tagel-Din, to aid in the analysis of highly nonlinear behaviour of structures, such as crack initiation, crack propagation, separation of structural elements, rigid body motion of failed elements and total collapse of the structure. Current available methods cannot deal with structural collapse accurately; however, AEM can simulate the behavior of a structure from an initial state of no loading until collapse of the structure [2]. Nonlinear dynamic analysis has been widely modelled using the finite element method for analysis of progressive collapse of structures; however, difficulties in the analysis were found at the presence of excessively deformed elements with cracking or crushing, as well as having a high computational cost, and difficulties on choosing the appropriate material models for analysis [2]. In this paper, the Gaussian based Applied Element Method [3], a modification to the applied element method spring distribution is implemented for enhancing the accuracy and increasing the computational efficiency by decreasing the number of springs required per analysis. The springs are distributed based on an adaptive method, where they depend on the elasticity of each element. The method is applied to dynamically loaded structures to enduce nonlinear behaviour and collapse. The next sections in this

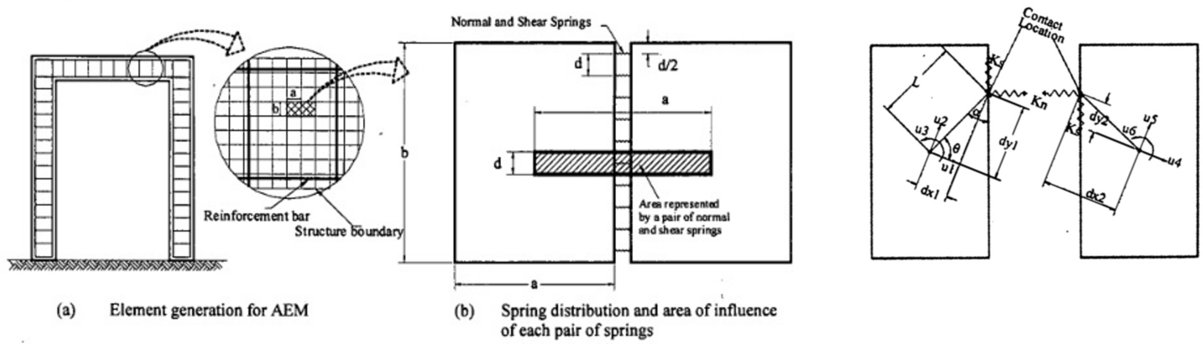


Figure 1: 2D AEM Elements [4]

paper breifly explain the generic AEM formulation, the Guassian-springs formulation and the dynamic model used. Structural beams and frames are analysed and the results are presented.

2 AEM Formulation for 2-D element

The elements in AEM are rigid body elements that are connected with sets of normal and shear springs along the edges of the elements. The springs represent the stresses and strains of the element in that region. The material properties are specified through the spring stiffness. For a 2-D element, three degrees of freedom are considered per element, deflection in x, deflection in y and rotation [2]. The stiffness matrix for a pair of elements is a 6×6 matrix. The upper left quadrant of the matrix is displayed in Equation 1 [2]. Each spring location in the elements is represented by a pair of normal and shear springs, with stiffness displayed in Equation 3 [2]. The elements are displayed in Figure 1. For each element, each pair of springs is analysed and the stiffness matrix is assembled into the global stiffness matrix. The AEM is implemented using MATLAB.

$$K = \begin{pmatrix} K_{n1} & 0 & -K_{n1}b_{n1} \\ 0 & K_{s1} & K_{s1}\frac{a}{2} \\ -K_{n1}b_{n1} & K_{s1}\frac{a}{2} & (K_{n1}b_{n1}) + (K_{s1}\frac{a}{4}) \end{pmatrix} \tag{1}$$

$$K = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \tag{2}$$

where,

$$K_n = \frac{Edt}{a}; K_s = \frac{GdT}{a} \tag{3}$$

3 Adaptive Gaussian AEM

Gaussian quadrature is used for finite element applications because of they have less function evaluation for given orders. The weights and evaluation points are determined so that the integration rule is exact to as high an order as possible [5].

$$\int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b-a}{2}x + \frac{b+a}{2}\right)dx = \frac{b-a}{2}\sum_{i=1}^{n} W_i f(x_i) \quad (4)$$

The Gaussian Quadrature formulation is used to determine the gaussian weights and coordinates using the number of springs. The location of the springs is determined by considering w as the width of the tributary area of each spring ('d' in AEM), and x as the spring location from Equation 4. Table 1 displays the calculated locations of the spring coordinates, along with d, the width of the tributary area of each spring for 5 points. Assuming the width of the section is 5, then x_i and w_i are multiplied by $5/2 = 2.5$.

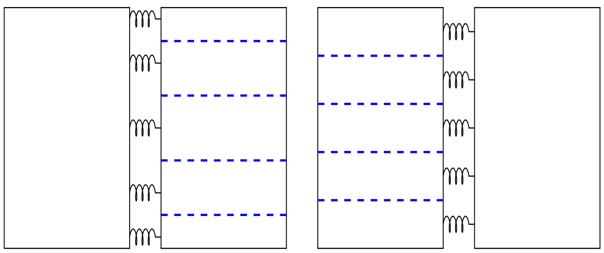


Figure 2: Comparison of Gaussian and Equal Springs Tributary Area.

Point	x_i	w_i	$2.5x_i$	$2.5w_i$
1	-0.906	0.237	-1.359	0.355
2	-0.538	0.479	-0.808	0.718
3	0	0.569	0	0.853
4	0.538	0.479	0.808	0.718
5	0.906	0.237	1.359	0.355

Table 1: 5 Gaussian Quadature Points.

Abdul Latif and Feng showed that for a simple cantilever beam with 30 elements an 1KN applied at the free end, convergance using Gaussian springs occured while using 2 springs per element. While with the conventional AEM 45 springs were required. In other words 1350 springs were required using the conventional AEM while only 60 springs were required using the Gaussian based AEM. This method is more computationally effictive and will be used in the scheme of fragmentation and collapse.

4 Damage and Fragmentation Modelling

In approaching damage and fragmentation, a pair of springs must first undergo elasticity, elasto-plasticity and finally damage. Once the material exceeds the yield criteria after plasticity, then the elements are considered to have failed. The following sections describe the models used for plasticity and how the failure criteria is taken into consideration.

4.1 Modified Return Mapping Algorithm

The damage is implemented into the 1D model shown by first checking if the strain is larger than the yield strain. If that is the case, then the stress, strain, internal force are set as 0, to simulate that the spring is not in the system anymore. The model used is derived from the 1D model for elasto-plasticity [6]. The yield crtieria in the 1D model is dependant on the stresses. However, a modified return mappng algorithim is introduced to model material softening, shown in Figure 3. Since the σ_{n+1} is smaller than the σ_n in the softening model, the yield criteria cannot depend on the stress, rather on the strain. The modified return mapping model is displayed as follows:

1. Evaluate elastic predictor

$$\epsilon_{n+1} = \epsilon_n + \delta\epsilon_{n+1} \tag{5}$$

$$\sigma_{n+1}^{trial} = E(\epsilon_{n+1} - \epsilon_{n+1}^{p}) \tag{6}$$

$$f_{n+1}^{trial} = |\sigma_{n+1}^{trial}| - (\sigma_y + H(\varepsilon_n - \varepsilon_Y)) \tag{7}$$

2. Check yield criterion

$$\begin{aligned} iff_{n+1}^{trial} \geq 0 \quad \& \quad \varepsilon_n > \varepsilon_Y \Rightarrow \text{spring failed} \\ iff_{n+1}^{trial} \leq 0 \Rightarrow \Delta\lambda = 0 \\ iff_{n+1}^{trial} \geq 0 \Rightarrow \Delta\lambda > 0 \end{aligned} \tag{8}$$

3. Plastic step (if $f_{n+1}^{trial} \geq 0$)

$$\Delta\lambda = \frac{f_{n+1}^{trial}}{E} \tag{9}$$

$$\sigma_{n+1} = \sigma_{n+1}^{trial} + H(\varepsilon - \varepsilon_y) \tag{10}$$

$$\epsilon_{n+1}^{p} = \epsilon_n^{p} + \Delta\lambda sign\left[\sigma_{n+1}^{trial}\right] \tag{11}$$

$$\alpha_{n+1} = \alpha_n + \Delta\lambda \tag{12}$$

where the nonlinear stiffness of the springs is:

$$K = \frac{CA}{L}\,;\; C = \frac{EH}{E+H} \tag{13}$$

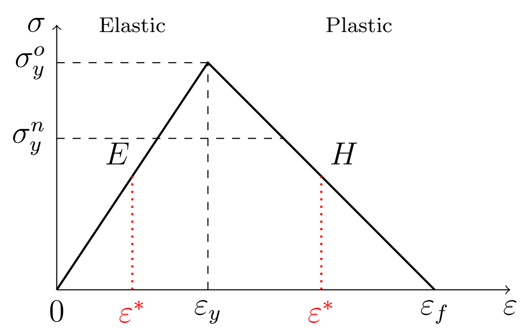


Figure 3: Softening Scheme

4.2 Simulating Collapse of Structures

Collapse of structure is presented in the AEM by checking the yield criteria of the spring. Once a spring has a strain larger than the yield strain, then the spring is considered to have detached from the system. This is represented by setting the stiffness of the failed spring to zero. Once all the springs between two elements fail, then the two elements are totally detached. The simplification in modelling the failure of springs is what makes using the AEM a reliable and efficient tool for modelling fracture.

5 Dynamic Model

AEM can also be applied to dynamic problems. A special case of the dynamic problem can results in convergance of solution of a static problem. In other words, a static problem can be presented as a special case of dynamic problem. dynamic relaxation The central difference method is the most commonly used time integration scheme in structural dynamic analysis. was found the most applicable model since it can avoid the use of the stiffness matrix. The method is derived below. The method is based on a finite difference approximation of the time derivatives of displacement [7]. Firstly, consider a damped structural system subjected to dynamic forces and experiencing nonlinear material behaviour modelled by;

$$M\ddot{u}(t) + C\dot{u}(t) + F_{int} = P(t) \tag{14}$$

where, x is the vector of displacements of structural coordinates, M is the positive definite mass matrix, C is a non-negative definite damping matrix, and K is a non-negative definite stiffness matrix. For representing the static case using the dynamic model, $F_{int} = KU$.

5.1 Central Difference Method

For constant time steps $\Delta t_i = \Delta t$, the expressions for the velocity and acceleration at time i are,

$$\dot{u}_i = \frac{u_{i+1} - u_{i-1}}{2\Delta t} \tag{15}$$

$$\ddot{u}_1 = \frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2} \tag{16}$$

Substituting these expressions for velocity and acceleration into the equation of motion gives;

$$M\left(\frac{u_{i+1} - 2u_i + u_{i-1}}{(\Delta t)^2}\right) + C\left(\frac{u_{i+1} - u_{i-1}}{2\Delta t}\right) + F_{int} = P_i \tag{17}$$

From time stepping, u_i and u_{i+1} are assumed known. Solving for u_{i+1} gives,

$$\left[\frac{M}{(\Delta t)^2} + \frac{C}{2\Delta t}\right] u_{i+1} = P_i - F_{int}\left[\frac{M}{(\Delta t)^2} - \frac{C}{2\Delta t}\right] u_{i-1} + \left[\frac{2M}{(\Delta t)^2}\right] u_i \tag{18}$$

From Equation 16, it is evident that u_0 and u_{-1} are needed to determine u_1. The initial displacement u_0 is known. For $i = 0$,

$$(\dot{u_0}) = \frac{u_1 - u_{-1}}{2\Delta t} \tag{19}$$

$$(\ddot{u_0}) = \frac{u_1 - 2u_0 + u_{-1}}{(\Delta t)^2} \tag{20}$$

solving for u_{-1} from Equation 19 and substituting in Equation 20,

$$u_{-1} = u_0 - \Delta t(\dot{u}_0) + \frac{(\Delta t)^2}{2}\ddot{u}_0 \tag{21}$$

The Equation of motion at time 0 shown in Equation 22, gives the acceleration in Equation 23,

$$M\ddot{u}_0 + C\dot{u}_0 + F_{int0} = P_0 \quad (22)$$

$$\ddot{u}_0 = \frac{P_0 - C\dot{u}_0 - F_{int0}}{M} \quad (23)$$

In this analysis a time step of 0.00005 is used for a solution convergance.

6 Results

Frames under different loading conditions are analysed in this paper. A pair of springs is considered to have failed when either one of the normal or shear spring fails. Also, at this stage, the contact between elements is not considered. The external forces applied to the frame is: a point load at the column, gravity effect, and a seismic load.

Consider a simple frame, with bases fixed in all three degrees of freedom. The frame has a span of 5 metres for both the columns and the beam. Ten elements per structural member is used.

The geometry of the frame is obtained from Ansys Mechanical APDL, and the remaining analysis on the developed MATLAB Code. The loading conditions on the frame is a point load in the x-direction at the top left corner, the 1940 El Centro earthquake data, and gravity. The material and geometry properties are shown in Figure 4 and Table 2.

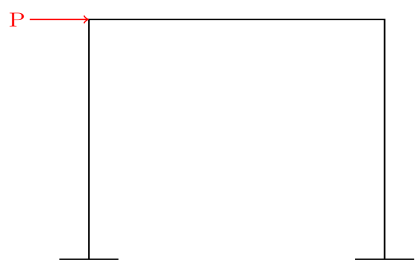


Figure 4: Frame with point load

Paramters	Value	Units
Modulus of Elasticity E	200,000	[MPa]
Shear Modulus of Elasticity G	76,923	[MPa]
Yield Stress	250	[MPa]
Beam span	1	[m]
cross-section width	0.15	[m]
cross-section thickness	0.15	[m]
Applied Load	1000	[N]

Table 2: Section Properties

Figure 5 displays the displacement, velocity and acceleration of the top left point of the frame where the frame is loaded, while Figure 6 is the displacement, velocity and acceleration after fracture occured. When collapse occured the element was totally free and this is the reason for the excitation in the acceleration. Figure 7 displays the deflection of the frame at every 0.05 seconds, until collapse occurs. Figure 8 shows the displacement time history of a multi-storey frame (40 m high) with a point load applied at the first storey in the x-direction, along with the seismic load. As can be seen, collapse occured at t=0.8 seconds. The frame was initially propagating due to the seismic load, and finally when damage occured at the bottom storey, the damage propagated progressively to the remainder of the building causing total collapse. The failed elements that have hit the ground are not shown in the figure.

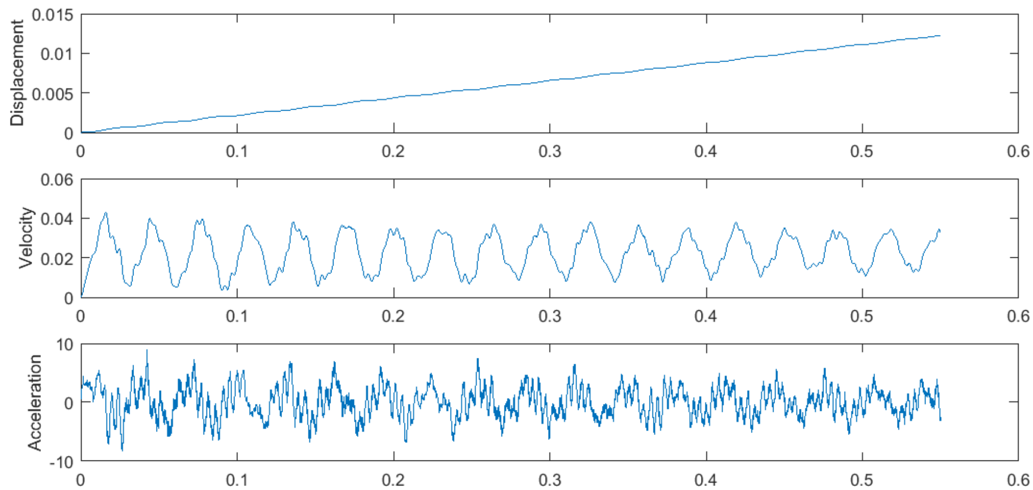


Figure 5: u,v,a before fracture

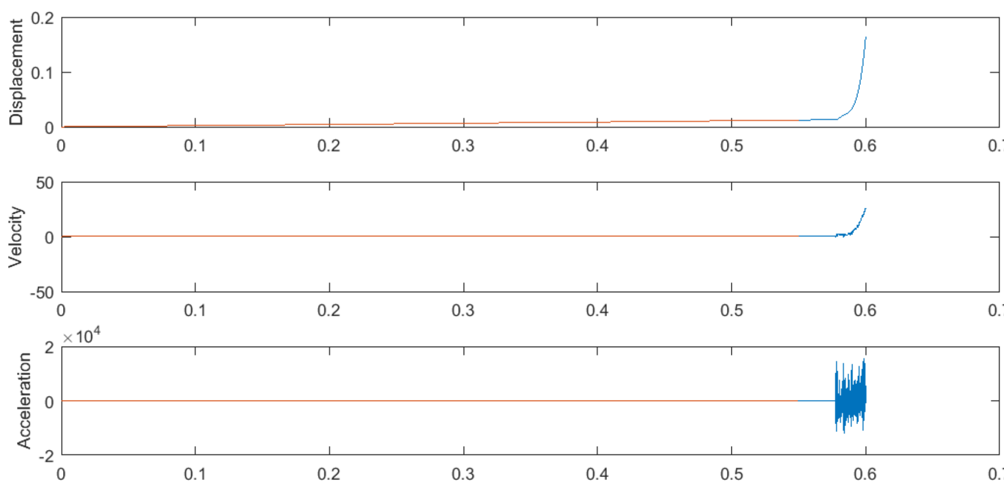


Figure 6: u,v,a after fracture

7 Conclusion

The conclusions of the study presented are as follows; (1) the Applied Element Method is a suitable and straight-forward method for modelling the damage and fragmentation of structures(2) the Gaussian based AEM is computationally less expensive than the conventional AEM (3) the developed return mapping algorithm displays a softening material behaviour that can easily represent material fracture. Future work entails expanding the collapse to larger structures, applying extreme loading to simulate extreme weather conditions and including contact between elements.

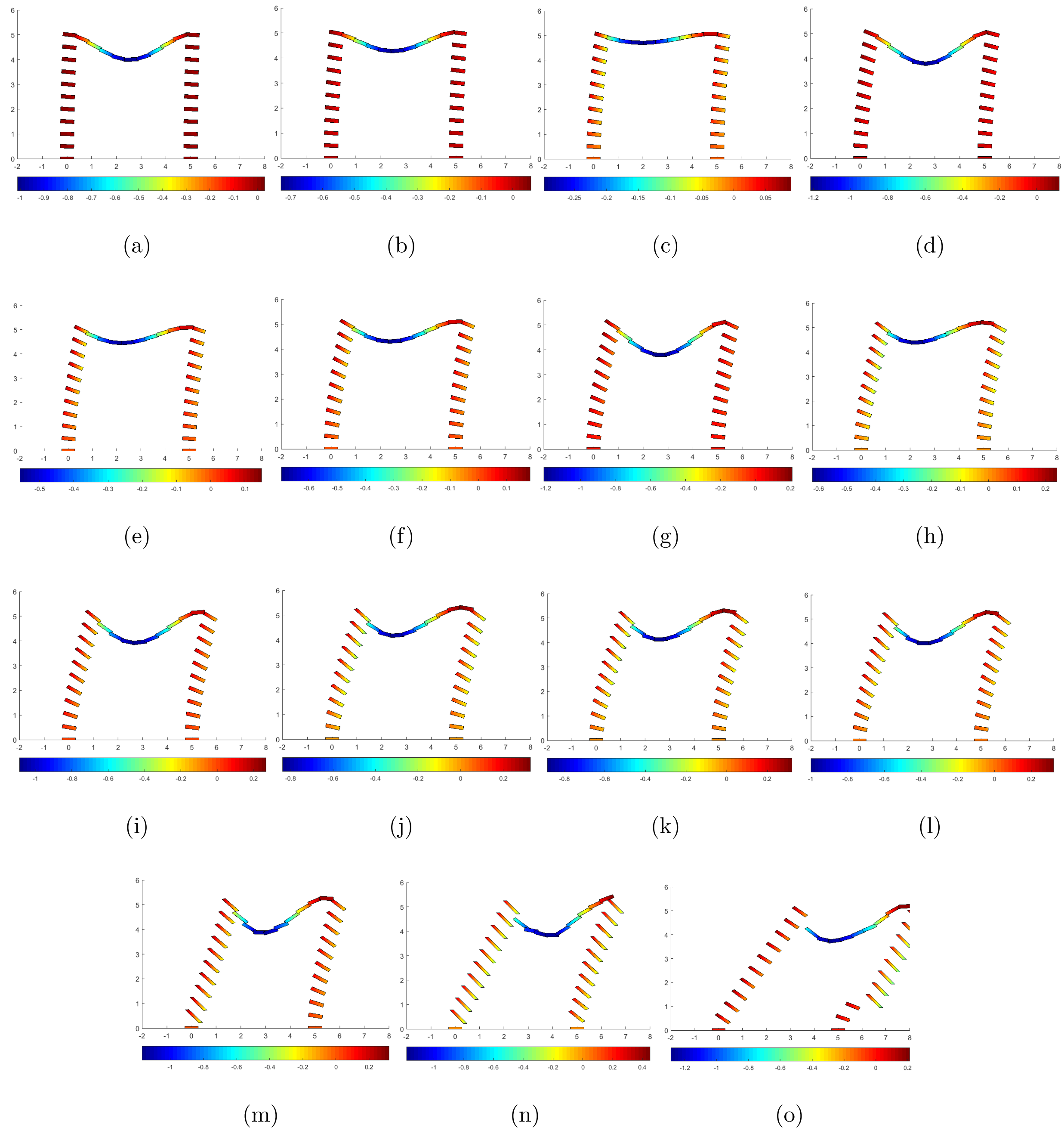
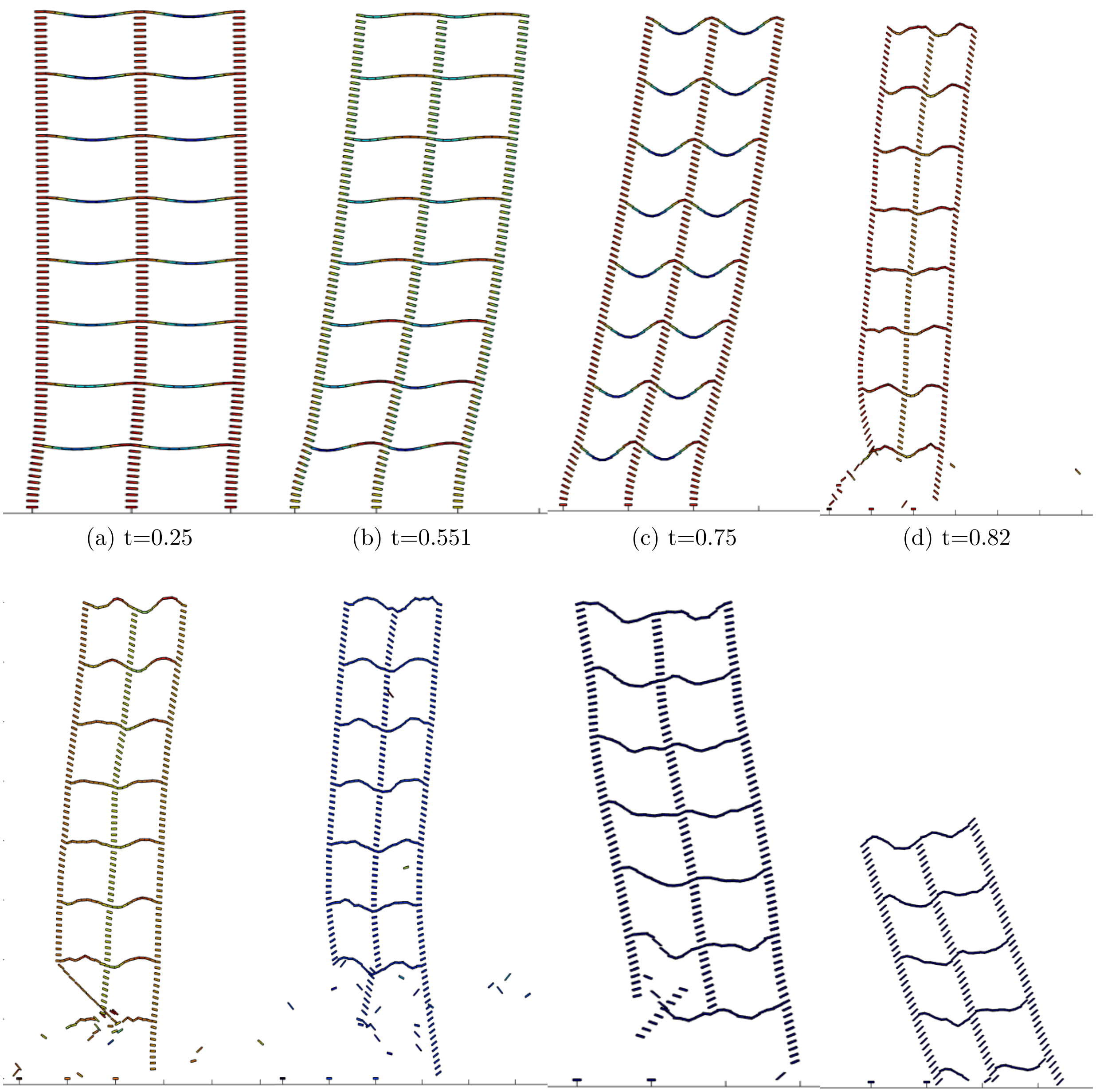


Figure 7: 1x1 frame undergoing seismic loading and point load at top left corner



(a) t=0.25 (b) t=0.551 (c) t=0.75 (d) t=0.82

(e) t=0.83 (f) t=0.84 (g) t=0.9 (h) t=1

Figure 8: Time history of a high rise frame undergoing seismic loading and point load at in positive x-direction at first floor

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