SURROGATE MODEL BASED RELIABILITY ANALYSIS FOR FUZZY CROSS-CORRELATED RANDOM FIELD MATERIAL DESCRIPTION

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Abstract. Heterogeneous materials can be described by random fields (RF), which depend on an auto-correlation function. Material interdependencies can be taken into account using cross-correlated RFs. Usually, these RF parameters (auto- and cross-correlation) are only vaguely known. In this contribution they are described by convex fuzzy sets yielding a polymorphic uncertainty approach, which combines basic uncertainty models into one variable for a more realistic model of uncertain input data. An appropriate reliability analysis scheme is introduced, which is based on a surrogate model, approximating the failure probability while reducing the computational costs. A hydro-mechanical coupled system of a masonry gravity dam cross-section, implemented as a 2D plain strain finite element model, serves an an application example. The resulting failure probability fuzzy sets using the surrogate are found to be in good agreement with re-calculated failure probabilities. The computational costs are reduced by a factor of several tens. A newly investigated domain decomposition approach suggests an effective reliability estimation even for large finite element models.

1 INTRODUCTION

Different concepts of uncertainty have been developed to describe input data of systems realistically, depending on the level of available information or knowledge. Basic approaches like probabilistic, fuzzy or interval uncertainty description are used in structural engineering. Polymorphic uncertainty models take more than one of the described basic uncertainty characteristics into account for uncertain parameters [1, 2]. Spatially heterogeneous material parameters (e.g. geo-science or aggregate materials) can be described by random fields (RF), where an auto-correlation of the spatially varying material parameters is incorporated. Karhunen-Loève expansion is widely used for RF generation. In multi-field situations, e.g. coupled thermal-hydro-mechanical systems like dams, dikes or geologic deposits, there are a series of sensitive heterogeneous material properties, which exhibit a visible cross-correlation [3]. These cross-correlated heterogeneous material properties can be modeled using cross-correlated random fields [4]. Both the auto-correlation function and the cross-correlation are usually barely known [3]. Therefore, this paper suggests a polymorphic uncertainty model based on cross-correlated random fields, where both the spatial auto-correlation and the cross-correlations (RF parameters) are modeled using a fuzzy approach, yielding fuzzy cross-correlated RFs. General aspects of fuzzy random fields or processes have been reported in [5, 6].

Reliability analysis procedures demand high number of system evaluations, which are usually computationally costly. Additionally, RF description of heterogeneous materials cause a high-dimensional stochastic input space. Therefore, reliability algorithms that are suitable for a high-dimensional input space have to be used [7], for example subset simulation. For the suggested fuzzy-probabilistic approach, an α-level formulation is typically used used as a discretization scheme leading to an α-level optimization scheme to obtain the results [8]. The high computational costs of such optimization tasks can be remedied using surrogate models. For a high-dimensional polymorphic input space, the generation of system response surrogate models is very challenging. In this paper, a surrogate model, approximating failure probabilities directly, is used. Hence, the design space of the surrogate model is drastically reduced to the uncertain RF parameters, which are modeled as fuzzy sets.

A reliability analysis scheme for polymorphic uncertain input data is introduced. The proposed procedure is applied to an hydro-mechanical coupled system of a gravity dam cross-section, which is implemented as a FE model. The resulting fuzzy failure probabilities obtained by the surrogate model are verified with failure probabilities determined with the original FE model.

2 METHOD

We introduce a structural reliability analysis scheme for a structural system with material description by fuzzy cross-correlated RF. The reliability analysis scheme comprises two main blocks, which are illustrated in Fig. 1: First, the creation of a surrogate model that approximates the failure probability $\hat{p}_F$ as a function of the RF parameters (correlation length θ and cross-correlation coefficients ρ_{ij}). Second, the execution of an α-level optimization on the surrogate model to find the minimum and maximum failure probabilities $\hat{p}_F$ on different α-levels for all input fuzzy sets.

2.1 Polymorphic uncertain material model

In this study, cross-correlated random fields (RF) are used to describe heterogeneous material parameters of the system. The RF parameters of the cross-correlated random fields, i.e. the correlation length θ and the cross-correlation values ρ_{ij}, are barely known for geo-science or masonry materials like the ones found in old gravity dams. Therefore, as one possible approach to comply with the vague information, fuzzy sets are defined to describe these parameters. Fuzzy theory have been introduced in [9]. Hence, for the RF parameters fuzzy sets are defined to describe these parameters. In this study, without loss of generality, only trapezoidal fuzzy set membership functions are considered. This yield a material description by fuzzy cross-correlated RF.

The N_F cross-correlated RFs Z_i ($i = 1, \dots, N_F$) are generated using an algorithm reported in [4]. It is based on the Karhunen-Loève expansion (KLE). The RFs are described by the auto-correlation structure and the point-wise cross-correlation coefficients

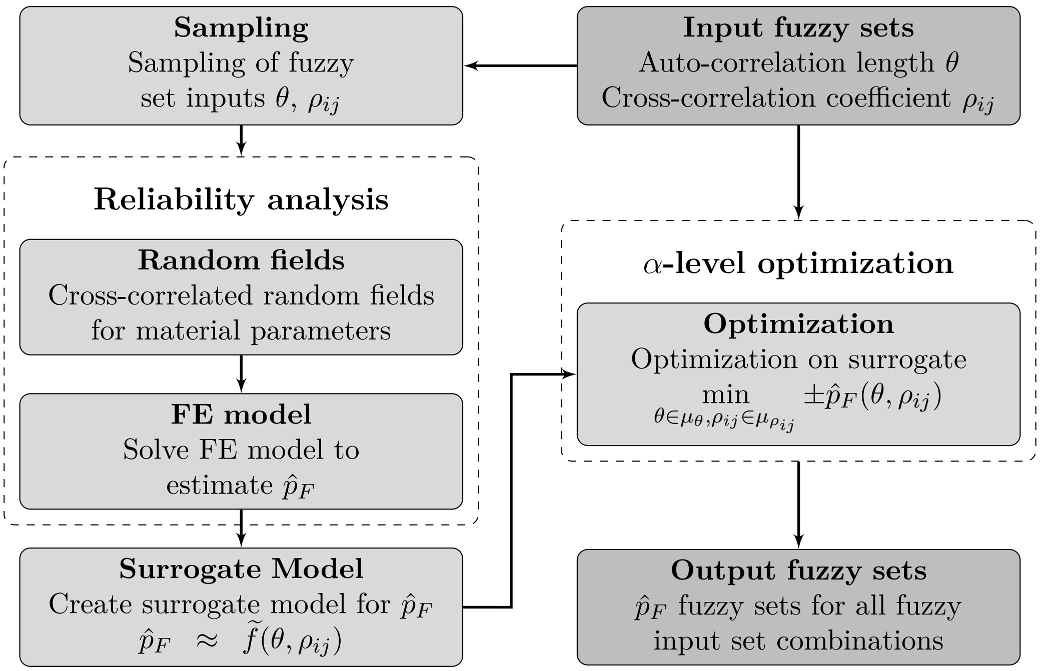


Figure 1: Scheme of a reliability analysis for polymorphic cross-correlated random field material description using a failure probability surrogate model

ρ_{ij}. The auto-correlation structure is discretized into the auto-correlation matrix $[\mathbf{C}]$ whereas the cross-correlation matrix $[\mathbf{\Gamma}]$ is formed by the cross-correlation coefficients $[\mathbf{\Gamma}]_{ij} = \rho_{ij}$ describing the point-wise cross-correlation between RFs Z_i and Z_j. A decoupled eigendecomposition of $[\mathbf{C}]$ and $[\mathbf{\Gamma}]$ enables an independent updating of the auto- or cross-correlation matrix, for changing θ or ρ_{ij}. For non-Gaussian fields, the performed Nataf transformation requires an appropriate correction of both the auto- and the cross-correlation [10, 4].

2.2 Surrogate model for failure probability

Within the proposed reliability analysis scheme (especially in the α-level optimization), the failure probability has to be estimated for a high number of RF parameter sets, where each reliability analysis needs a large number of FE system evaluations. Therefore, a surrogate model is introduced to approximate the failure probability $\hat{p}_F$ as a function of the RF parameters: $\hat{p}_F \approx \widetilde{f}\left(\theta, \rho_{ij}\right)$. In this contribution a Generalized Linear Regression Model (GLRM) with a binomial error term is considered [11], which ensures $\widetilde{f} \geq 0$. The sampling domain boundaries are determined by the minimum and maximum values of the corresponding fuzzy set membership functions for θ and ρ_{ij}. In order to ensure a positive definite cross-correlation matrix $[\mathbf{\Gamma}]$ for all support point samples, the "projection method" introduced in [12] is used to shift the support point to the nearest positive definite cross-correlation matrix, if necessary.

2.3 α-level optimization

The α-level optimization is used to estimate the resulting fuzzy set membership functions of failure probabilities $\mu(\hat{p}_F)$ for the input fuzzy sets of the RF parameters. This is accomplished by determining the minimum and maximum failure probability $\hat{p}_{F,\mathrm{mn}}$ and $\hat{p}_{F,\mathrm{mx}}$ at n_α α-cuts where $\alpha \in [0,1]$. At each α-level a constrained optimization is carried out to find the minimum ("best case") and maximum ("worst case") failure probability, respectively. In a one-dimensional case, where $\hat{p}_F$ only depends on parameter θ, it follows:

$$\hat{p}_{F,\mathrm{mn}} = \min_{\theta \in \mu_\alpha} \hat{p}_F(\theta) \quad \text{and} \quad \hat{p}_{F,\mathrm{mx}} = \max_{\theta \in \mu_\alpha} \hat{p}_F(\theta), \quad \text{where } \mu_\alpha = \{\theta \in \mathbb{R}^+ | \mu(\theta) \geq \alpha\}. \tag{1}$$

In the multi-dimensional case, this optimization problem is constrained by the combination of fuzzy sets defining the parameters θ and ρ_{ij} in an analogue manner.

3 NUMERICAL EXAMPLE

In the presented study, the introduced reliability analysis scheme was applied to a hydro-mechanical system of a gravity dam cross-section and its implementation as a FE model.

3.1 Model description

We consider the fluid (water) flow in a deformable porous saturated media. The governing equation and discretization into for FE can be found in [13]. The primary mechanical variables are the displacements u_x and u_y as well as the pore water pressure p for the hydraulic process. We assume a constant water density along the motion of water. A linear elastic behavior of the solid phase (Hooke's law) is assumed as well as an isothermal situation. For the considered isotropic case the permeability tensor is sufficiently described by a scalar permeability k_p. The linear elastic isotropic constitutive matrix is only a function of the Young's modulus E and the Poisson's ratio ν and can be found in various textbooks. A plain-strain situation is used for the dam cross-section model.

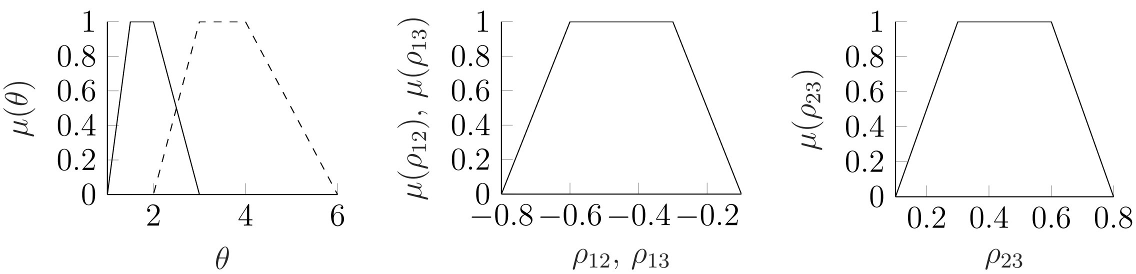
A two-dimensional FE model was chosen to simulate a cross-section of the Fürwigge gravity dam The geometry of the simulated cross-section of the Fürwigge gravity dam was simplified to a vertical upstream side and a bi-linear downstream side [14]. The foundation was fixed and the maximum water level was assumed. Surface loads according to the water and air pressure acting normal to the respective surface were specified in the mechanical domain. The body load was applied due to solid $\rho^s = 2600\,\mathrm{kg/m^3}$ and water $\rho^w = 1000\,\mathrm{kg/m^3}$ mass density, taking into account the porosity n. The gravitational driven water flow constituted the body load in the hydraulic domain. The applied flux in the hydraulic domain was set to zero. As a compromise between mesh convergence and computational performance a uniform mesh with around 1100 linear triangle elements was created using a Delaunay distance mesher.

Material parameters

Three spatially heterogeneous material parameters were chosen: Young's modulus E, the permeability k_p and the porosity n. They were described by fuzzy cross-correlated random fields, discretized at the triangular FE centers, i.e. the Gauss points. An identical isotropic exponential auto-correlation structure was defined for all fields. The truncation of the eigenvectors and eigenvalues obtained by the KLE was determined by the lowest possible correlation length θ, which was defined by the fuzzy sets in Fig. 2a. Log-normal distributions were assumed for the fields. Their respective means and standard deviations are based on literature values [14, 3]. The mean of Young's modulus E, permeability k_p and porosity n was set to $\mu_{\mathrm{E}} = 2.4\,\mathrm{GPa}$, $\mu_{k_p} = 1 \cdot 10^{-14}\,\mathrm{m}^2$ and $\mu_{\mathrm{n}} = 0.2$, respectively. The coefficient of variation (CoV) was fixed to 10 % for all. The Poisson's ratio was defined as $\nu = 0.15$.

Fuzzy sets for random field parameters

The correlation length was represented by two fuzzy sets ranging from a short to a long distance spatial correlation behavior, cf. Fig. 2a. The cross-correlation ρ_{12} (Young's modulus E and permeability k_p) and ρ_{13} (Young's modulus E and porosity n) can be denoted as "medium negatively" correlated, c.f. Fig. 2b. The cross-correlation ρ_{23} between k_p and n is positive instead, c.f. Fig. 2c, respectively. In this study the fuzzy set membership functions are discretized into $n_\alpha = 6$ equally distributed α levels for $\alpha = [0, 1]$ for $n_{\mathrm{fc}} = 2$ fuzzy set combinations.



(a) Auto-correlation length **(b)** Cross-correlation of E and k_p, E and n **(c)** Cross-correlation of k_p and n

Figure 2: Fuzzy set membership functions defining the variation of the cross-correlated random fields' parameters θ (a) and ρ_{ij} (b)-(c) associated with the material parameters Young's modulus E, permeability k_p and porosity n

Reliability limit state and surrogate model

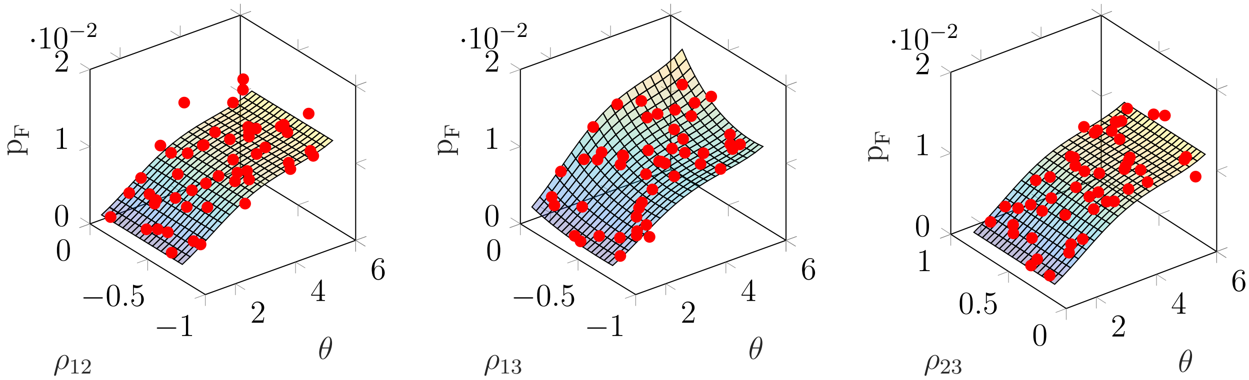
In this paper, the maximum horizontal displacement $u_{x,\mathrm{mx}}$ was chosen as a representative quantity of interest (QoI) with a horizontal displacement limit of $u_{x,\mathrm{lim}} = 21.35\,\mathrm{mm}$. For each RF parameter set $\{\theta, \rho_{ij}\}$ a minimum number of $n_S = 1000$ and a maximum number of $n_{\mathrm{S,mx}} = 10\,000$ realizations of cross-correlated RFs are generated and used as material parameter input for the FE model.

The GLRM surrogate model for the failure probability $\hat{p}_F$ corresponding to the maximum horizontal displacement $u_{x,\mathrm{mx}}$ was created with $n_{\mathrm{S,meta}} = 50$ support points using an adaptive regression scheme to prevent over-fitting.

3.2 Results and discussion

The reliability analysis was carried out according to the proposed scheme. The first step was to generate a surrogate model for failure probability $\hat{p}_F$ as a function of the random field parameters θ and ρ_{ij}. A parallelized execution on a computational cluster was used to estimate failure probabilities for different sets of RF parameters simultaneously. The subsequent α-level optimization could then be performed more efficiently on the surrogate model.

Response surfaces



(a) **(b)** **(c)**

Figure 3: Surface plots of the GLRM surrogate model for $u_{x,\mathrm{mx}}$ in the θ-ρ_{12}, θ-ρ_{13} and θ-ρ_{23} planes, respectively; dots represent 50 support points used as data basis

Surface plots of the GLRM surrogate model in the θ-ρ_{12}, the θ-ρ_{13} and the θ-ρ_{23} plane can be seen in Fig. 3. It can be noticed that the failure probability $\hat{p}_F$ increases with increasing correlation length θ. Especially in the long range correlation length regime, an increase of the failure probability can be observed for less negatively correlated material parameters E and porosity n, i.e. increasing ρ_{13}, cf. Fig. 3b. On the contrary, almost no dependence of the failure probability on the cross-correlation coefficients ρ_{12} can be observed.

Failure probability fuzzy sets

The surrogate model was used to carry out the α-level optimization. In average, one optimization using an interior-point algorithm [15] took ≈ 20 iterations and ≈ 100 objective function evaluations. The resulting fuzzy sets are plotted in Fig. 4a with their corresponding input fuzzy sets of the RF parameters θ and ρ_{ij} in Fig. 4b,c, respectively.

It is noticed, that the α-level optimization yield two distinct failure probabilities, describing the "best" (minimum $\hat{p}_F$) and "worst" case (maximum $\hat{p}_F$) scenarios on all

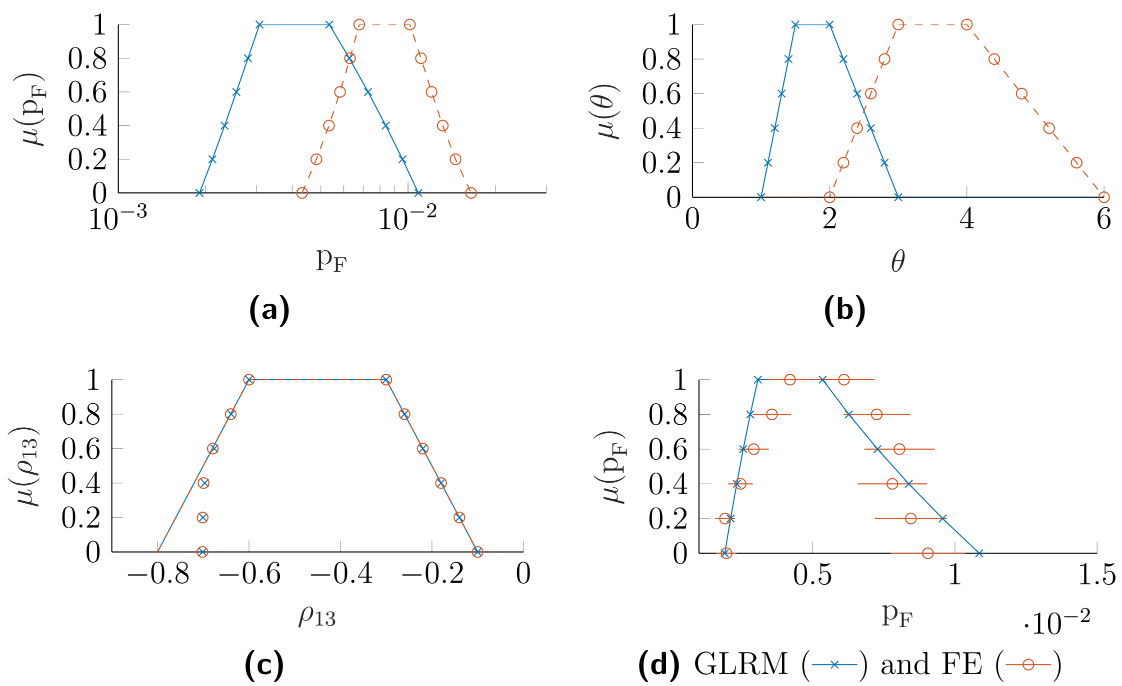


Figure 4: (a) Resulting fuzzy sets of failure probability $\hat{p}_F$ for $u_{x,\mathrm{mx}}$ derived from the GLRM surrogate model for both fuzzy set combinations $i_{\mathrm{fc}} \in \{1, 2\}$ (—×—, - ○ -); corresponding fuzzy sets of correlation length θ (b) and cross-correlation coefficients ρ_{13} (c); (d) Comparison to re-calculated failure probabilities of the FE model (—○—) for fuzzy set combination $i_{\mathrm{fc}} = 1$; horizontal bars indicate confidence intervals

6 α-levels for the two introduced fuzzy set combinations (short and long range auto-correlation), cf. Fig. 4a. Apparently, smaller failure probabilities are found in the case of long range auto-correlation for the used FE model configuration. While the corresponding maximum and minimum points of RF parameters θ, ρ_{ij} are found mostly on the edges of the specified fuzzy set membership functions for θ (cf. Fig. 4b) and ρ_{13} (cf. Fig. 4c), one observes, that the $\hat{p}_F$ extrema are found at the fuzzy set means for ρ_{12} and ρ_{23} (no figures).

In order to evaluate the quality of the resulting failure probability fuzzy sets derived from the surrogate model, the failure probabilities were re-calculated using the FE model at the optimized RF parameter sets $\theta_{\mathrm{mn/mx}}$ and $\rho_{ij,\mathrm{mn/mx}}$, that resulted from the α-level optimization on the surrogate models. An adaptive MCS scheme was used and the fuzzy sets derived from the GLRM surrogate model and the MCS re-calculation were found to be in good agreement, cf. Fig. 4d.

4 DOMAIN DECOMPOSITION

4.1 Method

Stochastic finite element systems usually are accompanied by high computational costs. Especially in the case of large multi-field coupled system with a large number of degree of freedoms (DOF), parallel computation approaches are necessary. One common approach is domain decomposition of a global domain into D smaller (non-)overlapping subdomains. In this contribution, we follow the approach from [16, 17] of non-overlapping subdomains Ω_d.

4.1.1 Random field by local KL

In the following, a brief summary of [16] is given for a better understanding. The eigenfunctions of the correlation $C(x, x')$, restricted to domain Ω_d, are introduced as local eigenfunctions $\phi_\beta^{(d)}$ with their corresponding eigenvalues $\lambda_\beta^{(d)}$. An approximation of the global modes is then given by

$$\Phi(x) \approx \hat{\Phi}(x) = \sum_{d=1}^{D} \sum_{\beta}^{m_d} a_\beta^{(d)} \phi_\beta^{(d)}(x), \tag{2}$$

where m_d is the number of dominant eigenpairs of subdomain Ω_d. We solve for $a_\beta^{(d)}$ using the Galerkin method, leading to a global discrete eigenvalue problem yielding the eigenvalue Λ_α and the entries $a_{\alpha,\beta}^{(d)}$ of the corresponding eigenvectors. A domain decomposition KL expansion in terms of the local eigenmodes is used to approximate the global RF U by

$$U \approx \hat{U}_{\hat{N}}(x, \omega) = \sum_{d=1}^{D} \left[\sum_{\beta=1}^{m_d} \sqrt{\lambda_\beta^{(d)}} \xi_\beta^{(d)}(\omega) \phi_\beta^{(d)}(x) \right], \quad \xi_\beta^{(d)}(\omega) = \sum_{\alpha=1}^{\hat{N}} \sqrt{\frac{\Lambda_\alpha}{\lambda_\beta^{(d)}}} a_{\alpha,\beta}^{(d)} \hat{\eta}_\alpha(\omega) \tag{3}$$

where $\eta_\alpha(\omega)$ denote the orthonormal stochastic coordinates of $\hat{U}_{\hat{N}}$ with $\hat{N}$ being the number of the dominant global eigenvalues Λ_α and $\{\xi_\beta^{(d)}, \beta = 1, \ldots, m_d\}$ are called the local random variables. Hence, the global random field U is represented on the subdomain level in terms of local random variables. The local stochastic dimension m_d is significantly reduced if the correlation length is around the subdomain size. This property is used in the following part regarding a FE solution.

4.1.2 FE solution

Any hydro-mechanical or structural FE model can be represented as a system of linear equations as

$$[\mathbf{A}]\, \mathbf{u} = \mathbf{b} \tag{4}$$

According to [17], we can distinguish between interior nodes $\mathcal{N}_{\text{in}}^{(d)}$ and boundary nodes $\mathcal{N}_\Gamma^{(d)}$, lying at the interface of more than one subdomain. The linear system in Eq. 4 can than be recast in the following form

$$\begin{bmatrix} [\mathbf{A}_{\boldsymbol{\Gamma},\boldsymbol{\Gamma}}] & [\mathbf{A}_{\boldsymbol{\Gamma},\text{in}}] \\ [\mathbf{A}_{\text{in},\boldsymbol{\Gamma}}] & [\mathbf{A}_{\text{in,in}}] \end{bmatrix} \begin{pmatrix} \mathbf{u}_\Gamma \\ \mathbf{u}_{\text{in}} \end{pmatrix} = \begin{pmatrix} \mathbf{b}_{\boldsymbol{\Gamma}} \\ \mathbf{b}_{\text{in}} \end{pmatrix} \tag{5}$$

which yields to a condensed problem at the subdomains' interfaces

$$\widehat{[\mathbf{A}]}\mathbf{u}_\Gamma = \widehat{\mathbf{b}} \tag{6}$$

The condensed operator can than be described by independent contributions of the subdomains. After solving the condensed problem for $\mathbf{u}_\Gamma$, the internal solution $\mathbf{u}_{\text{in}}$ for can be found by solving

$$\left[\mathbf{A}_{\text{in,in}}\right]\mathbf{u}_{\text{in}} = \mathbf{b}_{\text{in}} - \left[\mathbf{A}_{\text{in},\boldsymbol{\Gamma}}\right]\mathbf{u}_\Gamma \tag{7}$$

This step can also be performed independently for every subdomain.

For a stochastic problem, $\widehat{[\mathbf{A}]}$ and $\widehat{\mathbf{b}}$ become stochastic operators. In case of a direct MCS, they must be assembled for each sample, which is computationally demanding (cf. [17]).

4.1.3 Polynomial chaos expansion of the condensed problem

For large number of samples, the overall computational costs can be reduced by approximating the condensed operators by surrogate models using Polynomial Chaos Expansion (PCE) on a subdomain level [17]. The number of terms P in an expansion for N_c random variables and degree N_o is given by $\frac{(N_\mathrm{c}+N_o)!}{N_\mathrm{c}!+N_o!}$. It gets very large for large N_c, even for low degrees N_o. Now the property of the representation of the RF by local random variables can be exploited (Eq. 3). The low dimensionality in each subdomain enables the construction of PCE surrogates with reasonable computational effort.

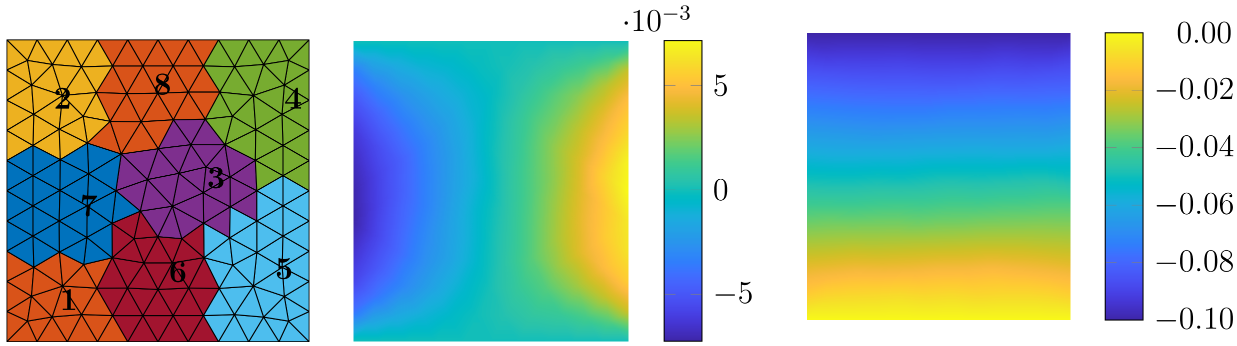
4.2 Reliability analysis

Reliability analysis of large stochastic finite element models is computationally demanding and a direct MCS approach is usually not feasible. Global surrogate models like PCE or Low Rank Approximation (LRA), approximating the model responses, have been investigated for example in [18]. Usually, such surrogate models used in reliability analysis tends to yield deteriorating results for decreasing failure probabilities. This is mainly due to the poor approximation of the system's response pdf in the low probability tails. The idea of PCE surrogate modeling was adopted to the described domain decomposition approach. Exploiting the low stochastic dimensionality on the subdomain level, PCE surrogates are constructed independently on each subdomain for the operators of the condensed problem, cf. Eq. 6 [17].

In this contribution, the effect of direct MCS, PCE surrogate modeling and a hybrid approach is investigated. Direct MCS is performed without any surrogate approximation of the local condensed problem and serves as a reference. An all-PCE approach is realized, where all local condensed operators are approximated using PCE. The hybrid approach assumes, that for a reasonable variation of material parameters, one can usually identify a critical region, where the determining quantity with respect to reliability is most critical. PCE surrogates are constructed for all subdomains, except those in the vicinity of the critical region, whereas direct MCS is used for those critical subdomains. The usage of direct MCS in the critical subdomains is supposed to remedy the affect of poor approximation of the low probability tails by PCE.

4.3 Example

As an demonstrative example, a 2D structural FE model on a unit square was used. Around 230 linear triangular elements constitutes the model with 270 degree of freedoms. The domain was divided into $D = 8$ evenly distributed subdomains, cf. Fig. 5a. The displacements are fixed on the lower edge $u_x(y = 0) = u_y(y = 0) = 0$, whereas $u_y(y = 1) = -0.1$. Additionally a vertical self-weight is applied as a body force. A random field is used to describe the Young's modulus in the domain. the domain decomposition approach described in Sec. 4.1 was used. The mean and standard deviation were chosen to $\mu_E = 2.4 \cdot 10^9$ Pa $\sigma_E = 0.24 \cdot 10^9$ Pa, the correlation length was set to 0.3.



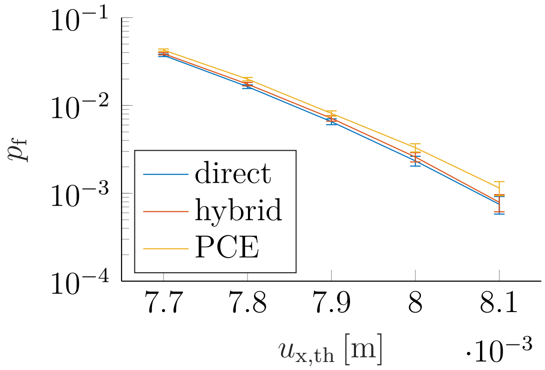
(a) **(b)** u_x [m] **(c)** u_y [m]

Figure 5: Domain decomposition of a 2D unit square domain into 8 subdomains

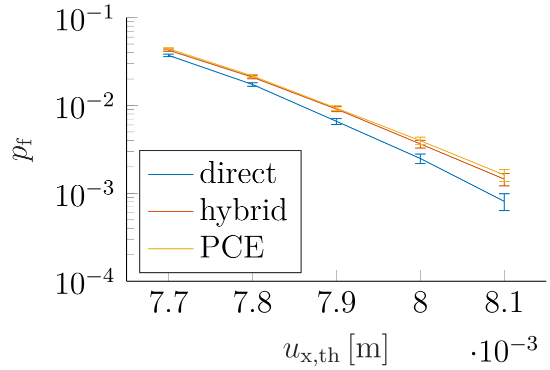
Exemplary displacement results are shown in Fig. 5b,c. It can be noticed, that the region of minimum horizontal displacement $u_{x,\text{mn}}$ lies in the subdomain 7. For this simple model, the three different reliability approaches have been performed. The total number of samples was set to $N_\text{S} = 100\,000$, whereas only the first $N_{\text{S,PCE}} = 100$ samples were used to train the PCE surrogate models in the subdomains. While PCE of degree $N_o = 2$ were used for all subdomains for the all-PCE approach, in the hybrid procedure subdomains $d_\text{dir} = \{1, 2, 7\}$ where calculated directly, whereas the other subdomains used a PCE surrogate to approximate the condensed operators.

Fig. 6 summarizes the results for different horizontal displacement limits $u_{x,\text{th}}$. The failure probabilities were estimated for the minimum ($u_{\text{x,mn}}$) and maximum ($u_{\text{x,mx}}$) horizontal displacement using three different thresholds $u_{x,\text{th}}$. This procedure was performed with all proposed methods: direct MCS (direct), hybrid and all-PCE (PCE), cf. Sec. 4.2. One can observe, that in the case of minimum horizontal displacement $u_{\text{x,mn}}$, the direct and hybrid method overlap, whereas the hybrid and PCE results coincide in the case of maximum horizontal displacement $u_{\text{x,mx}}$.

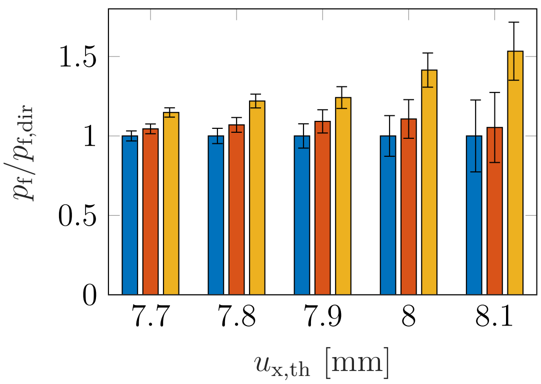
As we take the direct MCS method as a reference, it is obvious, that the all-PCE methods tends to overestimate the failure probabilities. This behavior is not uncommon for surrogate models, due to the poor low probability tail representation. In contrast to the all-PCE approach, it can be observed for the hybrid method, that the estimated failure probabilities are in good agreement with the reference for all considered displacement



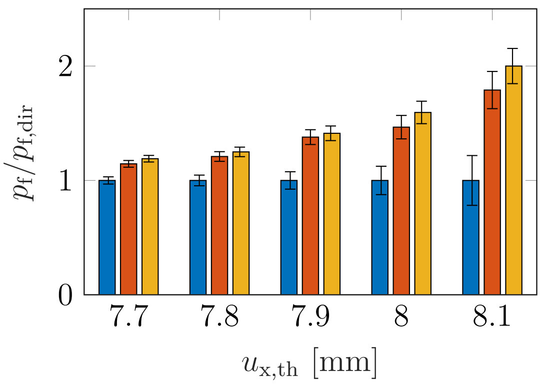
(a) $u_\mathrm{x,mn}$



(b) $u_\mathrm{x,mx}$



(c) $u_\mathrm{x,mn}$



(d) $u_\mathrm{x,mx}$

Figure 6: Failure probabilities including a 95 % confidence interval for minimum (a,c) and maximum (b,d) horizontal displacement for different horizontal displacement limits $u_{x,\mathrm{th}}$ calculated using three different methods: direct MCS, hybrid and all-PCE

limits in the case of the minimum displacement $u_\mathrm{x,mn}$, cf. Fig. 6a,c. Apparently, the direct MCS sampling in the subdomains near the critical region of minimum displacement ($d = 1, 2, 7$) compensates for the PCE approximation in the other subdomains. In the $u_\mathrm{x,mx}$ case the hybrid and all-PCE approach show similar results, cf. Fig. 6b,d, since the critical subdomains 4 and 5 are PCE approximated.

5 SUMMARY

Heterogeneous material parameters which show interdependencies can be described by fuzzy cross-correlated random fields (RF), which are based on cross-correlated random fields (RF), where the RF parameters (auto-correlation length and the cross-correlation coefficients) are described by convex fuzzy set membership functions. Performing a reliability analysis for systems with such a material description, the proposed reliability analysis scheme can be used resulting in failure probability fuzzy sets. The efficient procedure utilizes a surrogate model, which approximates the failure probabilities of the original system. Using only a relatively small number of support points for the surrogate

(50 for 3 material parameters), accurate results are obtained with an extremely reduced computational time (by a factor of ≈ 50). The domain decomposition approach shows promising results, which can be extrapolated to large complex models handled in an effective parallelized manner.

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