MULTI-MATERIAL LEVEL SET BASED TOPOLOGY OPTIMISATION OF CONVECTIVELY COOLED HEAT SINKS

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Abstract. This paper presents the application of Level-set based Topology Optimisation to a convectively cooled multi-material heatsink design problem. Two level set functions are used to model two solids and a single fluid with minimum thermal compliance considered as the optimisation objective. Both the level set functions are evolved by solving Hamilton Jacobi equations and are re-initialised at regular intervals. Topology optimisation is carried out for number of different fluid-solid conductivity ratios and, additionally, at a number of different solid volume fractions. Details of numerical modelling and results obtained are presented in this article.

1 INTRODUCTION

A heat sink is a structure intended to effectively transfer the heat received from a source into adjacent fluid medium through means of natural or forced convection. Heat sinks can be formed using either copper or aluminium alloys, with copper providing superior performance at higher cost. The use of both materials in an additively manufactured heatsink may provide an improved cost-performance trade-off over a single material design. The rapid development taking place in additive manufacturing, enables multi-material manufacturing, with spatial variation in material properties achieved through selective deposition. Topological optimisation (TO) techniques can be utilised to determine the optimal distribution of one or more materials within the given design space for the prescribed set of constraints [1]. The two most prevalent TO approaches are density method and Level-set (LS) methods, with the latter preferred in fluid flow problems due to the ability to sharply capture inter-material interfaces [2], [3].

Topology optimisation has been used for multi material structural optimisation for more than two decades [4]. TO is particularly useful for multi-material optimisation as it can simultaneously change the shape and layout of the materials. Sigmund [5] used the density method for TO of 3 phase thermal expansion materials. Wang [6] presented a level set based multi-material TO method for structural optimisation, wherein he used 'l' level sets for modelling 2^l distinct material phases. Furthermore, Wang [7] proposed a model wherein l-1 level sets are used to model 'l' phases of materials. This model has been adopted in this study

and applied to the design of two-material heat sinks for a number of different material conductivity and volume fraction ratios. A Level-set TO numerical model is formulated in Matlab and Comsol Multiphysics is used to solve the physics using Finite Element method. In this paper section 2 describes the two material level set TO formulation, section 3 describes the computational details. Results and discussion are given in section 4 and conclusions are given in section 5.

2 TWO-MATERIAL LEVEL SET TOPOLOGY OPTIMISATION MODEL

In this study, two Level-set functions (LSF) are used to model the two different solids and the fluid. A positive Signed Distance Function (SDF) (ψ_1) is considered to represent the solid and negative SDF (ψ_1) is considered to represent the fluid (Figure 1). A second level set function (ψ_2) is used to differentiate the two solids. Region where both ψ_1 and ψ_2 are positive represents solid2 and the other option indicates solid1 as illustrated in Figure 1.
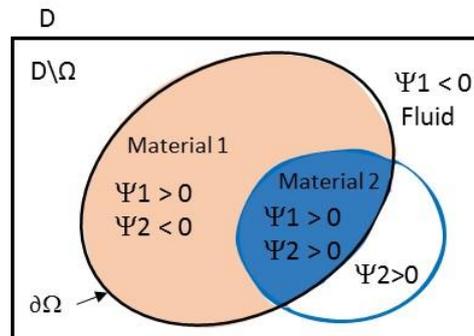

Figure 1: Design domain and level set function definitions

At any point within the design domain, the thermal properties k, Cp and ρ take values based on the values of ψ_1, ψ_2 and their corresponding Heaviside function (H) values (Table 1). H_1 and H_2 are the Heaviside functions which respectively correspond to ψ_1 and ψ_2. In this section, subscript 1 refers to a property corresponding to solid 1 and subscript 2 refers to a property of solid 2.

Table 1: Thermal properties interpolation formula in LSM

Design domain property	Name / Notation	Expression
Thermal conductivity	Kgam	$H_1*(H_2*ks_2+(1-H_2)*ks_1)+k_f*(1-H_1)$
Specific heat capacity	Cpgam	$H_1*(H_2*cps_2+(1-H_2)*cps_1)+cp_f*(1-H_1)$
Density	ρgam	$H_1*(H_2*\rho s_2+(1-H_2)*\rho s_1)+\rho_f*(1-H_1)$
Impermeability factor	α	$(\alpha_{max}-\alpha_{min})*H_1+\alpha_{min}$

In single material (1solid, 1 void) level set TO, level sets are convected by solving a

Hamilton Jacobi (HJ) equation. Similarly here, since two level set functions are used, two HJ equations are solved. The shape sensitivity of each of the LSF is calculated and the velocity of convection of LSF is equal to sum of shape sensitivity, Lagrange multiplier and area constraint terms are as per the Augmented Lagrangian method of optimisation.

$$\frac{\partial \psi_1}{\partial t} = V_{n1}|\nabla \psi_1| \tag{1}$$

$$\frac{\partial \psi_2}{\partial t} = V_{n2}|\nabla \psi_2| \tag{2}$$

The above HJ equations are solved using an explicit first order upwind scheme. The time step chosen for marching satisfies the CFL criterion for stability. Every time the physical problem is solved, the HJ equations are marched in time several time steps in order to obtain new shape or new level set functions. Velocity of convection of LSFs are obtained from augmented Lagrangian, as given below.

$$V_{n1} = F_1'(\Omega) + \lambda_1 + \lambda_2 H(\psi_2) + \Lambda_1(\int_\Omega H(\psi_1)d\Omega - V_1 * V_\Omega) \tag{3}$$

$$V_{n2} = F_2'(\Omega) + \lambda_2 H(\psi_1) + \Lambda_2(\int_\Omega H(\psi_1)H(\psi_2)d\Omega - V_2 * V_\Omega) \tag{4}$$

In the above equation $F_1'(\Omega)$, $F_2'(\Omega)$ are shape sensitivities, λ_1, λ_2 are Lagrangian multipliers and Λ_1, Λ_2 are volume penalty factors corresponding to Ψ_1 and Ψ_2 respectively. V_1, V_2 are volume constraints of total solid and solid2 and V_Ω is the design domain volume. The heat sink optimisation problem is stated as follows.

$$\text{Objective} \qquad F = \int_\Omega kgam * (\nabla T)^2 \, d\Omega \tag{5}$$

$$\text{Subjected to, } \rho(u.\nabla u) = -\nabla p + \nabla.\{\mu\{\nabla u + (\nabla u)^T\}\} - \alpha u \tag{6}$$

$$(\nabla.u) = 0 \tag{7}$$

$$\rho_{gam}C_{pgam}(u.\nabla T) = \nabla.(k_{gam}\nabla T) + Q \tag{8}$$

$$H(\Psi_1)u = 0 \tag{9}$$

$$\text{Volume constraint of total solid} = 0.40 * V_\Omega \tag{10}$$

$$\text{Volume constraint of solid2} = 0.20 * V_\Omega \text{ or } 0.32 * V_\Omega \tag{11}$$

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Shape sensitivities are obtained by differentiating the objective function with respect to LS functions.

$$F_1'(\Omega) = (H_2 * ks_2 + (1 - H_2) * ks_1 - k_f) * \delta_1 * (\nabla T)^2) \tag{12}$$

$$F_2'(\Omega) = (ks_2 - ks_1) * H_1 * \delta_2 * (\nabla T)^2) \tag{13}$$

Where δ_1 and δ_2 are Dirac-delta functions and they are derivatives of H_1 and H_2 respectively. The Lagrangian multiplier and volume penalty factor are updated as follows.

$$\lambda_k = \lambda_{k-1} - \Lambda_{k-1} (Voume\ Difference) \tag{14}$$

$$\Lambda_k = \frac{1}{\beta} \Lambda_{k-1} \tag{15}$$

The initial value of Lagrangian multipliers, and the area penalty factors are chosen appropriately. Both the LSFs are re-initialised at regular intervals by time marching the Eikonal equation given in Eqn. (16) and (17).

$$\frac{\partial \psi}{\partial t} + w . \nabla \psi = S(\psi o) \tag{16}$$

$$w = S(\psi o) \frac{\nabla \psi}{|\nabla \psi|} \tag{17}$$

Where S is the smoothed sign function. For details on solving the HJ equation and Eikonal equation, the reader may refer to [2] and [3] and for more details on numerical implementation of level set topology optimisation using Comsol and Matlab refer to [8] and [9].

3   COMPUTATIONAL DETAILS

The Level-set method numerical model is implemented for heat sink design using Comsol and Matlab. Comsol5.2 is used for solving the fluid flow and heat transfer while Matlab is used for solving the HJ equation and for re-initialisation of level set functions. The application of TO to heatsink design has been studied by number of researchers. Some of the notable works on single material heat sink design are Alexanderson [10] (density method) and works of Yaji [11] and Coffin [12] (LS method). Zhuang [13] presented a method for multi-material TO of heat conduction problems based on colour-level set approach, evaluating the shape sensitivity using the adjoint method. In the current two-material TO study, the material interface between two solids is assumed to be perfectly bonded. Michailidis [14] gives a description of different methods for modelling the material interface with relevant numerical examples.

The design domain is rectangular in shape, with heat source at the bottom of the domain and

liquid convection injected from the top of computational domain as shown in Figure 2. The two sides of the computation domain act as outlet.
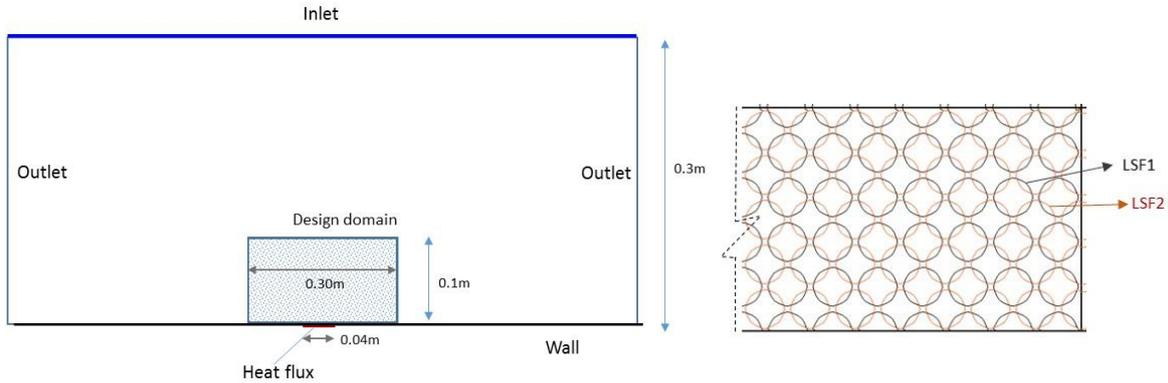

Figure 2: Computational domain and initial level set functions on the design domain

Design domain is discretised with 150x50 rectangular elements. The initial level set used for the computation is series of circles as shown in Figure 2. The level set function is evolved on a grid mesh with ghost elements. A liquid flow of velocity 0.002m/s and temperature 293K is applied at the Inlet. The inlet velocity corresponds to a Reynolds number of 600 and a heat flux of 3500W/m^2 is specified as heat source in the bottom wall and zero pressure boundary condition is applied at the outlet. The properties of solid and fluid used in this study are presented in Table 2.

Table 2: Material Properties

Property	Value
C_{ps1}	385 J/(kgK)
C_{ps2}	770 J/(kgK)
C_{pf}	4184 J/(kgK)
ρ_{s1}	8920 kg/m^3
ρ_{s2}	4460 kg/m^3
ρ_f	1000 kg/m^3
Volume fraction	0.4
Solid2 volume fraction	0.2 and 0.32
α_{max}	1e4

TO is carried out totally for 6 cases and the thermal conductivity and volume fractions of solid and fluid for each of the cases are listed in Table 3. Two different conductivity ratios are studied between the solids, i.e., 10 and 2. Also two different conductivity ratio is studied between the solid1 and fluid they are 1000 and 100. Total solid volume is constrained at 40% of design domain volume. Among the two solids, two different ratios are considered for solid1 to solid2 material volume, they are 50:50, and 20:80.

Typical convergence history of a two-material TO run is given below (Figure 3). At convergence, area constraint of both the solids are satisfied and both area and thermal compliance remains stationary.

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Table 3: Thermal conductivity values at different simulations

	$V_{solid1}=0.2V\Omega$, $V_{solid2}=0.2V\Omega$				$V_{solid1}=0.08V\Omega$, $V_{solid2}=0.32V\Omega$	
	Case1	Case2	Case3	Case4	Case5	Case6
k_{s1}	400	400	400	400	400	400
k_{s2}	40	200	40	200	40	200
k_f	0.4	0.4	4	4	0.4	0.4
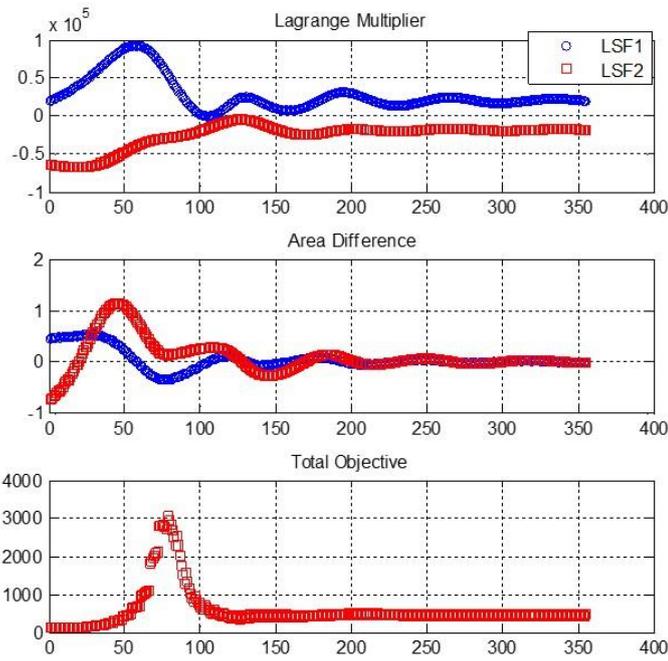
Figure 3: Convergence history of two material topology optimisation

4   RESULTS & DISCUSSION

The results obtained for equal and disparate solid volume ratios are segregated and presented in the following sections.

4.1 Equal solid volume ratio study

The optimised shape obtained for different cases along with their temperature distributions are given in Figure 4 to 7. The objective value achieved and maximum temperature in the design domain are tabulated and given in Table 4. The following points are observed from the results.

1. When the thermal conductivity ratio between solid1 to solid2 is 2, the resulting design resembles a tree-like dendritic structure with discrete/unconnected regions of the two materials. Whereas, if the thermal conductivity ratio is higher say, 10, the solid1 tends to have a branched structure and solid2 is mostly distributed near the centre to evenly distribute the heat in the design domain.

2. When solid to fluid thermal conductivity ratio is 100 (that is when fluid thermal conductivity is 4W/m/k), heat sink tends to have more flat base than the case with conductivity ratio 1000.

3. Optimised shape obtained for case4 is non-intuitive in nature with highly conductive solid placed at specific places discretely and fluid gaps are present within the heat sink structure. Formation of design with fluid gaps inside a solid is the drawback of modelling solid using porosity approach.
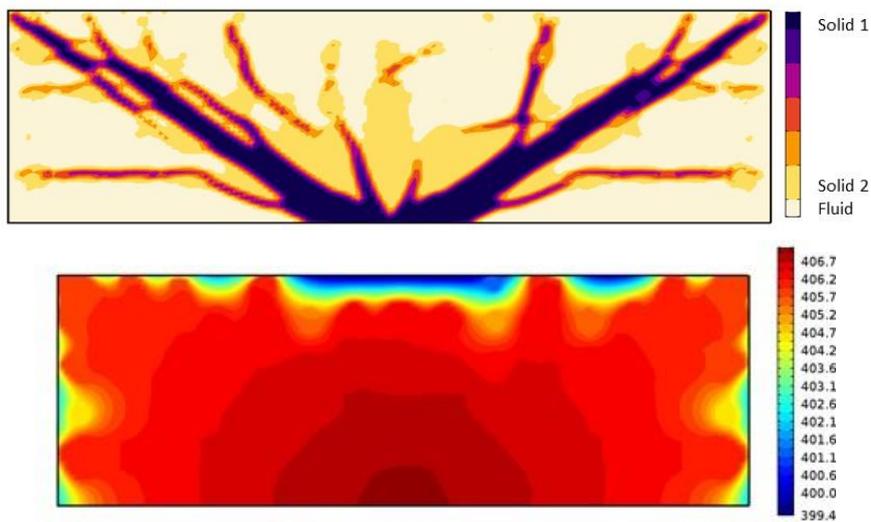

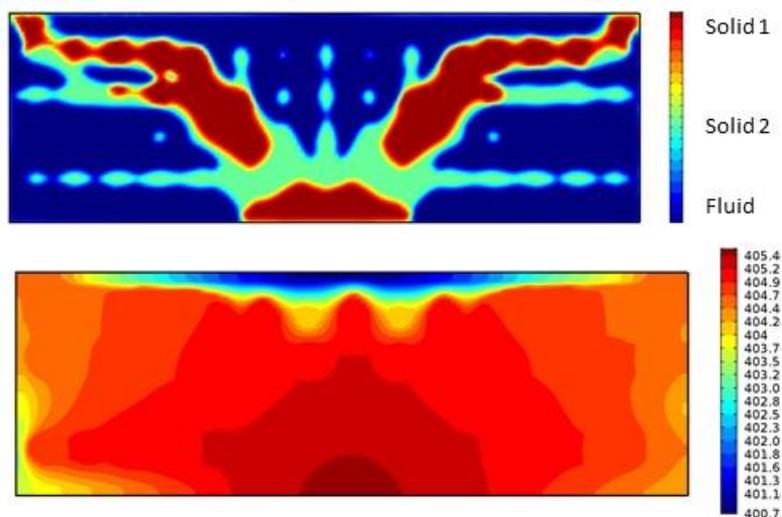**Figure 4**: Optimised shape and the Temperature contour (K) for Case1

Figure 5: Optimised shape and the Temperature contour (K) for Case2
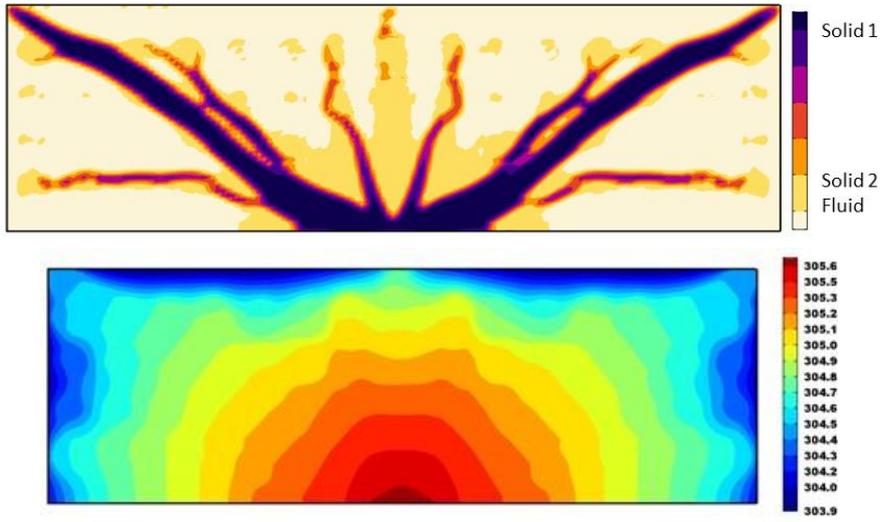

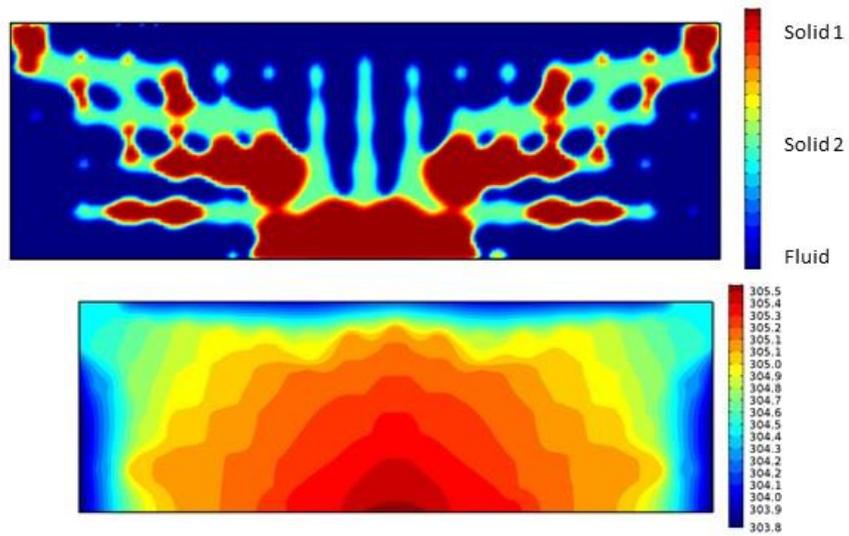**Figure 6**: Optimised shape and the Temperature contour (K) for Case3

Figure 7: Optimised shape and the Temperature contour (K) for Case4

Table 4: Summary of results

Case No.	Thermal Compliance	Maximum Temperature (K)
1	464.98	407.18
2	293.12	405.55
3	198.15	305.66
4	190.80	305.59
5	630.90	408.28

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| 6 | 358.46 | 406.29 |

4.2 Disparate solid volume ratio study

In these cases, solid1 and solid2 are constrained at a ratio of 2:8. These cases relates to economic heat sink design with small fraction of highly conductive solid used along with low cost medium conductivity solid. Results obtained for case5 and case6 are shown in Figure 8 and 9 respectively.

The results show that in spite of very low usage of highly conductive solid1, these two material heat sinks are performing on par with heat sink with equal volume solids (Section 4.1). The maximum temperature in the design domain is only 1°K higher than the equal volume solid case. Further it can be noted that shape of case 6 is to some extent similar to case 2 and case 5 is similar to case 1.
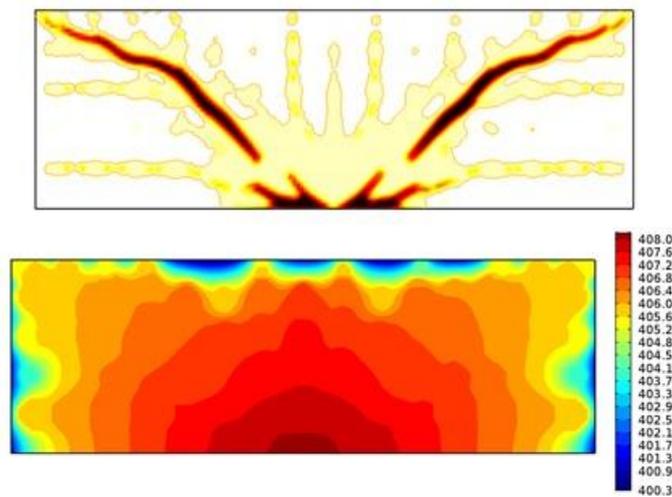

Figure 8: Optimised shape and the Temperature (K) contour for Case5
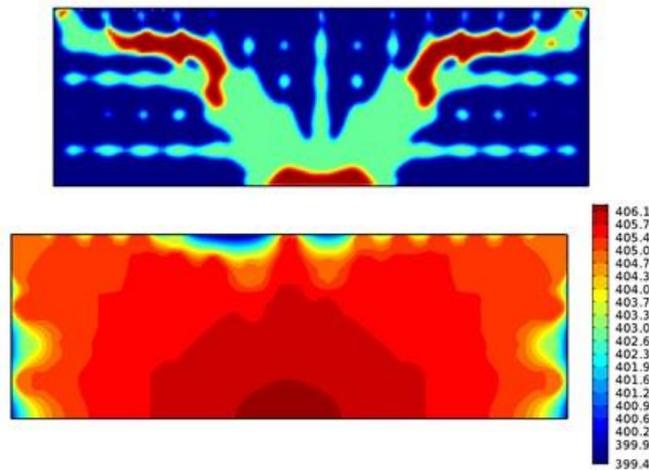

Figure 9: Optimised shape and the Temperature (K) contour for Case6

The results obtained in the LS TO depend on the initial LS distribution, indicating that many

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local minima appear to be present. So the shapes have to be investigated further to determine the global optima.

5   CONCLUSIONS

Design of two material convectively cooled heat sink is carried out for minimum thermal compliance objective using Level set topology optimisation. Two different solids and a fluid are modelled using two different level set functions. The level sets are evolved using a Hamilton Jacobi equation and they are re-initialised at regular intervals.

Topology optimisation is carried out for different solid-solid and solid-fluid conductivity ratios and for 2 different solid1 to solid2 volume fractions. When the solid1 to solid2 thermal conductivity ratio is higher say, 10, then in the optimised shape the solid 1 tends to have a branched structure and solid2 is mostly distributed near the centre to evenly distribute the temperature.

When solid to fluid thermal conductivity ratio is 100, heat sink tends to have longer flat base than the case with conductivity ratio 1000 indicating convective cooling is significant in the former case than later. Since the study minimizes only thermal compliance and doesn't consider the convective cooling, the shapes may not be the optimal for high Reynolds number flows.

This work demonstrates the design of two material heat sink which opens the possibility of using copper or other highly conductive metal at minimal amount in combination with aluminium to enhance its performance yet keeping the cost low.

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