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LINEAR OPTIMIZATION ALGORITHM FOR 1D HEMODYNAMICS PARAMETER ESTIMATION

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Abstract. An algorithm for estimating the parameters of 1D blood flow modelling is proposed. The following parameters are identified based on the blood flow velocity measurement at certain points: elasticity of arteries, hydraulic resistances of peripheral regions and cardiac output. The estimation method is based on the algorithm of linear optimization. This method was tested by adjusting 1D hemodynamics parameters to fit target values of systolic and diastolic blood pressures.

1 INTRODUCTION

Parameter identification problems are very common in the life sciences: we are given a mathematical model and a set of measured data, and we try to adjust the parameters in the model to fit the data. Such a problem arises constantly during construction of a patient-specific model. Many approaches were proposed to solve this task. Some of them based on Bayesian perspective [1], others on Kalman filter [2]. These are viable methods to solve an inverse problem.

We propose an approach based on linear optimization problem. It is based on the assumption that we have a number of similar tasks with almost identical sets of parameters. An example is a coronary blood flow model. Each task has similar aorta and boundary conditions. Left and right coronary arteries can be obtained from CT scans. We use one base task to teach our algorithm how blood flow model reacts to changes in each parameter. Then we use calculated reactions to changes in each parameter to adjust another task to fit some target values. In this work, we adjust parameters of the aorta and stroke volume to fit certain systolic and diastolic pressures. This allows us to get a model with adequate ranges of blood pressure that can later be used to study coronary blood flow under different conditions (stenosis, hyperemia).

An advantage of our method is its low dependence on computational resources. The first step of the algorithm does require significant amount of calculations to be made. We have to perform numerical simulations for each parameter. The second step involves utilizing the results of previously made calculations to adjust parameters of a new model. It requires a few iterations of numerical simulations.

2 Methods

2.1 Blood flow model

1D hemodynamics model used in this work is the model of viscous incompressible fluid in a network of elastic tubes. A network of arteries can be obtained from CT images or other clinical data. In this section, we present a brief description of the model, for details we refer to [3]. Blood flow in each vessel is described by hyperbolic set of mass and momentum balances

$$\partial A_k/\partial t + \partial (A_k u_k)/\partial x = 0, \tag{1}$$
$$\partial u_k/\partial t + \partial \left(u_k^2/2 + P_k/\rho\right)/\partial x = f_{fr}(A_k, u_k), \tag{2}$$

where k is an index of the vessel; t is time; x is distance along the vessel counted from the vessel's junction point; ρ is blood density (constant); $A_k(t,x)$ is vessels's cross-section area; p_k is blood pressure; $u_k(t,x)$ is linear velocity averaged over the cross-section; f_{tr} is a friction force given by

$$f_{tr}(A_k, u_k) = -4\pi\mu \frac{u_k}{A_k^2}\left(\eta_k + \eta_k^{-1}\right) \tag{3}$$

Elastic properties of the vessel wall material are described by the wall-state equation providing response to the transmural pressure (the difference between blood pressure and pressure in the tissues surrounding the vessel)

$$P_k(A_k) - P_{*k} = \rho c_k^2 f(A_k), \tag{4}$$

where S-like function $f(A_k)$ is approximated as

$$f(A_k) = \begin{cases} \exp\left(A_k/A_{0k} - 1\right) - 1, & A_k/A_{0k} > 1 \\ \ln A_k/A_{0k}, & A_k/A_{0k} \leqslant 1, \end{cases} \tag{5}$$

P_{*k} is pressure in the tissues surrounding the vessel; c_k is small disturbances propagation velocity; A_{0k} is unstressed cross-sectional area.

At the terminal point of the arteries we impose resistance R_k

$$R_k u_k A_k = P_k - P_v. \tag{6}$$

$p_v = 8mmHg$ is venous pressure. At the entry point of the aorta the blood flow is assigned

$$u(t,0)\,A(t,0) = Q_H\,(t)\,. \tag{7}$$

Here function $Q_H(t)$ corresponds to the heart rate value of 1 Hz and stroke volume of 65 ml [4].

We postulate continuity of total pressureat bifurcation points

$$p_i\,(A_i\,(t,\tilde{x}_i)) + \frac{\rho u_i^2\,(t,\tilde{x}_i)}{2} = p_j\,(A_j\,(t,\tilde{x}_j)) + \frac{\rho u_j^2\,(t,\tilde{x}_j)}{2}, \tag{8}$$

where i, j are indices of the vessels. $\tilde{x}$ is the coordinate of boundary point of the vessel. To close the system we add the mass conservation condition and compatibility conditions of hyperbolic set (1),(2) (see [5]).

2.2 Parameter estimation

An algorithm for estimating the parameters of 1D blood flow modelling is proposed. The following parameters are identified based on the blood flow velocity measurement at certain points: elasticity of arteries, hydraulic resistances of peripheral regions and cardiac output (heart rate HR and stroke volume SV).

The algorithm consists of two steps. In the **first step**, we use well-calibrated and detailed enough network of vessels. This network must include basic set of parameters $\vec{p} = \{p_1, p_2, ..p_n\}^T$. The set of parameters $\vec{p}$ produses set of target values $\vec{u} = \{u_1, u_2, ..u_m\}^T$. We suppose that $m \leqslant n$. Each value u_j can represent blood flow velocity, blood pressure, average flow, etc. Output set of target values $\vec{u}$ is produced by blood flow mode Ml with the input set of parameters $\vec{p}$

$$M(\vec{p}) = \vec{u}. \tag{9}$$

We introduce disturbances in parameters $\vec{p_k} = \vec{p} + \Delta\vec{p_k}, k = 1, 2..K$. In the simplest case $\Delta\vec{p_k} = \{0, 0, .., \Delta p_k, ..0\}^T$ and $K = N$. Each disturbance in parameters $\Delta\vec{p_k}$ corresponds to disturbance in target values $\Delta\vec{u}_k$

$$M(\vec{p} + \Delta\vec{p_k}) = \vec{u} + \Delta\vec{u}_k. \tag{10}$$

Second step of the algorithm involves identification of parameters of a new model $\tilde{M}$. We suppose that models $\tilde{M}$ and M are very close and correspondance between $\Delta\vec{p_k}$ and $\Delta\vec{u}_k$ holds

$$\tilde{M}(\vec{p} + \Delta\vec{p_k}) - \tilde{M}(\vec{p}) \approx \Delta\vec{u}_k. \tag{11}$$

The aim is to find set of parameters $\vec{p^*}$ that will lead to a known set of target values $\vec{u^*}$. This means that we want to find disturbance $\Delta\vec{p} = \sum_{k=1}^{K} x_k \Delta\vec{p_k}$ such as

$$\tilde{M}(\vec{p} + \Delta\vec{p}) - \tilde{M}(\vec{p}) = \Delta\vec{u}, \tag{12}$$

where $\Delta\vec{u} = \vec{u^*} - \tilde{M}(\vec{p})$. We propose to find coefficients x_k from

$$\Delta\vec{u} = \sum_{k=1}^{K} x_k \Delta\vec{u}_k, \quad x_k^{min} \leqslant x_k \leqslant x_k^{max}, \quad k = 1..K, \tag{13}$$

$$\sum_{k=1}^{K} |x_k| \rightarrow min, \tag{14}$$

where x_k^{min}, x_k^{max} can be defined based on the physiological range of parameters.

Task (13), (14) can be reformulated as linear optimization problem and solved with simplex method [6]. This will give us a set of calculated values $u_c\vec{alc}$. If difference $||u_c\vec{alc} - \vec{u^*}||$ is significant second step can be repeated. We substitute initial input set $\vec{p}$ with $\vec{p}+\Delta\vec{p}$ and $\Delta\vec{u}$ with $u_c\vec{alc} - \vec{u^*}$. After that we formulate new linear optimization problem (13),(14) and calculate new set of target values.

Condition (14) can take other forms, for e.g.

$$\sum_{k=1}^{K} x_k^2 \rightarrow min, \tag{15}$$

but this will force us to develop other methods to solve (13), (15).

3 Results

3.1 Aortic parameters estimation

In this example, we estimate parameters of the aorta to satisfy measured systolic and diastolic pressures. For the base model M we use a network of 4 vessels (Fig. 1, A). On the terminal points of the left coronary artery (LCA), right coronary artery (RCA) and aorta, we impose hydraulic resistance (6). Resistance of LCA is $R_{LCA} = 72\ kba{\cdot}s/cm^3$, resistance of RCA is $R_{RCA} = 86\ kba \cdot s/cm^3$, resistance of the aorta is $R_{RCA} = 2.3\ kba \cdot s/cm^3$. Elasticity (5) of both LCA and RCA is $c_{cor} = 1200\ cm/s$. Elasticity of aorta and aortic root is $c = 430\ cm/s$. On the inlet of the root blood flow is assigned (7). Heart rate $HR = 60\ bps$ and stroke volume $SV = 65\ ml$.

The input set of parameters consists of resistance at the terminal point of the aorta R, the elasticity of the aorta c and stroke volume SV. Target values are systolic and diastolic blood pressures in the aorta:

$$\vec{p} = \{R, c, SV\}^T, \quad \vec{u} = \{P_s, P_d\}^T. \tag{16}$$

We set Δp_k to 10% of base value of parameter p_k. We calculate disturbances $\Delta\vec{u}_k$ (10) and use obtained data for parameter estimation of new model $\tilde{M}$. Model $\tilde{M}$ uses same aorta and root but different LCA and RCA (Fig. 1, B). We calculate input set of parameters for the model $\tilde{M}$ for different target values of systolic and diastolic pressures. After that, we use this set of parameters to calculate systolic and diastolic pressures. Results are

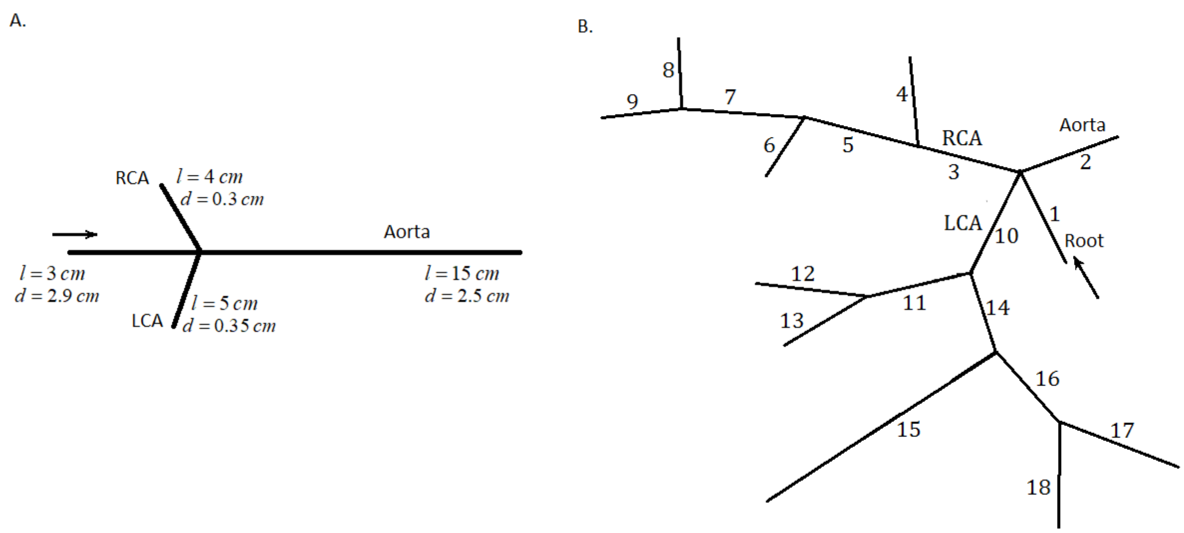

Figure 1: A: network used for the base model. B: network used for parameter estimation; this network is obtained from patient's CT-scans [7]

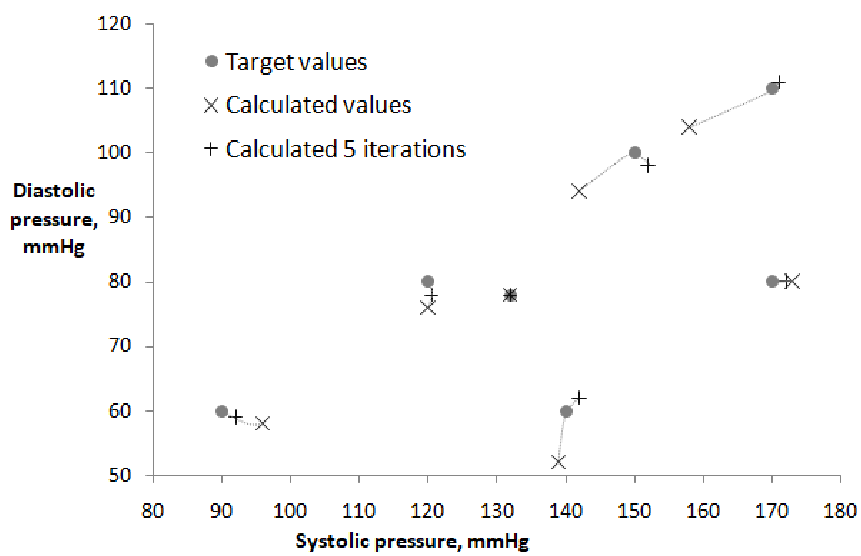

Figure 2: Target and calculated values of systolic and diastolic pressure. Two sets of calculated values are presented: after single iteration and after five iterations.

presented on Fig. 2. Corresponding target values and calculated values are connected with a dotted line.

For each target value of P_s and P_d we present two calculated values: after single iteration and after five iterations. Further iterations did not provide a substantial increase in accuracy. After one iteration root-mean-square error for systolic pressure is $\sigma_{sys} = 2.7mmHg$ and for diastolic pessure $\sigma_{dias} = 2.1mmHg$. Maximum deviations between calculated and target values are $\sigma_{sys}^{max} = 12mmHg$ and $\sigma_{dias}^{max} = 8mmHg$. After five iteration root-mean-square error for systolic pressure is $\sigma_{sys} = 0.7mmHg$ and for diastolic pessure $\sigma_{dias} = 0.6mmHg$. Maximum deviations between calculated and target values are $\sigma_{sys}^{max} = 2mmHg$ and $\sigma_{dias}^{max} = 2mmHg$.

4 CONCLUSIONS

Linear optimization provides a solid approach to parameter identification problems. With robust modelling and simulation, we obtain posterior values that correspond to target values of systolic and diastolic blood pressure.

In this work, we described only a basis of linear optimization approach. In its current form described algorithm might struggle with significant non-linearities and wider ranges of target values. Extending amount of preliminary simulations and generalizing to a non-linear optimization could solve these problems. Another limitation is a requirement for a robust basic task with known ranges of parameters. We proposed this algorithm to deal with a set of similar parameter identification tasks. The descrbed approach does not work well when investigating a new problem in its current form.

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