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IMPROVED WALL BOUNDARY CONDITIONS WITH IMPLICITLY DEFINED WALLS FOR PARTICLE BASED FLUID SIMULATION

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Abstract. We developed boundary conditions for particle based fluid simulation with implicitly defined walls. We employed moving particle semi-implicit as a particle simulation method, and all walls in the computational domain are defined by implicit function form. In existing particle based fluid simulation methods with implicitly defined walls, boundary conditions are formulated by assuming wall particles are arranged along flat planes. This formulation, however, causes particle clustering near walls and inaccurate pressure distribution because of the inappropriate assumption. We addressed this problem by considering contributions of non-planar wall shapes. We uniformly distributed dummy particles around the implicitly defined wall boundaries using the characteristics of implicit function form. The dummy particles are used only for precomputation, therefore no dummy particles are needed at the time of actual simulation. Since our proposed method takes the non-planar boundaries into account, wall weight functions are more accurately evaluated, and as a result, unnatural fluid behavior around walls are improved. Our test results show that the unnatural clustering and inaccurate pressure distribution are enhanced by our proposed method.

1 INTRODUCTION

We developed boundary conditions for particle based fluid simulation. As a particle simulation method, moving particle semi-implicit (MPS) [1] is employed, and all wall shapes are defined by implicit function form. Implicit function form is useful representation technique created not only from point sets generated by real world objects [2], but also from simple user interactive modeling [3]. In spite of the advantages of implicit function form, fluid simulation with implicit function form is under developing for industrial purposes. For example, existing particle based fluid simulation methods for implicit function form [4] assume wall particles arranged along flat planes when formulating boundary conditions. This assumption, however, causes particle clustering near walls and inaccurate pressure distribution.

In order to overcome this problem, some techniques are developed for polygon wall boundaries. Boundary conditions for polygon walls are firstly developed by [5]. Since this technique does not require any particles on and outside the walls, the total number of particles is reduced. Although the simulation with polygon walls is performed efficiently by reducing particles, inaccurate fluid motion is observed. Recently, this problem has been handled by taking non-planar wall shapes into account in [6], and improved in [7, 8].

We addressed the problems caused by the assumption of flat planes for implicit function form. In order to evaluate the contributions of non-planar boundary conditions, dummy particles are uniformly arranged around the implicitly defined walls using the characteristics of implicit function form. The generated dummy particles are used only for precomputation, therefore no dummy particles are needed at the time of actual simulation.

Since our proposed method takes the non-planar shapes into account, wall weight functions used in implicit function form are more accurately evaluated. Our test results show that the unnatural clustering and inaccurate pressure distribution are enhanced by our proposed method.

2 DISCRETIZATION OF GOVERNING EQUATIONS FOR IMPLICITLY DEFINED WALLS

2.1 Discretization of MPS with implicitly defined walls

We assume that fluid is incompressible, and such fluid motion is governed by the following Navier-Stokes equations:

$$\frac{\mathrm{D}\rho}{\mathrm{D}t} = 0, \tag{1}$$

$$\frac{\mathrm{D}\boldsymbol{v}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{v} + \boldsymbol{g}, \tag{2}$$

where ρ is the density, t is the time, $\boldsymbol{v}$ is the velocity, p is the pressure, ν is the dynamic viscosity, and $\boldsymbol{g}$ is the acceleration due to gravity. Here, Equation (1) represents the incompressible condition, and Equation (2) is the law of momentum conservation.

In MPS, fluid is represented by particles $\boldsymbol{x}_1^t, \boldsymbol{x}_2^t, \boldsymbol{x}_3^t, \cdots, \boldsymbol{x}_N^t$. When each particle is moving with velocities $\boldsymbol{v}_1^t, \boldsymbol{v}_2^t, \boldsymbol{v}_3^t, \cdots, \boldsymbol{v}_N^t$, the positions and velocities of fluid are updated through the following discretized equations:

$$\begin{aligned}
\boldsymbol{v}_i^* &= \boldsymbol{v}_i^k + \Delta t(\langle \nu\nabla^2\boldsymbol{v}\rangle_i + \boldsymbol{g}), \\
\boldsymbol{x}_i^* &= \boldsymbol{x}_i^k + \Delta t \boldsymbol{v}_i^*, \\
\boldsymbol{v}_i^{k+1} &= \boldsymbol{v}_i^* + \Delta t\langle -\frac{1}{\rho}\nabla p\rangle_i, \\
\boldsymbol{x}_i^{k+1} &= \boldsymbol{x}_i^* + \Delta t^2\langle -\frac{1}{\rho}\nabla p\rangle_i.
\end{aligned}$$

Since the walls are represented by implicit function form ($f(\boldsymbol{x}) = 0$: $f(\boldsymbol{x}) > 0$ inside and $f(\boldsymbol{x}) < 0$ outside) but not by particles, we divide the pressure and viscosity terms into the contribution of fluid and walls.

$$\begin{aligned} \langle -\frac{1}{\rho}\nabla p\rangle_i &= \langle -\frac{1}{\rho}\nabla p\rangle_{if} + \langle -\frac{1}{\rho}\nabla p\rangle_{iw}, \\ \langle \nu\nabla^2 \boldsymbol{v}\rangle_i &= \langle \nu\nabla^2 \boldsymbol{v}\rangle_{if} + \langle \nu\nabla^2 \boldsymbol{v}\rangle_{iw}, \end{aligned}$$

where $\langle -\frac{1}{\rho}\nabla p\rangle_{if}$ and $\langle \nu\nabla^2 \boldsymbol{v}\rangle_{if}$ represent the contribution of fluid, and $\langle -\frac{1}{\rho}\nabla p\rangle_{iw}$ and $\langle \nu\nabla^2 \boldsymbol{v}\rangle_{iw}$ represent the contribution of walls. Because fluid is represented by particles, we can discretize the terms, $\langle -\frac{1}{\rho}\nabla p\rangle_{if}$ and $\langle \nu\nabla^2 \boldsymbol{v}\rangle_{if}$, in the same way as original MPS [1].

$$\begin{aligned} \langle -\frac{1}{\rho}\nabla p\rangle_{if} &= -\frac{D}{\rho n^0}\sum_{i\neq j}\left[\frac{p_j - \hat{p}_i}{\|\boldsymbol{x}_j^* - \boldsymbol{x}_i^*\|^2}(\boldsymbol{x}_j^* - \boldsymbol{x}_i^*)w(\|\boldsymbol{x}_j^* - \boldsymbol{x}_i^*\|)\right], \\ \langle \nu\nabla^2 \boldsymbol{v}\rangle_{if} &= \nu\frac{2D}{\lambda n^0}\sum_{i\neq j}\left[(\boldsymbol{v}_j^k - \boldsymbol{v}_i^k)w(\|\boldsymbol{x}_j^k - \boldsymbol{x}_i^k\|)\right], \end{aligned} \tag{3}$$

where D is the number of spacial dimensions, n^0 is the particle number density constant, $\hat{p}_i$ is the minimum pressure within the effective radius of the weight function, $w(\|\boldsymbol{x}_j^k - \boldsymbol{x}_i^k\|)$ is a weight function, and λ is the Laplacian model coefficient. In contrast, the terms, $\langle -\frac{1}{\rho}\nabla p\rangle_{iw}$ and $\langle \nu\nabla^2 \boldsymbol{v}\rangle_{iw}$, need special treatment to evaluate the contribution without particles.

As mentioned above, MPS evaluates the interactions of particles using a weight function. In this paper, we employ the following weight function.

$$w(r) = \begin{cases} \frac{r_e}{r} - 1 & (0 \leq r \leq r_e) \\ 0 & (r_e < r) \end{cases},$$

where r_e is the effective radius. For Equation (3), the pressure values are evaluated through the pressure Poisson equation:

$$\langle \nabla^2 p\rangle_{if} + \langle \nabla^2 p\rangle_{iw} = -\frac{\rho}{\Delta t^2}\frac{n_i^* - n^0}{n^0} + \frac{p_i}{(\Delta t c_s)^2},$$

where the second term on the right hand side is added for the stability. The contribution of fluid is evaluated in the same way as original MPS

$$\langle \nabla^2 p\rangle_{if} = \frac{2D}{\lambda n^0}\sum_{i\neq j}\left[(p_j - p_i)w(\|\boldsymbol{x}_j^* - \boldsymbol{x}_i^*\|)\right],$$

but the contribution of walls, $\langle \nabla^2 p\rangle_{iw}$, has to be reformulated without particles. As a boundary condition in the pressure Poisson equation for free surface particles, the pressure of the particles is set to zero when $n_i^* < 0.97n^0$. The particle number density n_i^* is evaluated using the weight function as

$$n_i^* = \sum_{i\neq j} w(\|\boldsymbol{x}_j^* - \boldsymbol{x}_i^*\|) + n_{iw}^*.$$

The first term on right hand side is the contribution of fluid and can be calculated in the same way as original MPS. However, the contribution of walls, n^*_{iw}, needs special treatment again.

In MPS with implicitly defined walls, the contribution of walls is evaluated as

$$\begin{aligned} \langle -\frac{1}{\rho}\nabla p\rangle_{iw} &= \frac{l-d^*_i}{\Delta t^2}\frac{\nabla f(\boldsymbol{x}^*_i)}{\|\nabla f(\boldsymbol{x}^*_i)\|}, \\ \langle \nu\nabla^2\boldsymbol{v}\rangle_{iw} &= -\nu\frac{2D}{\lambda n^0}\boldsymbol{v}^k_i n^k_{iw}, \\ \langle \nabla^2 p\rangle_{iw} &= \frac{2\rho(l-d^*_i)d^*_i}{\lambda\Delta t^2}, \end{aligned}$$

where d^*_i is the distance between the fluid particle and the walls. We estimate the distance d^*_i using the following approximation:

$$d^*_i \approx \frac{|f(\boldsymbol{x}^*_i)|}{\|\nabla f(\boldsymbol{x}^*_i)\|}. \tag{4}$$

The particle number density is one of the important quantities for the reliable fluid simulation. The existing particle based methods with implicitly defined walls assume that the wall is locally flat [4]. Under this assumption, the dummy particles are arranged along the flat plane, and the particle number density of walls can be evaluated using the dummy particles as

$$W^k_{iw} \approx \sum_a w(\|\boldsymbol{x}^k_a - \boldsymbol{x}^k_i\|), \tag{5}$$

where the index a indicates the dummy particles. Note that Equation (5) can be the function of the distance from the walls, and we can make lookup tables beforehand. Our method inherits this feature from the existing methods, so no dummy particle is required during actual simulation.

2.2 Estimation of particle number density of walls without particles

In Section 2.1, the particle number density of walls, which is also called the wall weight function, is formulated assuming flat planes, and that wall weight function is inaccurate. As a result, this assumption causes unnatural clustering and inaccurate pressure distribution as shown in Figure 2(a) and 3(a).

We extend the method proposed in [7] for polygon walls to avoid this problem. In the polygon model [7], dummy particles are arranged on and outside walls using a uniform grid. Our proposed method arranges the dummy particles more uniformly using the characteristics of implicit function form.

If we assume that the dummy particles are arranged by our proposed method (See Section 3 for the arrangement technique), our method evaluates the wall weight function Z_m at every uniform grid point in the same way as the polygon model.

$$Z_m = \sum_b w(\|\boldsymbol{x}_b - \boldsymbol{x}_m\|),$$

where the index m indicates the grid points, and b indicates the generated dummy particles on and outside the walls. During the actual simulation, the wall weight function of each fluid particle is estimated as

$$n_{iw}^k = C_i W_{iw}^k,$$

where C_i can be evaluated by the linear interpolation of C_m at the neighboring grid points. As a precomputation, we evaluate C_m using the following equation

$$C_m = \frac{Z_m}{W_{mw}^k}.$$

The value C_i is used to transfer the wall weight function for flat plane, W_{iw}^k, into the one for non-planar walls, n_{iw}^k [7].

3 PARTICLE ARRANGEMENT FOR IMPLICITLY DEFINED WALLS

For the evaluation of the wall weight function, we have to arrange dummy particles on and outside the walls. In this section, we propose a dummy particle arrangement technique for implicitly defined walls using the characteristics of implicit function form by extending the method proposed in [7, 9].

The weight function is more influenced by closer particles, so the dummy particles on wall boundaries, which is the most internal ones, are the most important for the wall weight function. In MPS, the uniformity of particles is crucial factor for simulation accuracy. Firstly, we arrange the particles by Poisson disk sampling (PDS) [10] only on wall boundaries. However, the generated particles are not uniform (not equal distance to the neighboring dummy particles). For the uniformity of dummy particles on the walls, we relaxed the generated dummy particles using the surface relaxation method proposed in [9].

The surface relaxation method in [9] is designed for signed distance functions but not for general implicit function form. We propose the idea to extend their method for more general implicit function form. To relax the particles, the distance from the wall boundaries to the particle and its direction are required. We evaluate the distance using Equation (4), and the direction with the following approximation

$$\boldsymbol{d} \approx \frac{\nabla f(\boldsymbol{x})}{\|\nabla f(\boldsymbol{x})\|}.$$

This assumptions of the distance and direction are accurate only around the wall boundaries [4]. The distance and direction are required only around the boundaries in [9], so our approximation here is appropriate for the surface relaxation.

After arranging the surface dummy particles, we distribute the dummy particles outside the walls in the same way as [9]: Distribute dummy particles with volume PDS, and then apply the volume relaxation. At this time, the distance and direction are also required, and using the distance, we distribute the particles within the distance $1.5r_e$ from the walls to cover the effective region and maintain the uniformity of particles. We use the same definitions as surface sampling for the distance and direction. The whole process of the dummy particle generation is shown in Figure 1.

As shown in Figure 1, although the particle distribution of surface and volume without relaxation is not uniform (not equal distance to the neighbors), the relaxed dummy particles are more uniformly distributed. In Figure 1(d), the most outer layer of particles looks less uniformly, but this ununiformity does not affect the simulation results because these particles are beyond the effective radius of simulation. For this purpose, we generated the dummy particles beyond the effective radius.

4 RESULTS

We tested the effectiveness of our proposed method with a two dimensional hydrostatic problem. We defined the wall boundary using the following equation

$$x^2 + y^2 - r^2 = 0$$

with $r = 0.4$. We generated fluid particles in the region $y < 0$ inside the wall with $l = 0.01$, and we use $\rho = 1000$, $r_e = 3.1l$ and $\Delta t = 0.003$ in this numerical test. For the numerical stability, we apply the collision model used in [8].

Figure 2 shows the results of the previous model with implicit function form explained in Section 2.1, and of our proposed method at 5000 frame. From Figure 2, we can confirm that the previous method suffers from unnatural clustering of fluid particles while our method appropriately eases this problem.

Figure 3 shows the evaluated pressure values at 5000 frame. As shown in Figure 3, the previous method suffers from the errors caused by the inaccurate wall weight function and the clustering of fluid particles near walls. Our method reproduced the pressure values following the analytical solution.

5 CONCLUSION

We proposed a boundary condition of MPS with implicitly defined walls. In the previous method, only flat walls are considered for the particle number density and result in unnatural clustering and inaccurate pressure distribution.

For the avoidance of this problem, we developed a method to evaluate the wall weight functions considering non-planar wall shapes. To effectively estimate the contribution of walls for particle number density, we extended the method proposed in [7, 9]. Our proposed method uniformly arranges dummy particles on and outside the walls. Since the uniformity of dummy particles is the important factor for accuracy, we carefully distribute the dummy particles using the characteristics of implicit function form. Our test results show that the unnatural clustering and inaccurate pressure distribution in the previous method are improved by our proposed method.

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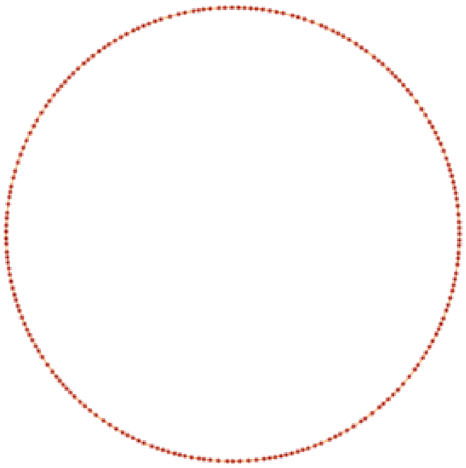
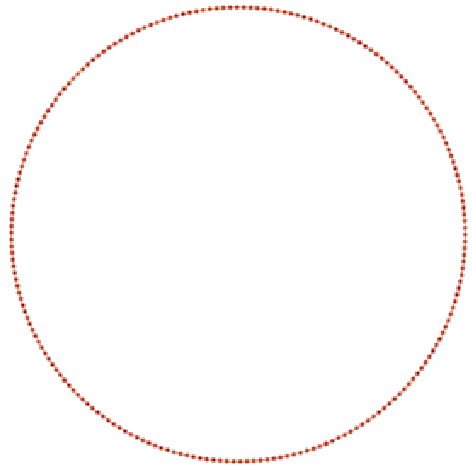
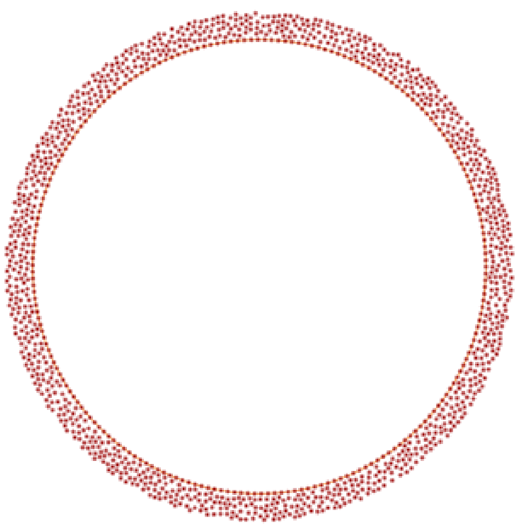


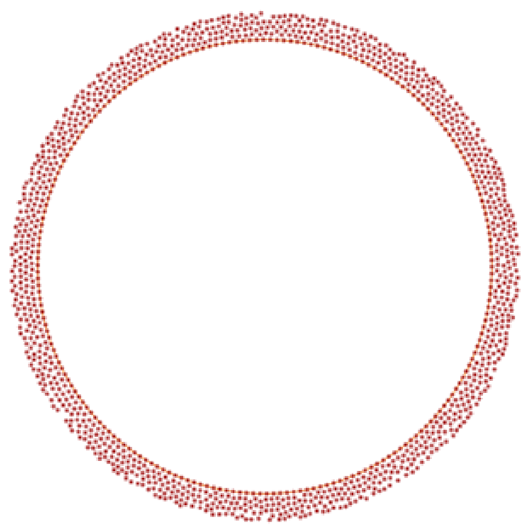
(a) Generated dummy particles by surface PDS



(b) Relaxed dummy particles by surface relaxation

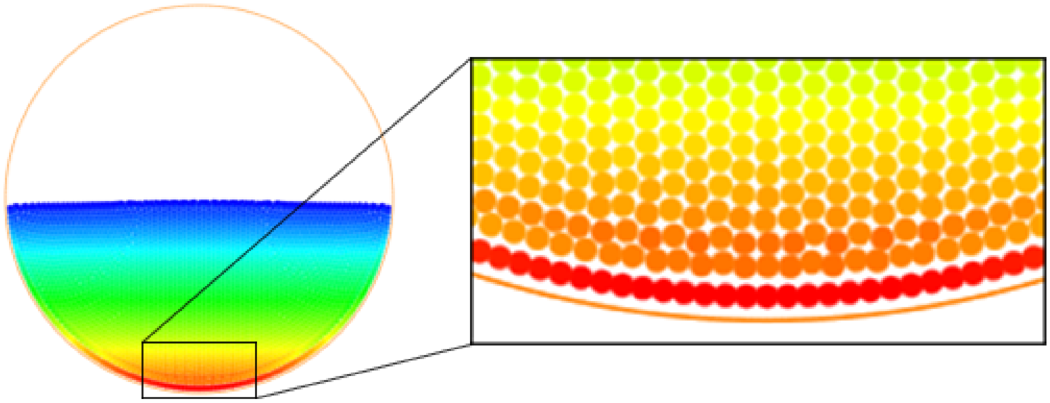


(c) Dummy particles after volume PDS

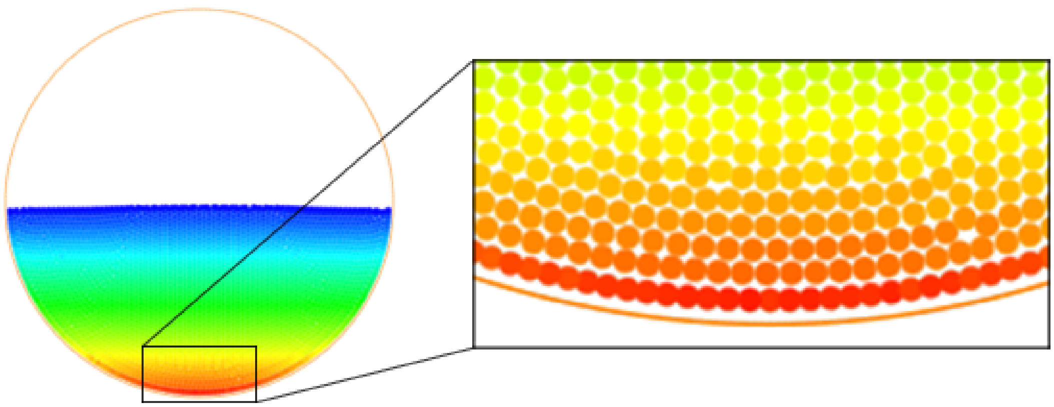


(d) Final distribution of dummy particles after volume relaxation

Figure 1: Generation process of dummy particles. The orange line indicates the wall boundary, and the brown points show the dummy particles.

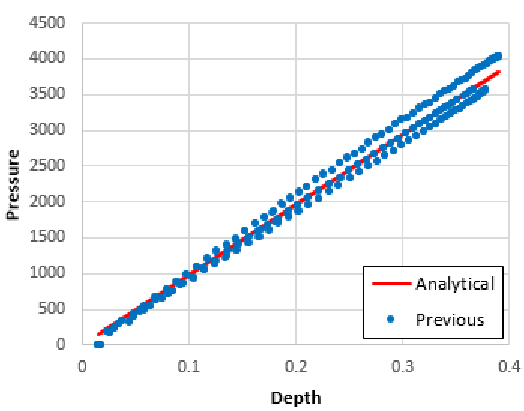


(a) Previous method

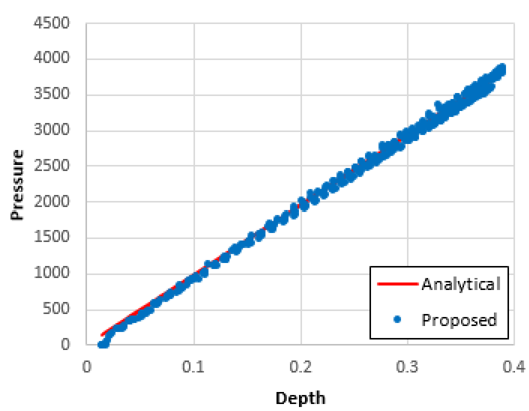


(b) Proposed method

Figure 2: Simulated results of a hydrostatic problem. The color of the fluid particles represents the pressure distribution (blue is low and red is high).



(a) Previous method



(b) Proposed method

Figure 3: Pressure values within the effective radius from the walls evaluated by the previous method and the proposed method. The red lines indicate the analytical solution and the blue points indicate the evaluated values.