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FOUR TYPES OF DEPENDENCIES FOR THE FUZZY ANALYSIS

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Abstract. Uncertain structural analysis is the computation of uncertain structural response for uncertain input quantities, based on a computational model ξ. If only rare data is available, the usage of the uncertainty model fuzziness is common. The computation of fuzzy responses with respect to predefined fuzzy inputs is called fuzzy analysis. In terms of structural analysis, four kinds of dependencies are identified.
Firstly, the input parameters $\underline{x}^{\mathrm{f}}$, i.e. material characteristics or loading conditions, may vary with time τ and/or space $\underline{\theta}$. This feature is called functional input dependency $\underline{x}(\tau, \underline{\theta})$ and could be described by fuzzy processes or fuzzy fields.

Secondly, the set of input parameters $\underline{x}^{\mathrm{f}}$ can be internally dependent, which means not all combinations of parameter values are permissible with regard to the computational model. This pre-condition is the prior dependency, which mainly depends on the definition of the multi-dimensional membership function $\mu(\underline{x})$. For random variables, the term correlation is established in this context.

Thirdly, the result quantities (e.g. stresses, strains, damages, ...) are time and spatially dependent as well. This functional output dependency $\underline{z}(\tau, \underline{\theta})$ is always given, if the finite element method is used as fundamental solution and the results are not reduced to a small amount of Quantities of Interest.

Fourthly, the result parameters $\underline{z}^{\mathrm{f}}$ can be internally dependent, called posterior dependency. This fact is commonly ignored, because fuzzy result quantities are computed for separated deterministic results. For instance, stresses $\mu(z_1 = \sigma)$ and strains $\mu(z_2 = \varepsilon)$ (at one point in space and same time) are computed independently, which subsequently yields a significant overestimation of uncertainty, since for most of undamaged elastic solid materials high stresses come alone at high strains. The challenge is to compute the membership function, which depends on a vector of result quantities $\mu(\underline{z})$.

The goal of this contribution is to discuss the different kinds of dependencies and present possible solution strategies.

1 INTRODUCTION

Uncertainty Numerical analysis enables the simulation of the reality based on physical models. The simulation model represents the physical behaviour of a structure or a process at specific boundary conditions and various parameters representing the properties of the observed phenomena. These parameters can be distinguished in resistance (yield stresses, friction coefficients, ...) and actions (external forces, support, ...) parameters. The identification of deterministic values for these parameters is challenging or even not possible, because of uncertainty. Uncertainty has various reasons, the main reason is the inherent variability of material parameters and natural variability of loadings (e.g. wind, snow). Due to limitations in observing and evaluating this variability, the uncertainty characteristics imprecision and incompleteness exist [2]. These characteristics are classified into aleatoric (variability) and epistemic (imprecision and incompleteness) uncertainty.

The involvement of uncertainty into structural analysis results in an uncertainty model, beside the existing deterministic simulation model, whereas the uncertainty model describes the uncertainty of the simulation model parameter based on existing data and information with validatable assumptions. Therefore, various numerical models are developed. Aleatoric uncertainty is mainly modelled by random variables and epistemic uncertainty is considered by interval or fuzzy numbers [13]. These models are the basis for many advanced models within the context of imprecise probability (e.g. p-boxes and fuzzy randomness) [2]. In this contribution, the uncertainty model fuzziness is addressed.

Fuzzy variable A normalized fuzzy variable A^{f} is the gradual assessment of a crisp set within the interval $[0,1]$ indicated by the superscript $\square^{\mathrm{f}}$ (alternatively $\tilde{\square}$). A membership function defines the fuzzy variable as mapping $\mu\colon \mathbb{R} \to [0,1]$. The normalisation condition: $\exists\, x \mid \mu(x) = 1$ needs to be fulfilled. Due to numerical aspects, the fuzzy variable is separated into α-levels. Another representation of a fuzzy variable is the introduction of α-levels $A^{\mathrm{f}}_{\alpha} = \{x \in \mathbb{R} \mid \mu_{A^{\mathrm{f}}}(x) \geq \alpha\}$, $\alpha \in (0,1]$. Each A^{f}_{α} is an interval $A^{\mathrm{f}}_{\alpha} \subseteq \mathbb{R}$, $A^{\mathrm{f}}_{\alpha} = [x_{\alpha,l}, x_{\alpha,r}]$.

Fuzzy analysis The fuzzy analysis is the computation of n_{z} fuzzy result quantities $\underline{z}^{\mathrm{f}}$ for n_{x} predefined fuzzy input variables $\underline{x}^{\mathrm{f}}$ utilizing a mapping

$$\xi^{\mathrm{f}}\colon \mathcal{F}(\mathbb{R},[0,1]) \to \mathcal{F}(\mathbb{R},[0,1])\colon \underline{x}^{\mathrm{f}} \mapsto \underline{z}^{\mathrm{f}}. \tag{1}$$

The result membership function $\mu_j(\underline{z}_j)$ – defining the fuzzy result quantity – can be found by an α-level optimisation by computing the minimum (left bound) $z_{\alpha_k,\mathrm{left}}$ and maximum (right bound) $z_{\alpha_k,\mathrm{right}}$ value on a discrete number of α-levels n_{α} with $\alpha \in (0,1]$. The bounds are computed by solving two optimisation problems for each α-level and for each result z_j

$$z_{j,\alpha_k,\mathrm{left}} = \xi(\underline{x}) \to \min \mid \underline{x} \in \underline{\mathbf{X}}_{\alpha} \text{ and } z_{j,\alpha_k,\mathrm{right}} = \xi(\underline{x}) \to \max \mid \underline{x} \in \underline{\mathbf{X}}_{\alpha}. \tag{2}$$

To solve the $n_{\text{opt}} = 2 \cdot n_\alpha \cdot n_{\text{z}}$ optimisation tasks, various optimisation algorithms can be applied. The efficiency as well as accuracy of the fuzzy analysis depends only on the efficiency of the optimisation algorithms. The input space

$$\underline{\mathbf{X}}_\alpha = \{\underline{x} \mid \underline{x} \in \mathbb{R}^{n_{\text{x}}} \wedge \mu_{K^{\text{f}}}(\underline{x}) \geq \alpha\} \tag{3}$$

for the optimisation is defined on the basis of the α-level cut of the Cartesian product $K^{\text{f}} = \left(x_1^{\text{f}} \times x_2^{\text{f}} \times \ldots \times x_i^{\text{f}} \times \ldots \times x_{n_{\text{x}}}^{\text{f}}\right)$ of the fuzzy input quantities. If independence between the input variables is assumed [12], the multidimensional membership function $\mu_{K^{\text{f}}} \colon \mathbb{R}^{n_{\text{x}}} \to [0,1]$ can be defined as

$$\mu_{K^{\text{f}}} \colon\ (x_1, x_2, \ldots, x_i, \ldots, x_{n_{\text{x}}}) \mapsto \min\left(\mu_{x_1^{\text{f}}}(x_1), \mu_{x_2^{\text{f}}}(x_2), \ldots, \mu_{x_i^{\text{f}}}(x_i), \ldots, \mu_{x_{n_{\text{x}}}^{\text{f}}}(x_{n_{\text{x}}})\right). \tag{4}$$

Dependencies An extension of Eq. (1) indicates four types of dependencies

$$\xi^{\text{f}} \colon \mathcal{F}(\mathbb{R},[0,1]) \to \mathcal{F}(\mathbb{R},[0,1]) \colon\ \underbrace{\underline{x}^{\text{f}}}_{\text{prior dependency}} \overbrace{(\tau, \underline{\theta})}^{\text{functional dependency}} \quad \mapsto \quad \underbrace{\underline{z}^{\text{f}}}_{\text{posterior dependency}} \overbrace{(\tau, \underline{\theta})}^{\text{functional dependency}}. \tag{5}$$

The prior dependency describes the inherent relations of the fuzzy input variables $\underline{x}^{\text{f}}$ and can be interpreted as counterpart of correlation in terms of random variables. The extension of the input variables for consideration of time $\tau \in \mathbb{R}$ and spatially $\underline{\theta} \in \mathbb{R}^3$ dependent characteristics yields the functional input dependency $\underline{x}^{\text{f}}(\tau, \underline{\theta})$. This aspect is used to model uncertain processes (e.g. time dependent loading), uncertain fields (e.g. spatially dependent material characteristics) or combinations (e.g. time dependent wind field).

For the result variables $\underline{z}^{\text{f}}$, the posterior dependency describes the inherent relations of the results. This characteristic is usually not considered in fuzzy analysis, because the fuzzy result quantities are computed on the basis of α-level optimisation, separately for each result quantity z_j^{f}, see Eq. (2). Furthermore, the functional output dependency $\underline{z}^{\text{f}}(\tau, \underline{\theta})$ is applicable if time and spatially dependent fuzzy result quantities (stress or damage fields) are computed by e.g. finite element analysis.

The functional dependencies are discussed firstly, followed by the prior dependency. A detailed explanation of the posterior dependency is given and a new numerical approach on the basis of multiobjective optimisation is proposed. Two examples show the applicability and necessity of advanced fuzzy analysis methods.

2 FUNCTIONAL INPUT DEPENDENCY

To consider functional input dependency in context of random variables, various methods for random processes and random fields exist. For the fuzzy analysis, only few method are available. A fuzzy field defines a fuzzy quantity at each point in space $T^{\text{f}} \colon \mathbb{R}^3 \to \mathcal{F}(\mathbb{R})$, a one-dimensional example ($\theta_1 \in \mathbb{R}$) is depicted in Fig. 1. The fuzzy field can be developed by a finite series expansion as $T^{\text{f}}(\underline{\theta}) = \sum_{p=1}^{n} c_p^{\text{f}} \cdot \phi_p(\underline{\theta})$. The basis functions $\phi_p(\underline{\theta})$ can be modelled as radial basis functions [8] or on the basis of a modified Karhunen-Love expansion using the eigenvectors of an interaction matrix, see [6], [1].

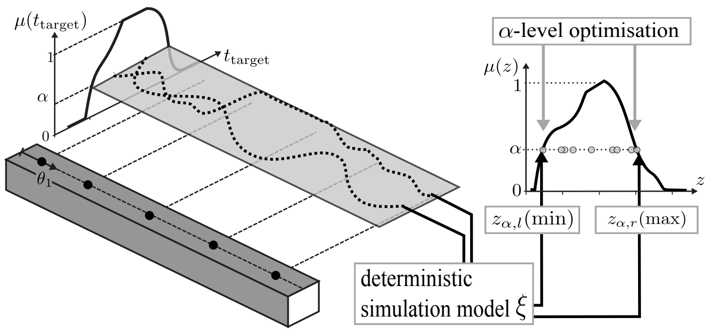


Figure 1: One-dimensional fuzzy field

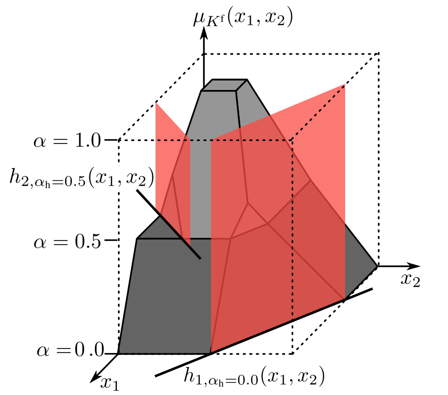


Figure 2: Prior dependency of two fuzzy input quantities

3 FUNCTIONAL OUTPUT DEPENDENCY

As indicated in Eq. (1), the fuzzy results vector $\underline{z}^{\mathrm{f}}$ can be independent of time and space. Examples therefore are general parameters such as mass of a system or overall values as maximum displacement or damage. But in general, the results of finite element computations, e.g. stress or displacement fields, are time and spatially dependent. The dependency structure can be found on the basis of the spatial finite element discretization and the time steps. The aspect of functional output dependency is to compute the fuzzy result quantities with respect to these relationships $\underline{z}^{\mathrm{f}}(\tau, \underline{\theta})$. This means, for each node in a finite element mesh n_{node} and for each time step n_{time}, the amount of n_{z} fuzzy result quantities needs to be computed. It is obvious, that the large amount of fuzzy result quantities ($n_{\text{fuzzy analysis}} = n_{\mathrm{node}} \cdot n_{\mathrm{time}} \cdot n_{\mathrm{z}}$) cannot be handled by the α-level optimisation approach, see Eq. (2). To overcome this problem, an enhanced fuzzy structural analysis is proposed in [7]. The main idea is to separate the point sampling and the fuzzy analysis, such that the feedback loop within the optimisation algorithm is not existing. The sampling points have to be placed in all α-level cuts $\underline{\mathbf{X}}_{\alpha}$ separately. Otherwise, the quality of the resulting membership function would decrease for higher α-levels, due to the curse of dimensionality. The proposed method allows the computation of a large amount of fuzzy result quantities in an approximative way.

The resulting fuzzy quantities for each time step and at each point in space are shown in Fig. 3. It is obvious, that the visualisation possibilities are limited. Therefore, information reduction measures (also called defuzzification) can be applied, whereas information reduction is the real valued representation of a fuzzy quantity $\mathfrak{R}^{\mathrm{f}} \colon \mathcal{F}(\mathbb{R}, [0, 1]) \to \mathbb{R}$. It is necessary to distinguish between characteristic measures, such as the centroid value and uncertainty-quantifying measures such as the area of the membership function. This classification is highly important. Both types are necessary for a holistic evaluation of the uncertain results.

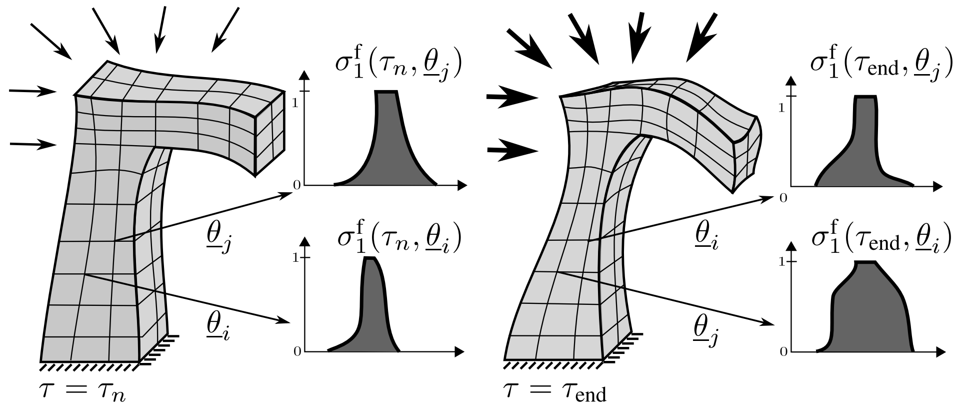


Figure 3: Discrete fuzzy stress field (σ_1) for two time steps

4 PRIOR DEPENDENCY

The main characteristic of $\underline{\mathbf{X}}_\alpha$ according to Eq. (3) is the independence of all parameters, i.e. each parameter can be selected without the knowledge of the others. Prior dependencies are inherent constraints/relations in the fuzzy input vector $\underline{x}^{\mathrm{f}}$. Conceivable cases are, physically infeasible combinations, e.g. low/high concrete Youngs modulus comes always along with the pendant of compression strength. The combinations of low Youngs modulus and high compression strength is infeasible. Another reason becomes obvious, if the parameter of the simulation model are identified by uncertain parameter identification methods. These methods result in a set of good fitting parameter combinations and non applicable sub-domains. The independent modelling of each parameter would yield to wrong results.

The question arises, what is the multidimensional membership function $\mu(\underline{x})$ if the assumed independence in Eq. (4) cannot be hold. In [10] the prior dependency structure is formulated on the basis of a t-norm which is similar to the copula approach in probability theory. Here, a simpler approach is described, based on a confined α-Level domain $\underline{\mathbf{X}}_\alpha$, see [9]. The interaction of fuzzy quantities can be observed by a constrained input space $\underline{\mathbf{X}}_\alpha^+$. The unconstrained input space Eq. (3) changes to

$$\underline{\mathbf{X}}_\alpha^+ = \{\underline{x} \mid \underline{x} \in \mathbb{R}^{n_{\mathrm{x}}} \bigwedge \mu_{K^{\mathrm{f}}}(\underline{x}) \geq \alpha \ \bigwedge \ h_{i,\alpha_{\mathrm{h}}}(\underline{x}) > 0 \mid \alpha_{\mathrm{h}} \leq \alpha \ \forall \ i \in \{0, \ldots, n_{\mathrm{h}}\} \tag{6}$$

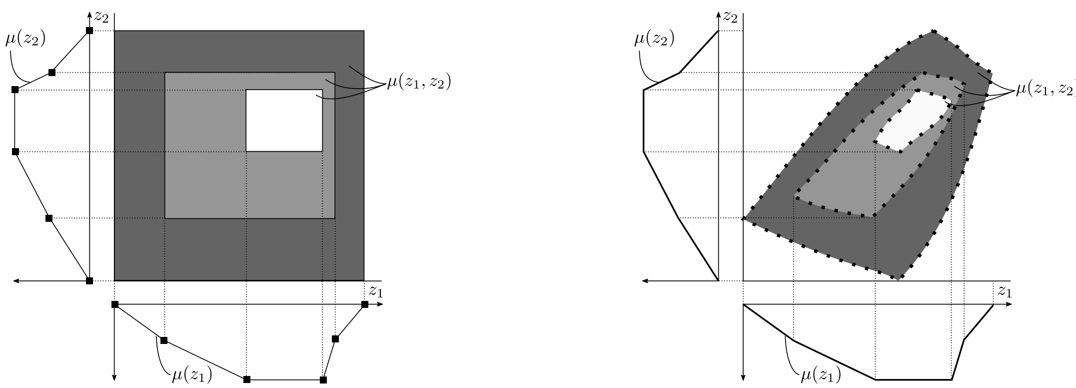
considering n_{h} inequalities $h_{i,\alpha_{\mathrm{h}}}(\underline{x})$. These inequalities define, whether a deterministic input vector may be possible or impossible. The subscript α_{h} indicates, that the definition of permissibility can be defined independently for each α-level, which yields to non-continuous membership functions. This definition is equal to the extension of Eq. (4) – assuming the independence of input quantities – using a multidimensional definition of $\mu(\underline{x})$, which is not only based on the one-dimensional membership functions $\mu_i(x_i)$ [12].

In Fig. 2, two linear constraints in a two-dimensional input space are shown. The definition can be applied on the basis of the α-level discretization.

5 POSTERIOR DEPENDENCY

The fuzzy analysis described in Section 1 is based on α-level optimisation, such that for each result quantity z_j^{f} the optimisation tasks according to Eq. (2) have to be executed. The resulting multidimensional membership $\mu(\underline{z})$ is a posteriori assembled in the same manner as shown in Eq. (4), based on the one-dimensional results $\mu_j(\underline{z}_j)$. The assumed independence of the results can be seen in Fig. 4a) as rectangular result domains.

The direct computation of $\mu(\underline{z})$ is discussed in [10]. The availability of the dependency between the results z_1^{f}, z_j^{f} and $z_{n_{\mathrm{z}}}^{\mathrm{f}}$ is shown in Fig. 4b). The existing computation methods are based on a convex hull concept for multivariate interval uncertainty [5], [4] and a sampling approach with adaptive decomposition of the output space [10].



a) Multidimensional membership function assuming independent results

b) Multidimensional membership function considering posterior dependency

Figure 4: Examples for two-dimensional fuzzy result quantities

Here, another method for computing the multidimensional membership function is proposed. The formulation is based on multiobjective optimisation tasks. Thus, the membership function is indicated by the relation to an α-level domain of result quantities

$$\mu(\underline{z}) = \min \alpha \mid \underline{z} \in \underline{\mathbf{Z}}_\alpha. \tag{7}$$

Therefore, a resulting point set, based on the union of the PARETO-Frontiers $\underline{\mathcal{P}}_{i,\alpha}$

$$\underline{\mathcal{Z}}_\alpha = \bigcup_{i=1}^{n_{\text{PARETO}}} \underline{\mathcal{P}}_{i,\alpha}, \tag{8}$$

is the basis for a continuous domain $\underline{\mathbf{Z}}_\alpha$. This domain is defined as polytope (multidimensional polygon) of the point set $\underline{\mathbf{Z}}_\alpha = \text{polytope}(\underline{\mathcal{Z}}_\alpha)$. If the number of results in a PARETO-Front is $\#\{\underline{\mathcal{P}}_{i,\alpha}\} = 1$, the multiobjective optimisation is equal to the single objective optimisation result (i.e. no conflicting objectives). The maximum number of PARETO-Frontiers is $n_{\text{PARETO}} = 2^{n_{\mathrm{z}}}$. The PARETO-Frontiers for two-dimensions are the solutions of the $n_{\text{PARETO}} = 4$ multiobjective optimisation tasks

$$\begin{aligned} &\underline{\mathcal{P}}_{1,\alpha} = \min_{\mathbf{x}_\alpha} (+z_1, +z_2), \quad \underline{\mathcal{P}}_{2,\alpha} = \min_{\mathbf{x}_\alpha} (+z_1, -z_2), \\ &\underline{\mathcal{P}}_{3,\alpha} = \min_{\mathbf{x}_\alpha} (-z_1, -z_2) \text{ and } \underline{\mathcal{P}}_{4,\alpha} = \min_{\mathbf{x}_\alpha} (-z_1, +z_2), \end{aligned} \tag{9}$$

as can be seen in Fig. 5. The positive results $+z$ and the negative results $-z$ are related to a minimum or maximum task, respectively. The multiobjective optimisations can be solved by e.g. the 'NSGA-II' algorithm [3].

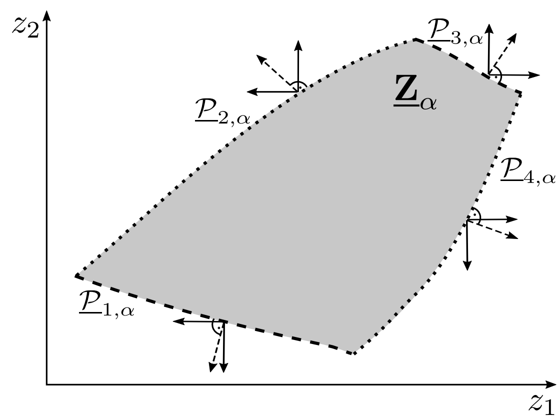


Figure 5: PARETO-Frontiers for a two-dimensional result quantity.

6 EXAMPLES

6.1 Uncertain pavement simulation

Model description and uncertainty modelling In this example, the influence of unknown environmental conditions and varying loading scenarios to an asphalt structure is observed. The numerical model includes the material modelling of asphalt, the discretization of a truck tyre and the consideration of a interface layer between these systems [11]. The asphalt material parameters are identified on the basis of 30 cyclic triaxial material tests with different loading conditions. The identification of the six material parameters was performed by the 'minimum root mean square error' method, separately for each sample. In Fig. 6, the identified material parameters are shown as scatter plot.

Due to the unknown loading conditions – especially frequencies – uncertainty modelling provides the basis for a generalized material description. Therefore, uncertainty exists due to the unknown loading conditions. This kind of uncertainty can be modelled by fuzziness sufficiently. In the following, two modelling scenarios are compared: without and with prior dependencies of the fuzzy input quantities.

The modelling of the fuzzy material parameters is shown in Fig. 6. The definition of the membership functions (blue colour) is derived from the histogram and expert knowledge gained on further investigations. The second modelling scenario considers 16 linear constraints h_i (see Eq. (6)), containing parameter combinations, which cannot be physically motivated. The definition of h_i is done on the basis of two input variables each identified by the scatter plot. The resulting non-permissible parameter combinations are marked in Fig. 6 by red colour. The constraints is applied to all α-levels.

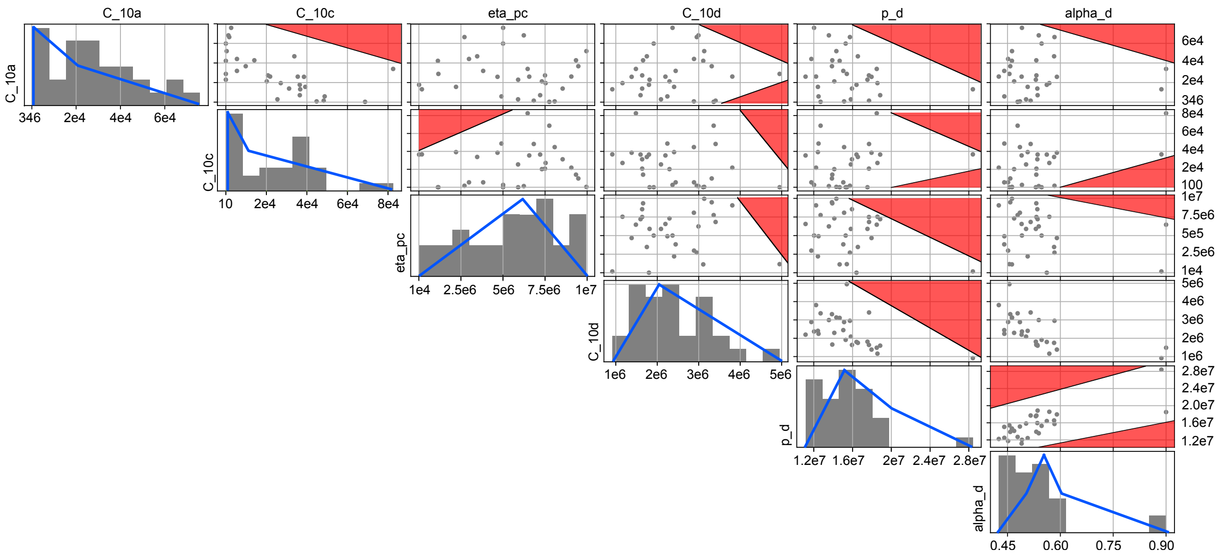


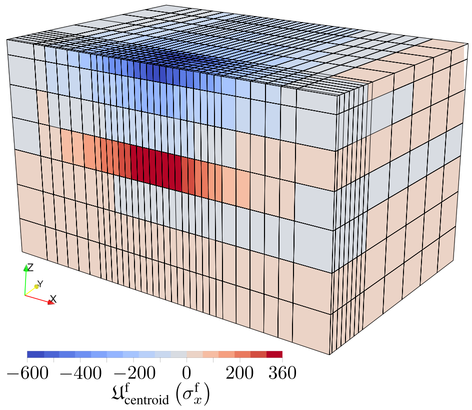
Figure 6: Scatter plot of the identified material parameters (including histograms) and modelling of fuzzy input quantities (including interaction)

Uncertainty analysis and results The analysis is carried out by the method proposed in Section 3. The fuzzy sampling scheme was applied for four α-levels $\alpha \in \{0, 0.5, 0.75, 1\}$ by in total $n_{\text{sim}} = 1680$ simulations. The computation of the relevant fuzzy result stresses $\sigma^{\text{f}}_{x,y,z}$ needs the computation of 38250 fuzzy quantities. In the following, the evaluation of these results is carried out for the region of high significance to the pavement subsystem (below the tyre contact area). The relevant parameter is the stress σ^{f}_{x}. The visualization of spatially dependent fuzzy result quantities requires information reducing measures. The entire evaluation of the uncertain structural response needs both, representative and uncertainty characterising measures. In Fig. 7, these results are shown for both modelling scenarios.

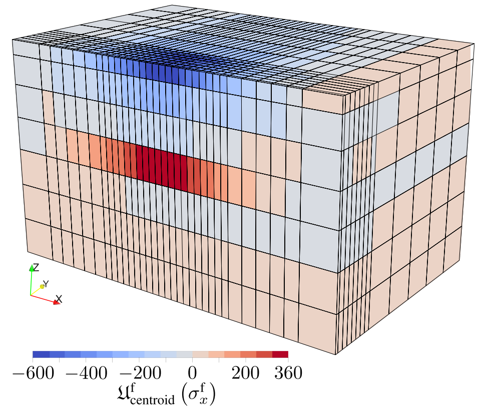
It can be seen, that the centroid values (Fig. 7a) and Fig. 7b)) can be interpreted as equivalent to a deterministic solution. The highest stresses are observed at the surface layer and at the bottom of the asphalt base layer, which stands for the characteristic bending like behaviour.

The area value (Fig. 7c) and Fig. 7d)) indicates areas of high and low uncertainty, which is the main benefit of an uncertainty analysis. Noticeable is the large gradient within the top surface in the tyre contact area. This phenomenon is not recognizable without uncertainty analysis. Furthermore, it can be seen, that the centroidal and the area value are independent.

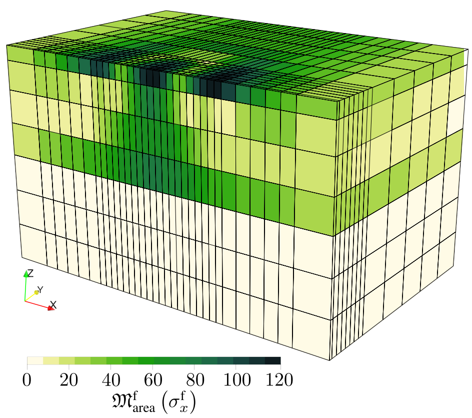
The comparison of the two modelling scenarios shows, that there is no influence of the consideration of the parameter interaction with respect to the centroidal value, see (Fig. 7a) and Fig. 7b)). A difference can be seen for the area value in (Fig. 7c) and Fig. 7d)). Mainly, the uncertainty of the stress σ_x decreases, if parameter interaction is considered. In other words, if a non-physical parameter combination is considered during the fuzzy analysis (modelling scenario 1), the uncertainty of the results is overestimated.



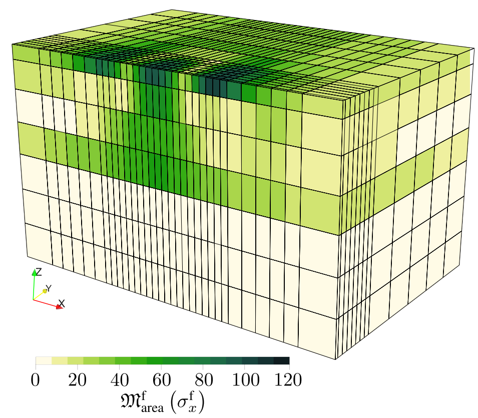
a) Centroidal value (without interaction)



b) Centroidal value (including interaction)



c) Area value (without interaction)



d) Area value (including interaction)

Figure 7: Comparison on fuzzy results σ^{f}_x

6.2 Demonstrative example for posterior dependency

In this example, the relevance for considering posterior dependency, see Section 5, is demonstrated on the basis of a test example proposed in [10]. The observed model is a complex eigenvalue problem of a synthetic dynamical system: $\mid \underline{\mathbf{A}}(\underline{p}^{\mathrm{f}}) - \lambda^{\mathrm{f}} \cdot \underline{\mathbf{I}} \mid = 0$ with the system matrix

$$\underline{\mathbf{A}}(\underline{p}^{\mathrm{f}}) = \begin{pmatrix} \underline{\mathbf{0}} & \underline{\mathbf{I}} \\ -\underline{\mathbf{a}}_1(p^{\mathrm{f}}_1, p^{\mathrm{f}}_2) & -\underline{\mathbf{a}}_2(p^{\mathrm{f}}_1, p^{\mathrm{f}}_2, p^{\mathrm{f}}_3) \end{pmatrix} \tag{10}$$

including the components

$$\underline{\mathbf{a}}_1 = \frac{1}{2}\frac{p_1^{\mathrm{f}}}{p_2^{\mathrm{f}}} \cdot \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \text{ and } \underline{\mathbf{a}}_2 = \frac{3}{4}\frac{p_2^{\mathrm{f}}}{p_1^{\mathrm{f}}} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + p_3^{\mathrm{f}} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - p_2^{\mathrm{f}} \cdot p_3^{\mathrm{f}} \cdot \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \quad (11)$$

The uncertain parameters are the three fuzzy triangular variables

$$p_1^{\mathrm{f}} = \langle 0.85, 1.0, 1.15\rangle,\ p_2^{\mathrm{f}} = \langle 0.85, 1.0, 1.15\rangle \text{ and } p_3^{\mathrm{f}} = \langle -0.1, 0.0, 0.1\rangle. \quad (12)$$

The computation of two-dimensional fuzzy result quantities is done with three different methods, whereby the pairs of real and imaginary part of the first two eigenvalues are considered, thus $\mu(\mathrm{real}(\lambda_1), \mathrm{imag}(\lambda_1))$ and $\mu(\mathrm{real}(\lambda_2), \mathrm{imag}(\lambda_2))$ are computed.

The results of a sampling-based approach (with 2000 samples per α-level) are depicted in Figs. 8a) and 9a). Additionally, the results of the convex hull, computed by the 'Qhull' algorithm, are shown. The two two-dimensional membership functions are visualized by coloured membership areas. It can be seen, that with the convex hull approach non-convex bounds of the membership function cannot be identified precisely. Furthermore, the sparsely distributed points indicate regions of high-sensitivity, such that the results are not fully trustable.

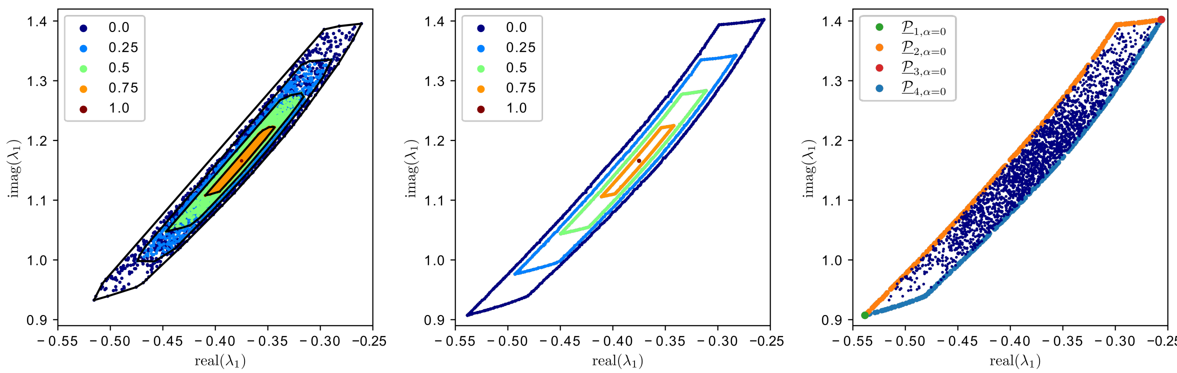
The third results are computed by the proposed multiobjective optimisation approach. The four membership bounds are computed for each α-level and for both membership functions. The optimization tasks were solved by the 'NSGA-II' algorithm [3] using a population size of 200 within 25 generations. In Figs. 8b) and 9b) these results are given. It has to be remarked, that the real Pareto results (point set) are interpolated for the continuous representation. To compare the results with the sampling approach $\alpha = 0$ is selected, see Figs. 8c) and 9c). It can be seen, that the regions of sparse point density are clearly surrounded by the Pareto frontiers. Another noticeable result is, that the Pareto frontiers $\underline{\mathcal{P}}_{1,3}$ (according to Fig. 5) containing one point only and are not relevant for this result.

It can be seen, that the computation of multidimensional fuzzy results is highly important to evaluate the influence of result quantities. By neglecting these posterior dependencies, the strong overestimation of uncertainty leads to unrealistic results.

7 CONCLUSIONS

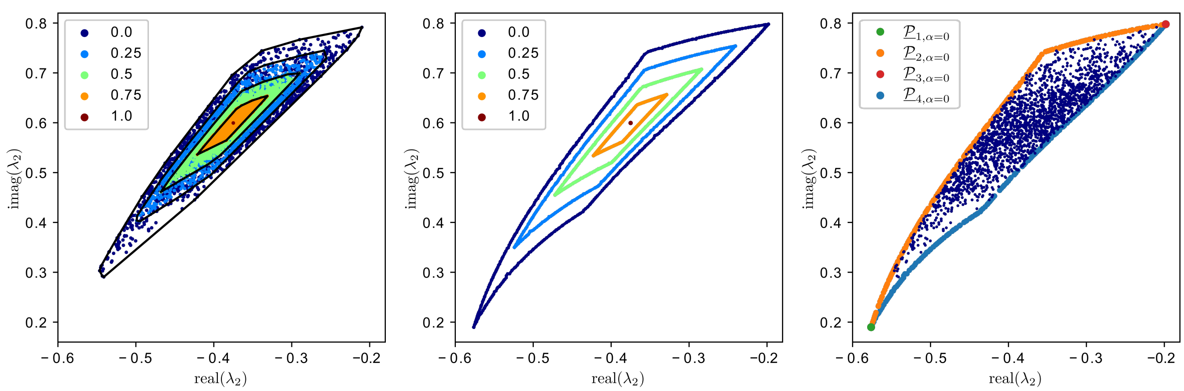
In this contribution dependencies within fuzzy analysis are addresses. The four types of dependency are described and numerical approaches are given. Functional input dependencies can be handled by time dependent fuzzy processes or spatially dependent fuzzy fields. Prior dependency describes the relations within the fuzzy input vector, e.g. due to non-physical domains and can be considered by confined α-level domains. If independence is assumed, the uncertainty is overestimated. For result quantities, the functional output dependency allows the computation of time dependent fuzzy result fields, e.g. in the context of finite element analysis. This ability for evaluating the uncertain results in the same manner as the deterministic pendant (time-dependent 3D visualisation) is highly beneficial. An additional advantage is, that no a priori identification of any quantity of interest (QoI) is necessary. For the posteriori dependency a new algorithm is proposed, by multiple solving of multiobjective optimisation tasks. This optimisation based method enables identifying dependencies within the fuzzy results, such that the overestimation of uncertainty is limited.

The advanced consideration of dependencies within fuzzy analysis yields more realistic results, due to a reduced overestimation of uncertainty. Furthermore, the additionally gathered information can be used to increase the understanding of the simulation and uncertainty models by a more comprehensive discussion of the uncertain results.



a) Convex-Hull approach b) Multiobjective optimisation c) Comparison for $\alpha = 0$

Figure 8: Two-dimensional fuzzy result for λ_1 discretised to five α-level



a) Convex-Hull approach b) Multiobjective optimisation c) Comparison for $\alpha = 0$

Figure 9: Two-dimensional fuzzy result for λ_2 discretised to five α-level

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