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TWO-PHASE FLOW MODELING FOR GAS-SOLID GRANULAR MIXTURES IN DENSE AND DILUTE CONCENTRATION

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Abstract. We consider modelling gas-solid granular mixture flow in a wide range of solid volume fractions. In such flows, concentration of the solid phase changes from dilute (low concentration) to dense (high concentration). Several macroscopic phenomenological models have been developed for gas-solid granular medium. The 6-wave 7-equation Baer-Nunziato model (BN) has been designed for gas-solid granular mixtures with dense concentration of the solid phase. Dilute mixtures are commonly modeled with the one-pressure model that is not hyperbolic in the whole range of model parameters. In the paper we are concerned with the problem of combaining the two models. We develop a new model of two-phase compressible flows which is hyperbolic and thermodynamically consistent and covers a wide range of solid phase concentrations in an unique approach. We tets the proposed model on several problems where the flow regime is changed from dilute to highly packed mixtures. The numerical calculations are carried out with moving adaptive Eulerian grids. The Godunov method is used with approximation of the non-conservative numerical flux with the HLLEM method.

1 INTRODUCTION

Nowadays, there are several phenomenological mechanical models developed for simulating compressible two-phase flows. Among them the 6-wave 7-equation Baer-Nunziato model (BN) [1] has been designed for gas-solid granular mixtures with dense concentration of the solid phase. This model is hyperbolic and has 6 characteristic velocities which represent material velocities and also plus/minus acoustic velocities in each phase. For dilute flow regimes where the volume fraction of solid phase is small, the one-pressure model [2-4] is employed. However, the one-pressure model is not hyperbolic in the whole range of model parameters and deteriorates as the concentration of solid granules increases.

Modeling two-phase flows is closely related to the notation of phase connectivity. For example, in the BN model both the solid and the gas phase are treated as connected and perturbations can propagate in each of the phases. On the other hand, in the model [4] only the gas phase is connected; the solid phase is considered as the number of non-connected dilute granules, and acoustic wave should not exist in this phase.

Phase connectivity for flows of gas-solid mixtures can be treated depending on the local value of phase volume fraction. For high concentrations of the solid phase, particles form the connected close-bed structure. For low concentrations, solid particles are rarefied in gas and do not interact each other. Therefore, one can suggest a simple criteria to distinguish phase connectivity: if the solid volume fraction is less than a critical value α^* than the solid phase is not connected, and the model of [4], or the Marble model [5] can be implemented. Otherwise, the solid phase is treated as connected, and the BN model is applied. However, realization of this approach is not trivial as the merging models have different number of equations and use different closure relations.

The absence of a general model that can describe the whole range of two-phase granular flow regimes is discussed in [6]. In this paper, an alternative approach to the BN model with improved acoustic properties is proposed. However, the thermodynamic consistency for this model is proven only for the stiff pressure relaxation limit. Once we admit the model derived from this limit, the entropy condition will be held, but the conservation of total energy will not because the limit model equations are derived from the internal energy equations. To remove this defect, authors of [6] suggest to replace one internal energy equation with the equation for total mixture energy conservation. However, in this case the entropy condition will be violated.

A thermodynamically consistent hyperbolic model is proposed in [7] based on the generalization of the BN model. This is a 6-wave model with the linear approximation of the interfacial pressure and velocity. As the BN model, the model [7] covers only one case of phase connectivity when the both phases are connected.

In the present paper we develop an approach alternative to the models [6] and [7]. It is based on the conservation laws for the phases. The model is hyperbolic and satisfies the entropy inequality. The model covers all cases of the phase connectivity in a unique approach. Moreover, the proposed approach allows constructing smooth transition between different cases of phase connectivity without losing the properties of hyperbolicity and thermodynamical consistency. The model is applicable for simulating flows with change in phase connectivity.

In Section 2, we give a general framework of the model to be considered. Next we consider phasic entropy equations and derive a closure relation for the solid volume fraction that ensures the mixture entropy inequality to be held. Finally we show results of some test calculations which demonstrate capability of the method to calculate two-phase transitional flows with change in phase connectivity.

2 BASIC PRINCIPLES OF THE MATHEMATICAL MODEL

We employ the phenomenological approach to model two-phase dispersed flow problems. The phases (mixture components) are indicated by the subscript 1 and 2. Each phase is characterized by volume fraction α, material density ρ^0, average density $\rho = \alpha\rho^0$, pressure p, internal energy e, velocity vector $\mathbf{u}$. The general system of equations that represents conservation laws for mass, momentum, and total energy of the mixture components is written in the following form:

$$\begin{aligned}
&\frac{\partial \rho_i}{\partial t} + \nabla \rho_i \mathbf{u}_i = m_{ij} \\
&\frac{\partial \rho_i \mathbf{u}_i}{\partial t} + \nabla \cdot \left(\rho_i \mathbf{u}_i \otimes \mathbf{u}_i\right) = \nabla \sigma_i + \rho_i \mathbf{f}_i + \mathbf{P}_{ij} \\
&\frac{\partial \rho_i E_i}{\partial t} + \nabla \left(\rho_i E_i \mathbf{u}_i\right) = \nabla \left(c_i - q_i\right) + \rho_i \mathbf{f}_i \mathbf{u}_i + E_{ij} + Q_i
\end{aligned} \tag{1}$$

where $E = e + 0.5\mathbf{u}^2$ is the total energy, σ is the stress tensor representing surface forces acting on the phase, c is the vork done by the surface forces, m_{ij}, $\mathbf{P}_{ij}$, and E_{ij} are mass, momentum, and energy exchange terms (the quantity transfering from *i* to *j* component), $\mathbf{f}$ and q are external volumetric force and heat flux, respectively, Q is the energy released due to mass transfering. The exchange terms satisfy the conservation relations,

$$m_{12} + m_{21} = 0\,, \quad \mathbf{P}_{12} + \mathbf{P}_{21} = 0\,, \quad E_{12} + E_{21} = 0 \tag{2}$$

For simplicity we will assume in what follows that $m_{ij} = 0$, and also $\mathbf{f}_i = q_i = Q_i = 0$.

The surface force acting on the phase across a surface element $dS = dS_1 + dS_2$ is represented in terms of two tensors:

$$\sigma_i dS = \sigma_i^0 dS_i + \sigma_{ji} dS_j = \left(\alpha_i \sigma_i^0 + \alpha_j \sigma_{ji}\right) dS$$

where σ_i^0 is the phasial stress, and σ_{ij} is the interphasial stress which must satisfy the following consistency condition:

$$\alpha_1 \sigma_{12} + \alpha_2 \sigma_{21} = 0 \tag{3}$$

The term $\mathbf{P}_{ij}$ in (1) represents the volumetric exchange of momentum between the phases. To model the energy exchange terms, we introduce an additional vector parameter $\mathbf{v}_*$ which defines the characteristic interface velocity so that

$$E_{ij} = \left(\mathbf{P}_{ij}, \mathbf{v}_*\right) + q_{ij} \tag{4}$$

with q_{ij} being the heat transfer flux from the *i* to *j* phase, $q_{12} + q_{21} = 0$. The work of surface forces can be then written as

$$c_i = \alpha_i \sigma_i^0 \mathbf{u}_i + \alpha_j \sigma_{ij} \mathbf{v}_* \tag{5}$$

Introducing the material derivative for each phase,

$$\dot{\varphi}_i = \frac{\partial \varphi_i}{\partial t} + \left(\mathbf{u}_i, \nabla\right) \varphi_i$$

and taking into account the above notations, the system of equations (1) can be recast in the following form:

$$\begin{aligned}
&\alpha_i \dot{\rho}_i^0 + \rho_i^0 \dot{\alpha}_i + \alpha_i \rho_i^0 \nabla \mathbf{u}_i = 0 \\
&\rho_i \dot{\mathbf{u}}_i = \nabla \left(\alpha_i \sigma_i^0 + \alpha_j \sigma_{ji}\right) + \mathbf{P}_{ji} \\
&\rho_i \dot{e}_i = \left(\alpha_j \sigma_i^0, \nabla\right) \mathbf{u}_i + \left(\mathbf{v}_* - \mathbf{u}_i\right), \nabla \left(\alpha_j \sigma_{ji}\right) + \left(\alpha_j \sigma_{ji}, \nabla\right) \mathbf{v}_* + \mathbf{P}_{ji}, \left(\mathbf{v}_* - \mathbf{u}_i\right) + q_{ji}
\end{aligned} \tag{6}$$

We assume the hydrodynamic approximation and take the phasial stress tensor in the diagonal form, $\sigma_i^0 = -p_i \mathbf{I}$. Next step in constructing the model is very important. So far the model has been symmetrical, the phases have been equivalent, and we could invariantly change the indexes *1* and *2* in the equations (2). Now we want to distinguish the phases assuming that one of them consists of small dispersed non-connecting inclusions (granules).

Let, for example, the subscript *i=2* indicate this dispersed phase. Then we can introduce another additional parameter P_* which will define the characteristic interphasial pressure so that

$$\sigma_{21} = -P_* I\ , \qquad \mathbf{P}_{12} = -\alpha_2 \nabla P_* + \mathbf{f}_* \tag{7}$$

where $\mathbf{f}_* = K(\mathbf{u}_1 - \mathbf{u}_2)$ is the viscous force between the phases (viscous friction), and K is a positive parameter ($K \geq 0$).

In terms of the interphasial pressure, the phasial momentum equations are written in the form

$$\begin{cases} \rho_1 \dot{\mathbf{u}}_1 = \nabla\left(-\alpha_1 P_1\right) - P_* \nabla \alpha_2 - \mathbf{f}_* \\ \rho_2 \dot{\mathbf{u}}_2 = \nabla\left(-\alpha_2 P_2\right) + P_* \nabla \alpha_2 + \mathbf{f}_* \end{cases}, \tag{8}$$

and the internal energy equations take the form

$$\begin{cases} \rho_1 \dot{e}_1 = -\left(\alpha_1 P_1, \nabla\right)\mathbf{u}_1 - \left(\alpha_2 P_*, \nabla\right)\mathbf{v}_* - \left(\mathbf{v}_* - \mathbf{u}_1, \mathbf{f}_*\right) + \left(\mathbf{v}_* - \mathbf{u}_1, -P_* \nabla \alpha_2\right) \\ \rho_1 \dot{e}_2 = -\left(\alpha_2 P_2, \nabla\right)\mathbf{u}_2 + \left(\alpha_2 P_*, \nabla\right)\mathbf{v}_* + \left(\mathbf{v}_* - \mathbf{u}_2, \mathbf{f}_*\right) + \left(\mathbf{v}_* - \mathbf{u}_2, P_* \nabla \alpha_2\right) \end{cases} \tag{9}$$

Phasial densities, pressures, and internal energies are supposed to be in functional dependency by means of appropriate equations of state describing thermodynamical properties of materials, $e_i = e_i(\rho_i^0, p_i)$. The considered system of governing equations (the first equation in (6), (8), and (9)) is not closed. For its closure, we need to define interphase parameters $\mathbf{v}_*$ and P_*, and also derive an additional equation to qualify the evolution of the volume fraction. With this purpose, we proceed to the analysis of mixture entropy that may give us a clue to how these closure issues should be settled.

3 ANALYSIS OF ENTROPY EQUATIONS

By using the fundamental thermodynamical relation that relates variations in entropy, specific volume, end internal energy, $T\dot{s} = \dot{e} + p\dot{\rho}/\rho^2$, where T is the temperature and s is the specific entropy, one can derive phasial entropy equations as consequence of the above conservation laws,

$$\begin{aligned} \rho_1 T_1 \dot{s}_1 &= \dot{\alpha}_1 P_1 - \alpha_2 P_* \nabla \cdot \boldsymbol{v}_* - \left(\boldsymbol{v}_* - \boldsymbol{u}_1, \boldsymbol{f}_*\right) - \left(\boldsymbol{v}_* - \boldsymbol{u}_1, P_* \nabla \alpha_2\right) \\ \rho_2 T_2 \dot{s}_2 &= \dot{\alpha}_2 P_2 + \alpha_2 P_* \nabla \cdot \boldsymbol{v}_* + \left(\boldsymbol{v}_* - \boldsymbol{u}_2, \boldsymbol{f}_*\right) + \left(\boldsymbol{v}_* - \boldsymbol{u}_2, P_* \nabla \alpha_2\right) \end{aligned} \tag{10}$$

These two equations define the change in time of the total entropy of the mixture,

$$\begin{aligned} \rho_1 \dot{s}_1 + \rho_2 \dot{s}_2 = &\left(\frac{P_2}{T_2} - \frac{P_1}{T_1}\right)\frac{\partial \alpha_2}{\partial t} + \left[\frac{P_2 - P_*}{T_2}\boldsymbol{u}_2 - \frac{P_1 - P_*}{T_1}\boldsymbol{u}_1\right]\nabla \alpha_2 + \\ &+ P_*\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\nabla\left(\alpha_2 \boldsymbol{v}_*\right) + \left(\frac{\boldsymbol{v}_* - \boldsymbol{u}_2}{T_2} - \frac{\boldsymbol{v}_* - \boldsymbol{u}_1}{T_1}\right)K\left(\boldsymbol{u}_1 - \boldsymbol{u}_2\right) \end{aligned} \tag{11}$$

which must be non-negative thanks to the second law of thermodynamics, $\rho_1 \dot{s}_1 + \rho_2 \dot{s}_2 \geq 0$.

To meet this condition, we first define the interphasial velocity to be an average between phasial velocities, $\boldsymbol{v}_* = \omega \boldsymbol{u}_1 + (1-\omega)\boldsymbol{u}_2$, where ω is the weight coefficient, $0 \leq \omega \leq 1$. Such a closure relation will ensure positivity of the last term in the r.-h.s. of (11). Then, to assure the entropy inequality, we require other terms in the r.-h.s. of (11) to vanish, i.e.,

$$\left[\frac{P}{T}\right]\frac{\partial \alpha_2}{\partial t} + \left[\frac{P - P_*}{T}\boldsymbol{u}\right]\nabla \alpha_2 + P_*\left[\frac{1}{T}\right]\nabla \cdot \left(\alpha_2 \boldsymbol{v}_*\right) = 0 \tag{12}$$

where the square brackets denote the difference between phasial parameters, $[\bullet]=(\bullet)_2-(\bullet)_1$.

Eq. (12) is considered as the closure relation to the model developed which defines how the volume fraction of the dispersed phase is evolving in space. It can be written as the advection equation for α_2 with a source term,

$$\frac{\partial \alpha_2}{\partial t}+\boldsymbol{u}_J\nabla\alpha_2=-\alpha_2\theta\nabla\cdot\boldsymbol{v}_*,$$
$$\mathbf{u}_J=\left[\frac{(P-P_*)\mathbf{u}+\mathbf{v}_*P_*}{T}\right]\Big/\left[\frac{P}{T}\right],\quad \theta=\left[\frac{P_*}{T}\right]\Big/\left[\frac{P}{T}\right] \tag{13}$$

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Thus, the system of governing equations for modeling two-phase compressible dispersed flows is given by two mass conservation equations of (6), momentum equations (8), energy equations (9), and volume fraction equation (13). Further qualification of this model is based on the analysis of eigenvalues for the matrix of this system. In order for the model developed to be evolutionary, the system of governing equations has to be hyperbolic, i.e. it has to possess a set of real eigenvalues. Full analysis of eigenvalues of the model for arbitrary interphasial velocity and pressure is out of scope of the presented paper. We are concerned here with only simple choice $\boldsymbol{v}_*=\boldsymbol{u}_2$ and $P_*=p_1$. With this choice, the advection velocity in Eq.(13) $\boldsymbol{u}_J=\boldsymbol{u}_2$, and the parameter $\theta\approx 1$ when the phasial pressures are near to equilibrium, $p_1\approx p_2$. In this limit the equation for α_2 in (13) takes the form of conservation law, and as consequence the density of the dispersed phase is kept constant. This situation relates to the regime of dilute flow.

4 NUMERICAL RESULTS

In this section, we consider application of the proposed model to simulation of dense-to-dilute gas-solid flows. We employ the second order Godunov-type method with uniform Euler grids. The HLLEM scheme is used to approximate the Riemann problem solution [8].

The first problem to be considered is the interaction of a shock wave propagating in air with a semi-infinite cloud of small aluminum particles. Diameter of particles is 1 mkm.

The problem can be formulated as a Riemann Problem. The calculation domain is an interval $[0,10]cm$. The grid consists of 1000 computational cells. Initial discontinuity is located at $x=6cm$. The initial data are: $\rho_1=1.16\cdot10^{-3}\ g/cm^3$, $\rho_2=2.71\ g/cm^3$, $u_1=u_2=0$, $P_1=P_2=1\ bar$, $\alpha_2=\alpha_2^0$ on the left from the initial discontinuity, and $\rho_1=4.47\cdot10^{-3}\ g/cm^3$, $u_1=-772\ m/s$, $P_1=10.334\ bar$, $P_1=10.334\ bar$ on the right. We consider four cases of initial volume fraction α_2^0 in the cloud: α_2^0=0.001, 0.01, 0.1, and 0.5.

The ideal gas EOS for air is used with the adiabatic exponent $\gamma=1.4$. For aluminum particles, the Mi-Gruneisen EOS is used:

$$P_2=\Gamma_2\rho_2(e_2-e_x)+\frac{\rho_{20}C_{20}^2}{n}\left(\left(\frac{\rho_2}{\rho_{20}}\right)^n-1\right)$$
$$e_x=\frac{C_{20}^2}{n}\left(\frac{1}{n-1}\left(\frac{\rho_2}{\rho_{20}}\right)^{n-1}+\frac{\rho_{20}}{\rho_2}-\frac{n}{n-1}\right) \tag{14}$$

with parameters: $\rho_{20} = 2.71\ \ g/cm^3$, $C_{20} = 5333\ \ m/s$, $n = 3.5$, $\Gamma_2 = 2.13$.

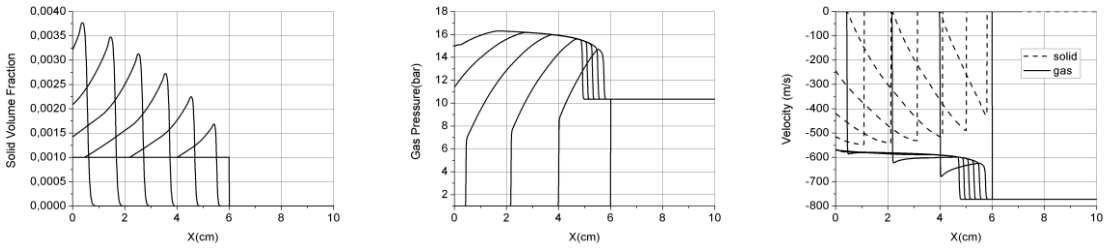

Figure 1: Solid volume fraction, Pressure and Velocities distributions for the problem of shock wave interaction with aluminum dust cloud with initial dust volume fraction 0.001.

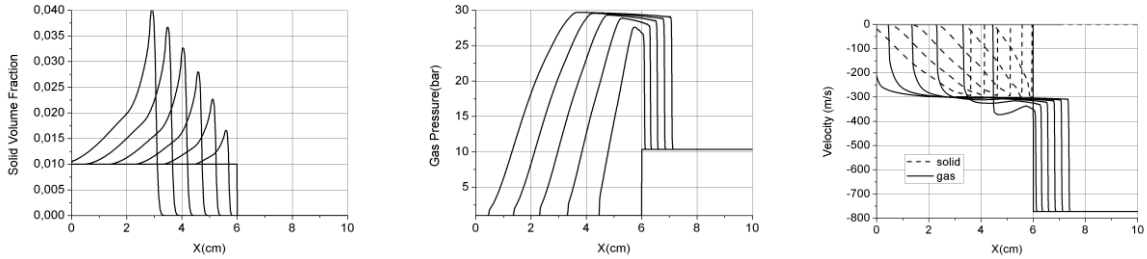

Figure 2: Solid volume fraction, Pressure and Velocities distributions for the problem of shock wave interaction with aluminum dust cloud with initial dust volume fraction 0.01.

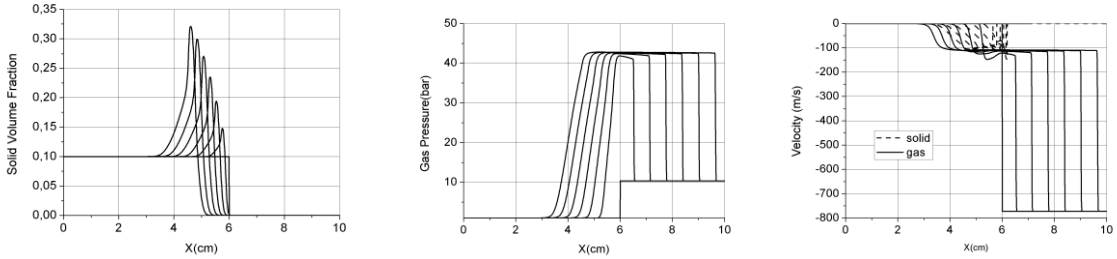

Figure 3: Solid volume fraction, Pressure and Velocities distributions for the problem of shock wave interaction with aluminum dust cloud with initial dust volume fraction 0.1.

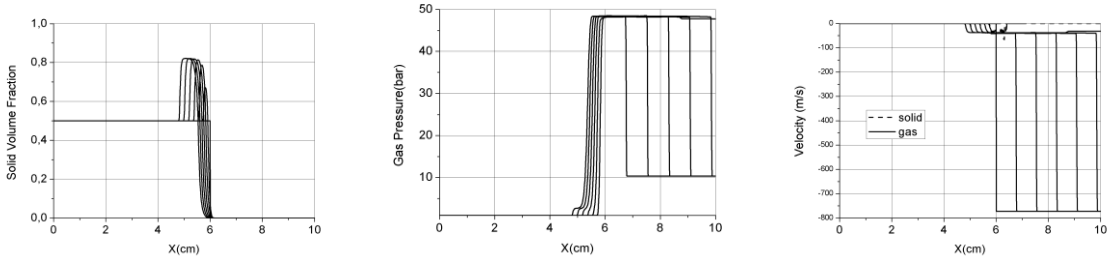

Figure 4: Solid volume fraction, Pressure and Velocities distributions for the problem of shock wave interaction with aluminum dust cloud with initial dust volume fraction 0.5.

In Figs. 1 – 4 we show numerical distributions of solid volume fraction, pressure, gas and solid velocity at time moments from 0 to 120 μs with the step 20 μs for four cases of

initial volume fraction in the dust cloud. As can be seen from this results, in the case of low dust volume fraction (0.001, 0.01) (see Figs. 1 and 2), the transmitted into the cloud shock wave propagates with a clear front followed by the relaxation zone where pressure is further increased. Estimations show that the velocity of the leading front of the transmitted shock is well correlated with the Wood's formulae. For time 60 μs, it propagates a distance 5.6 cm and 3.7 cm for the case of volume fraction 0.001 and 0.01, respectively. This corresponds the front velocity $93.3 m/s$ and $61.6 m/s$.

The front amplitude decreases while the region of relaxation increases as the initial volume fraction becomes larger. Fig. 3 shows results for $\alpha_2^0 = 0.1$. Here we can't distinguish the leading shock front; transmitted perturbation propagates in the dusty cloud with a lower velocity without any sharp front. The case of $\alpha_2^0 = 0.5$ is displayed in Fig. 4. It can be seen that in this case solid particles are compacted into the dense layered structure which further moves as a solid piston.

The second problem is the piston problem in a dust gas. The velocity of the piston is $10 m/s$. The mixture consists of small aluminum particles and air. We consider four cases when the initial volume fractions ϕ_2^0 equals to 0.001, 0.01, 0.1, and 0.5, respectively. Calculations are performed in the system of coordinates fixed with the piston. The calculation domain is an interval $[0,10] cm$. The EOSs for air and solid particles are taken the same as in the Problem 1.

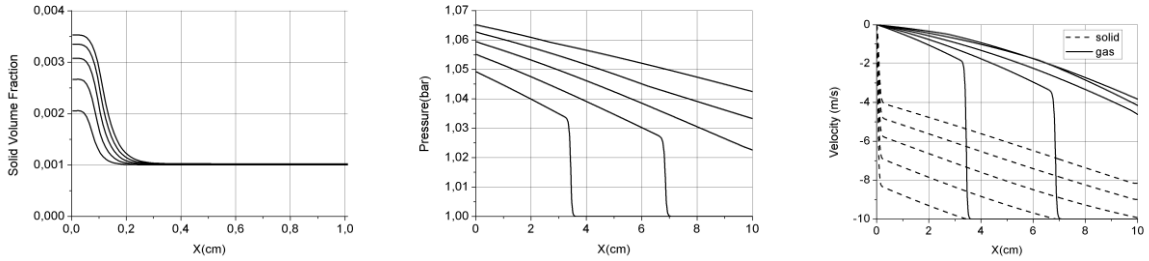

Figure 5: Solid volume fraction, Pressure and Velocities distributions for the problem of piston impact on the aluminum dust cloud with initial dust volume fraction 0.001.

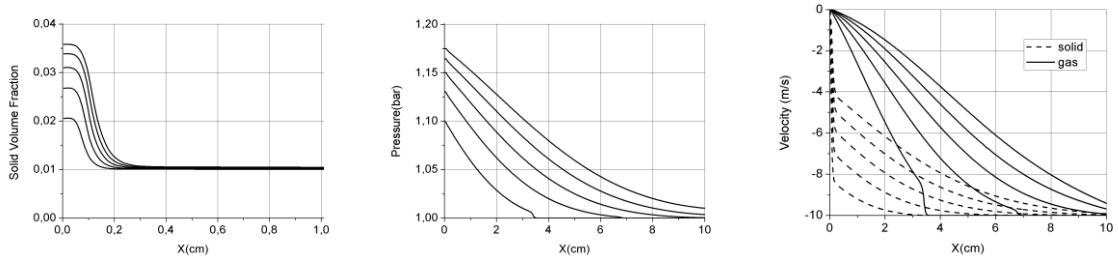

Figure 6: Solid volume fraction, Pressure and Velocities distributions for the problem of piston impact on the aluminum dust cloud with initial dust volume fraction 0.01.

In Figs.5-8 we show numerical distributions of solid volume fraction, pressure, gas, and solid velocity for several time moments from 0 to 500 μs with a step of 100 μs. The character of perturbation propagation is quite similar to that in previous calculations. When particles constitutes low fraction, the perturbation in the gas phase has the typical form of sharp shock

front with a region of relaxation. The amplitude of this front decreases with as the particle volume fraction becomes higher. The shock front velocity is estimated as $350m/s$. Perturbation in gas doesn't exhibit any shock front when the piston moves in the dusty air with higher packing of particles, $\alpha_2^0 = 0.1,\ 0.5$. A compaction wave is formed so that particles make dense close-bed structure near the piston. This dense layer prevent any perturbations run ahead of the compaction front. The phase velocity are rapidly come to equilibrium, and the perturbation region looks as a compaction wave that moves with a velocity of $25m/s$ relative to the piston for the case considered.

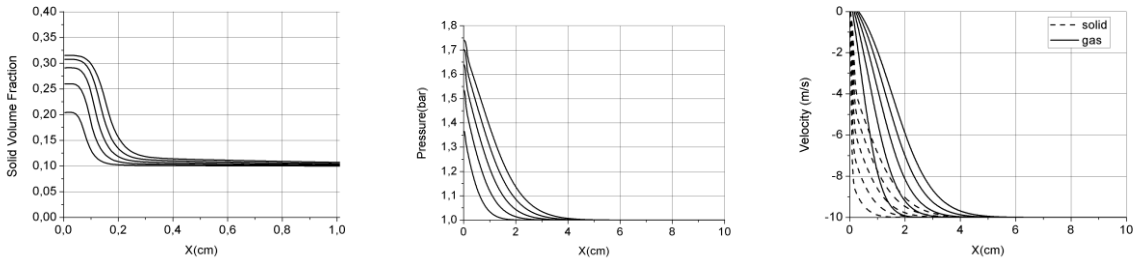

Figure 7: Solid volume fraction, Pressure and Velocities distributions for the problem of piston impact on the aluminum dust cloud with initial dust volume fraction 0.1.

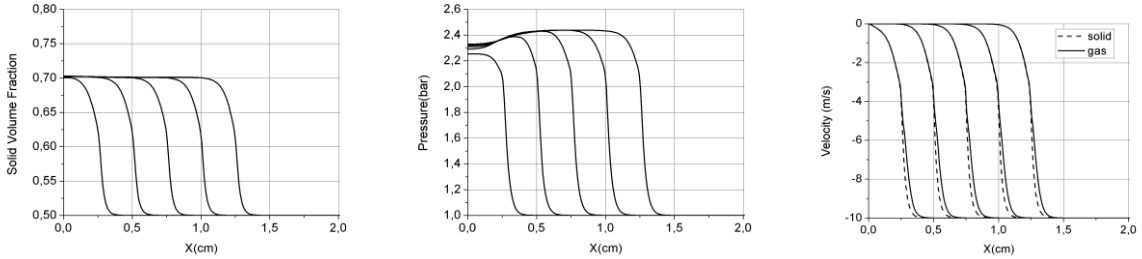

Figure 8: Solid volume fraction, Pressure and Velocities distributions for the problem of piston impact on the aluminum dust cloud with initial dust volume fraction 0.5.

5 CONCLUSIONS

- We have developed a new model for description of two-phase compressible dispersed (granular) flows that can be applied to the whole region of granular compositions, from dilute to dense packed. The model is based on basic conservation equations for mass, momentum, and energy. The closure relation for the volume fraction of the dispersed phase has been derived from the analysis of the mixture entropy equation. We test the proposed model on several problems where the flow regime is changed from dilute to highly packed mixtures.
- The model proposed has been tested on two problems that involve shocked flows in the mixture of air and small aluminum particles with different dense-to-dilute particle compositions. The results obtained have shown capability of the method to solve two-phase compressible flow problems in the wide range of volume fractions.

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REFERENCES

[1] Baer, M.R. and Nunziato, J.W. A two-phase mixture theory for the deflagration-to-detonation transition (DDT) in reactive granular materials. *Int. J. Multiphase Flow(* 1986) **12**:861–889.

[2] Korobeinikov, V.P., Markov V.V., Menshov, I.S. *Mathematical modeling of blast wave propagation in gas particle mixtures.* III International school on Exposibility of Industrial Dust, Turawa 5-7 Nov., (1982) Papers. /P. Wolanski (Ed.). - Turawa, 1982.

[3] Fedorov, A.V., Fomin, V.M., Khmel, T.A. *Wave processes in gas suspension of metal particles.* Novosibirsk (2015).

[4] Khomenko, Y P, Ischenko, A. N., and Kasimov, V. Z. *Mathematical modelling of interior ballistic processes in barrel systems* Novosibirsk: Publishing House of SB RAS *(*1999).

[5] Marble, F. E., *Combustion and Propulsion* (5th AGARD Colloquium) Pergamon Press, (1963), Vol. 175.

[6] Saurel, R., Chinnayya, A., and Carmouze, Q. Modelling compressible dense and dilute two-phase flows. *Physics of fluids* (2017)**29**:063301.

[7] Ambroso, A., Chalons, C., Raviart, P.-A. A Godunov-type method for the seven-equation model of compressible two-phase flow. *Computers & Fluids* (2012)**54**:67–91.

[8] Dumbser, M. and Balsara, D. A new efficient formulation of the HLLEM Riemann solver for general conservative and non-conservative hyperbolic systems *J. of Computational Physics* (2016)**304**:275 – 319.