

11 – 15 June 2018, Glasgow, UK

FE MODELS FOR THE EVALUATION OF HYDRODYNAMIC PRESSURE ON CONCRETE GRAVITY DAMS DURING EARTHQUAKES

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Key words: Concrete dams, FE Models, Fluid Structure Interaction, Frequency response analysis, 3D modelling.

Abstract: Earthquakes are a significant factor in the design and safety evaluation of dams and, in this context, an accurate prediction of forces acting on the structure is needed. The hydrodynamic action plays a key role in this regard and its accurate evaluation is a major issue. Most of the earthquake analyses of gravity dams in the past have ignored the interaction between dam and reservoir. The equivalent body of hydrodynamic forces, the “added mass” model, derived from the well-known Westergaard theory, is nowadays the most widespread approach still adopted in modern design codes. Nevertheless, taking into account Fluid Structure Interaction (FSI) is sometimes more appropriate, though more complicated.

In this work, hydrodynamic pressures on a large Italian gravity dam have been calculated using different modelling approaches: rigid barrier, deformable dam with added masses and FSI. Frequency domain analyses have been carried out both on plane strain models and on more refined 3D models, by applying a horizontal acceleration to the dam base. The comparison among frequency response curves obtained from 2D and 3D models highlights noticeable differences concerning resonance frequencies and peak response values, even with the same modelling approach.

This study demonstrates that simplified 2D added-mass models may be over-conservative compared to 3D FSI simulations.

Concluding, it can be argued that full 3D coupled analyses should be preferred to simplified 2D ones to estimate the hydrodynamic pressure acting on the upstream face of the dam during earthquakes.

1 INTRODUCTION

Hydrodynamic forces evaluation on water retaining structures during earthquakes is a challenging issue. In the past, hydrodynamic loads exerted by the reservoir on the upstream face of the dam were obtained from simplified theories, such as Westergaard’s [1] and

Zangar's [2], dating back to the first half of the twentieth century. Such theories, initially based on very simplistic hypotheses and successively developed by Chopra [3], are nowadays the base of main international codes and guidelines.

The well-known Westergaard simplified theory was developed in 1933 and originated the widespread added masses approach. This useful simplification adopted in highly sophisticated and irregular 3D models could however lead to inaccurate predictions. The possible differences are to be estimated in this work.

In this paper, the physical phenomenon has been investigated using Finite Element models by removing some of the simplifications underlying the aforementioned theories. Hydrodynamic pressures on an Italian large gravity dam have been calculated using different modelling approaches. The analysis has been carried out using two and three-dimensional models, and a parametric variation of boundary conditions has been performed.

2 ANALYTICAL SOLUTIONS

H.M. Westergaard's paper [1] dating back to 1933 is considered as the milestone for the evaluation of hydrodynamic loads on water retaining structures, given its simple yet elegant solution and physical interpretation of the phenomenon. The key assumptions of his solution are the following:

- 1) small displacements of water particles, neglecting global mass flow;
- 2) non-viscous fluid;
- 3) infinite upstream extension of the water reservoir;
- 4) no surface waves;
- 5) perfect waves reflection of the reservoir bottom;
- 6) horizontal and rigid water reservoir bottom surface;
- 7) rigid, sinusoidal motion of the dam;
- 8) plane symmetry of the problem;
- 9) earthquake dominant period higher than dam natural vibration period.

The first three hypotheses allowed obtaining a substantial simplification of the Navier-Stokes equation. These equations, valid for general fluid motion, turn into the more manageable linear wave equation, namely D'Alembert's equation

$$\nabla^2 p - \frac{1}{c^2} \cdot \frac{\partial^2 p}{\partial t^2} = 0 \quad (1)$$

where p is the pressure and c is the sound speed in the medium equal to $\sqrt{k/\rho}$, where k is the water bulk modulus and ρ its density. The solution is expressed as particle horizontal and vertical displacement, provided by Fourier series. The vertical displacements tend to infinity as the excitation period reaches the values:

$$T_n = 4h/(c \cdot n) \quad (2)$$

where h is the water height. If that is the case, resonance occurs and the corresponding values of frequency $f_n = 1/T_n$ are defined as reservoir *eigenfrequencies*. The solution of

equation (1) can be accomplished both in the time and in the frequency domain via the well-known Fourier transform.

In the same paper, Westergaard also introduced a simplified expression for the displacement and a physical interpretation of the hydrodynamic forces. Their effect can be estimated by neglecting their dependence on the vibration period and by approximating the pressure distribution by a parabolic expression. If one accepts the trade off, their behaviour is identical to that of a mass distribution $m(y)$ rigidly attached to the upstream face of the dam, according to the following distribution along the height y , measured downwards from the free surface

$$m(y) = \frac{7}{8}\rho\sqrt{hy} \quad (3)$$

In a later paper, Chopra [3] extended Westergaard's solution to the whole range of frequencies, by removing hypothesis 9, providing an analogous and more complicated Fourier series solution.

Analytical solutions of the problem did not have any major improvement over the decades, mainly because of the increasing availability of computing resources. Westergaard's solution remained the benchmark and the reference for the international codes up to modern days. In this paper the last three hypotheses are removed, and the arising consequences are analysed.

3 CASE STUDY

The aforementioned investigation is developed by means of a case study regarding an existing Italian dam. The structure is an ordinary concrete gravity dam made up of 27 concrete monoliths, with a maximum height of 65 m, a straight shape in plan, a vertical upstream face and a downstream face with a slope of 0.72. Concrete material parameters are summarized in table 1 while water level is supposed at maximum allowed of 60 m.

Table 1: Concrete material parameters.

<i>Young's Modulus (MPa)</i>	20550
<i>Poisson's ratio</i>	0.18
<i>Density (kg/m³)</i>	2450
<i>Structural damping ratio</i>	0.05

3.1 FEM analyses of 2D and 3D dam models

3.1.1 Modeling approach

2D and 3D finite element models have been created in COMSOL Multiphysics 5.3 software [4], as illustrated in the Figures 1 and 2. Each of the two models is studied under different hypotheses

- *Full reservoir – added masses*: hydrodynamic actions are introduced according to Westergaard's simplified expression (3) for equivalent masses rigidly connected to the structure. The correct implementation of such masses requires their value to be assigned only to the horizontal degree of freedom of the corresponding mesh element, leaving the

vertical contribution to inertial forces unmodified.

- *Full reservoir – fluid-structure interaction*; the fluid domain is explicitly modelled and an acoustic-structural interface is introduced at the dam upstream face.
- *Full reservoir – rigid structure*; this hypothesis is investigated mainly for comparison and interpretation of the previous models.
- *Empty reservoir*: the acoustic part of the model is ignored. This model is used for comparison purposes only, to appreciate the structural effects of the hydrodynamic force.

The 2D model (Fig. 1) is made up of 300 triangular second order Serendipity elements for the solid mechanics part and 624 quadrilateral second order Lagrange elements for the acoustics part, for a total of 841 nodes. The 3D model (Fig. 2) is made up of 8837 tetrahedral second order Serendipity elements for the solid mechanics part and 42384 tetrahedral second order Lagrange elements for the acoustics part, for a total of 17086 nodes.

The two models are analysed by performing a frequency domain study under harmonic base acceleration excitation.

3.1.2 Boundary conditions

On the solid mechanics component, a unit, horizontal acceleration is imposed at the dam base nodes, considering a rigid foundation motion. As for the acoustic component, a zero pressure condition is imposed at the water free surface and a rigid wall condition at the bottom of the reservoir. The fluid-structural interaction condition at the interface between the two domains is the following

$$\begin{cases} -n \left(\frac{1}{\rho} \nabla p \right) = -n \cdot u_{tt} \\ F_A = p \cdot n \end{cases} \quad (4)$$

where \mathbf{n} is the unit vector normal to the interface, \mathbf{F}_A is the acoustic force on the structure and \mathbf{u}_{tt} is the solid acceleration. The first equation transfers the structural acceleration to the fluid, while the second applies the fluid pressure load on the structure. Such a system of equations expresses a *fully coupled* problem, where the solid and the acoustic parts have to be solved simultaneously.

The original Westergaard formulation of semi-infinite domain is a reasonable practical assumption of a sufficiently long reservoir. This implies that none of the upstream waves is ever reflected back to the structure during the seismic event. It follows that a non-reflecting boundary condition is necessary in the upstream direction, namely the well-known Sommerfeld radiation condition. This can be accomplished in a finite element model through the *Perfectly Matched Layer (PML)* [5]. It consists in a complex coordinate stretching function $f_r(\xi)$ of an auxiliary absorbing domain

$$f_r(\xi) = s\xi \left(\frac{1}{3p(1-\xi) + 4} - \frac{i}{3p(1-\xi)} \right) \quad (5)$$

with ξ is the auxiliary stretched coordinate and s and p the scale and curvature parameters, respectively. In the present case an auxiliary PML domain has been added on the upstream side of the reservoir (Fig. 1) in the FE models.

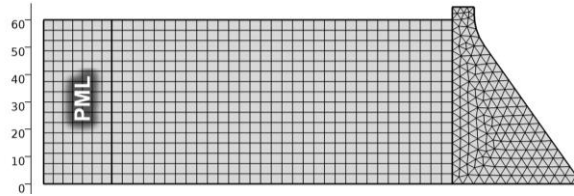


Figure 1: 2D Model.

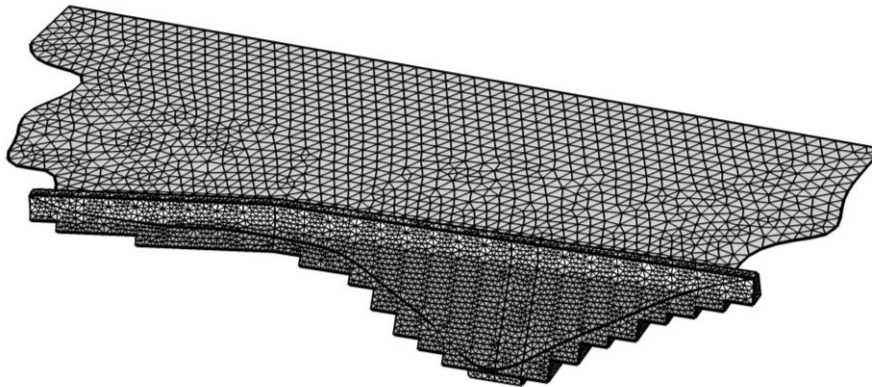


Figure 2: 3D model.

3.1.3 Analysis results

The 2D model introduces the deformability of the structure and extends the excitation frequency to the whole range of interest for seismic study purposes, thus removing hypotheses 3 and 9. The results are displayed in the form of acoustic pressure magnitude, being a complex quantity in a frequency domain solution. In Fig. 3 the pressure modulus is plotted over the domain.

The hydrodynamic load distribution along the dam upstream face is represented by the red curves and compared with Westergaard's theory prediction (black curves).

By examining the pressure distributions, it can be clearly seen that Westergaard's solution is reproduced only for frequency values tending to zero (top image). In fact, in this case the pressure distribution on the upstream face follows the theoretical quarter-wave shape, in good agreement with Westergaard's parabolic assumption. Higher excitation frequency values (bottom image) lead to the development of higher mode shapes for the pressure distributions as in the lower image, where a near three-quarter-wave distribution is displayed.

The hydrodynamic load distribution on the dam face is computed also for the 3D model, thus removing hypothesis 8 of planar behaviour. In Figure 4, in analogy with Figure 3, the blue surface represents the equivalent distributed load on the upstream face of the dam deduced by inertial added mass forces, while the red one is the acoustic pressure load obtained from the FE interacting model.

The 3D case displays a significant pressure variation along the cross valley direction,

which is impossible to be captured in a 2D modelling context. In addition, a significant phase difference between pressures on each monolith reduces the overall synchronism of hydrodynamic load and thus the amplification of its overall action on the structure.

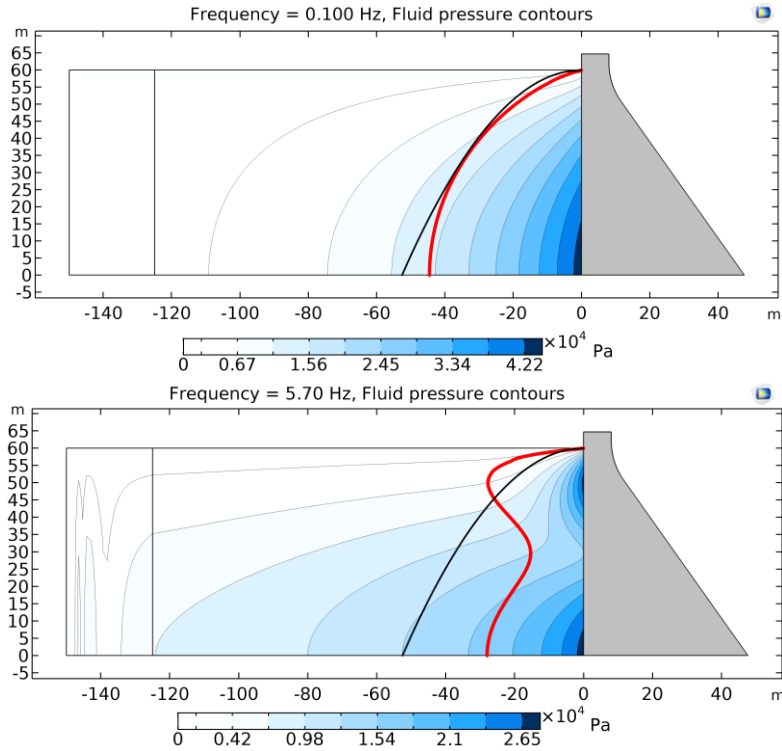


Figure 3: 2D model response for 0.1 Hz (upper image) and 5.7 Hz (lower image). The linear distributed forces on the upstream face are represented in red, in comparison with Westergaard's solution (black curve).

Model response is evaluated by plotting the amplitude of hydrodynamic resultant forces for the tallest monolith. The plots in Figures 5 and 6 show a noticeable difference between the 2D and 3D analyses; the main differences are the following.

- The frequency of the resonance peaks in the 3D case is higher than in the 2D case; a reasonable explanation could be found in the higher degree of constraint given by the adjacent, shorter monoliths, providing additional stiffness to the structure and the corresponding minor water depth.
- A slightly higher intensity of the hydrodynamic resultant force for the 2D model is observed.
- In the 2D case the response for near-zero frequencies of the added masses model tends to the same value as the FSI analysis, matching the theoretical solution. This does not occur in the 3D case, where the difference is still noticeable, being roughly 20% higher. This results may induce to believe that a direct added masses model transposition to three dimensional analysis be arbitrary and lead to errors.
- The added masses simplified model provides a reasonable estimation of the first resonant frequency, while it does not match the FSI analysis for superior modes, providing a much higher amplification.

- In the 2D case the rigid barrier peaks (blue curve) are still visible in the FSI analysis, meaning that specific reservoir dynamic properties are let unmodified by the structural coupling. In fact, the theoretical reservoir eigenfrequencies deduced from equation 2 are equal to 6.17 Hz and odd multiples in 2D, which are not affected by the structure. The phenomenon can be explained by noticing that at a certain distance from the dam, a local pressure variation at the upstream face does not influence the far-field response. In fact the latter is governed only by the reservoir depth itself, which is constant in 2D. In the 3D model such effect is less evident.

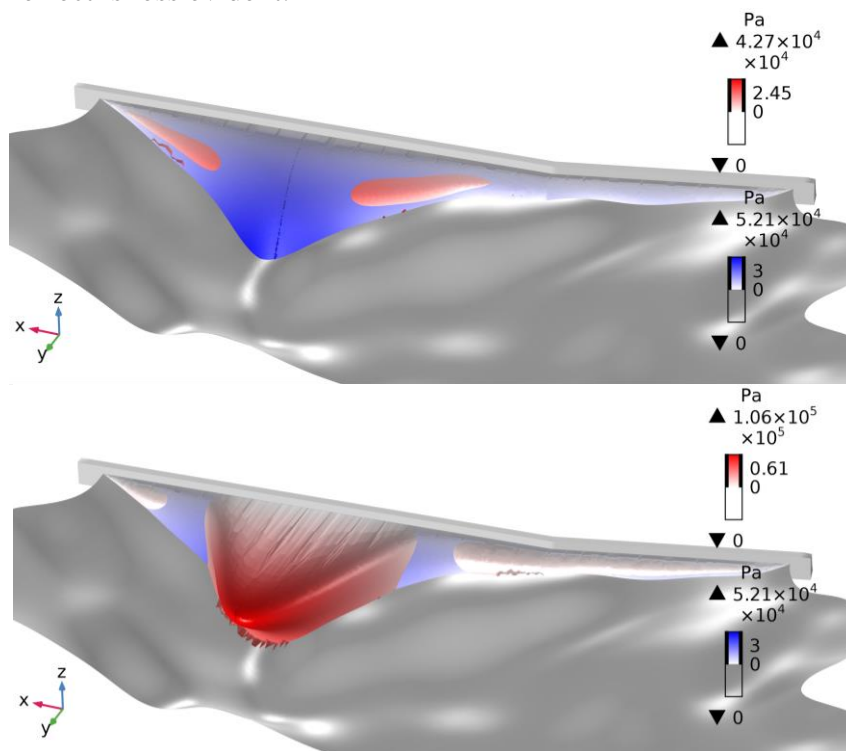


Figure 4. 3D model response for 0.1 Hz (upper image) and 5.7 Hz (lower image): pressures relative to the FS interaction (red surface) are shown, in comparison to the equivalent added masses inertial load (blue surface).

In Figures 7 and 8 the 3D model pressure contours are plotted for different excitation frequencies. The figures clearly display the variability of pressure distribution along the cross valley direction, even for low excitation frequency values.

Since Westergaard's expression is strictly valid in 2D cases, its extension to 3D can be accomplished in different ways: both by fixing h in equation (3) as the overall reservoir maximum depth, or by continuously varying it along the cross valley direction. The first solution leads to major errors in estimating both peak frequencies and response magnitudes. The second solution, adopted in this case, is quite adequate in matching the FSI simulation for low frequency ranges. Nevertheless, it remains inappropriate for higher modes, where pressure variations in the cross valley direction are significant due to excitation of superior longitudinal harmonics.

The same figures also plot the streamlines for acoustic intensity, defined as $I = p \cdot v$, with p is the fluid pressure and v the particle velocity. By comparing the figures for a low and a high value of excitation frequency, a remarkable difference is observed, and the energy flux is at

least two orders of magnitude greater for high frequencies. This energy is radiated upstream of the reservoir and it is thus subtracted from the dam body. Such effect is equivalent to consider an additional damping, referred to as *acoustic radiation damping*.

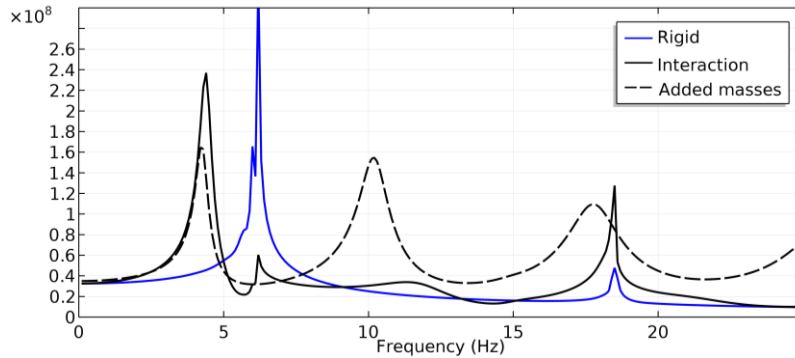


Figure 5: Hydrodynamic force - 2D central monolith model.

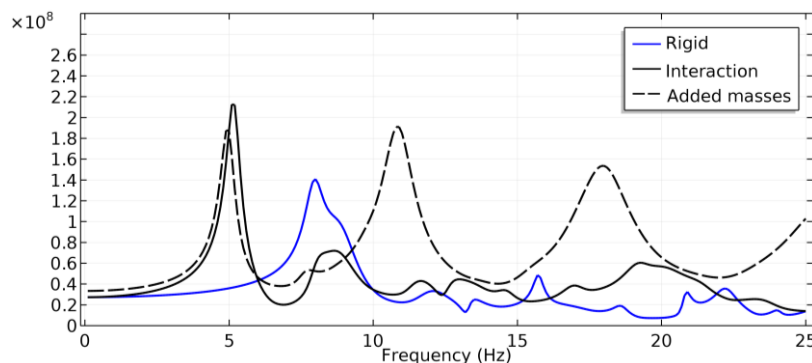


Figure 6: Hydrodynamic force - 3D model, central monolith.

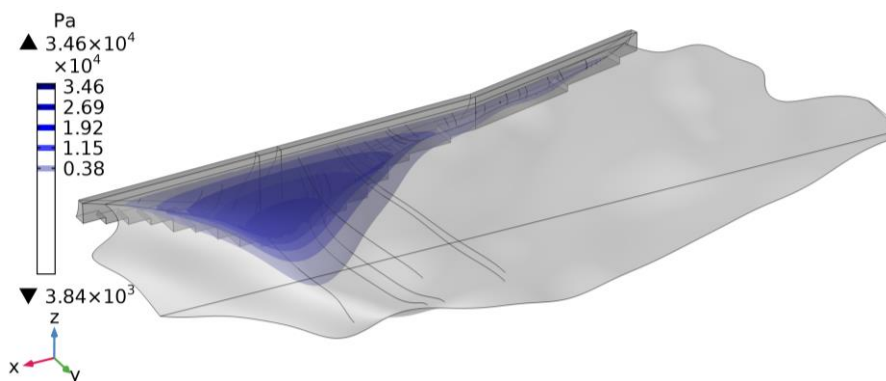


Figure 7: Hydrodynamic pressure contours and acoustic intensity streamlines - 3D model - excitation of 0.1 Hz.

The acoustic intensity is integrated over the upstream model boundaries and the total radiated power is plotted in figure 9 on a semi logarithmic scale for 2D and 3D models. It can be observed that the radiated energy for excitation frequencies below the first eigenfrequency of the reservoir is substantially negligible, compared to that for higher frequencies. The first reservoir eigenfrequency may thus be interpreted as a cut-off limit for energy radiation

upstream of the reservoir. It is also shown that the 2D FSI models provides a dissipation throughout the whole range which is orders of magnitude lower than the rigid barrier case. This behaviour is not replicated in the 3D case, where the rigid and FSI values are comparable.

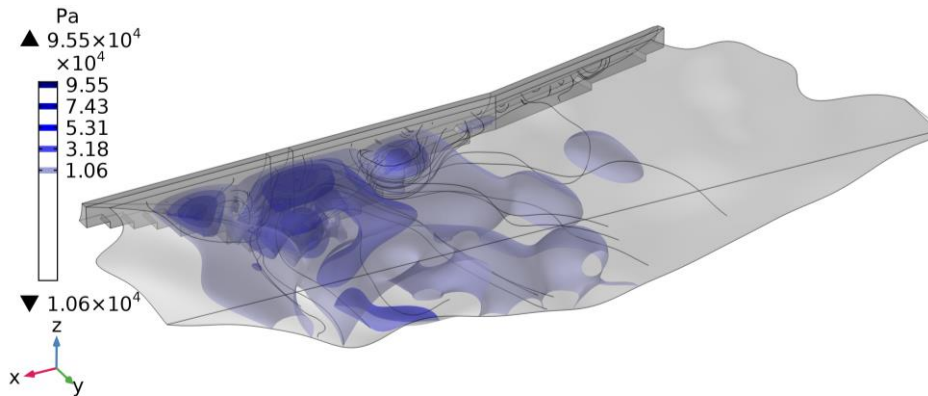


Figure 8: Hydrodynamic pressure contours and acoustic intensity streamlines - 3D model - excitation of 20 Hz.

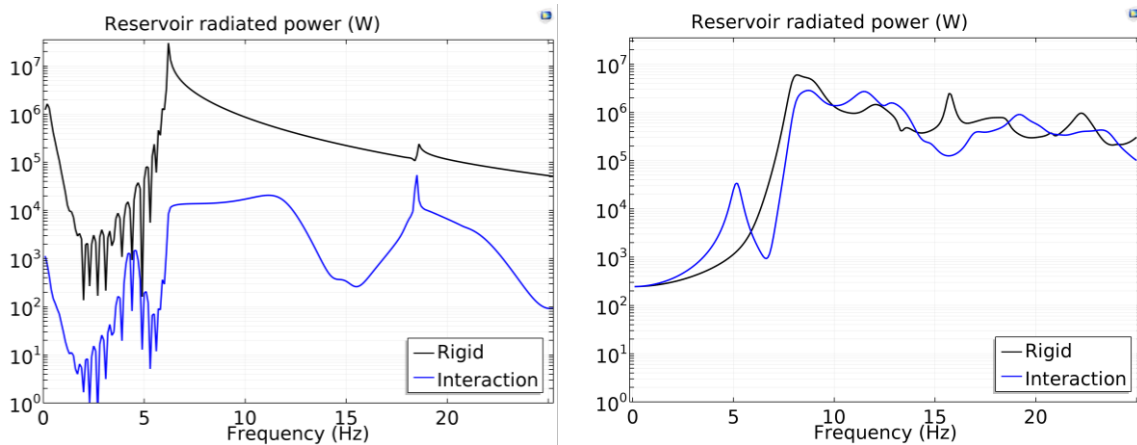


Figure 9. Total reservoir radiated power (semi-logarithmic scale) for 2D (left) and 3D model (right).

3.2 Parametric investigations

The reasons for the different response between 2D and 3D models in terms of peak frequency and resultant force intensity has been investigated through a parametric study involving the shape of the canyon and the angle between lateral dam embankments. Such investigation is performed on two simplified 3D study models.

3.2.1 Influence of basin shape

The first simple FE 3D model simulates the basin between the lateral embankments. It is a cylindrical sector of angle 2α with a deformable dam on the downstream side and an auxiliary PML domain on the upstream side. The mesh of the fluid domain has a maximum of 11346 tetrahedral quadratic Lagrange elements and 3158 nodes. The mesh of the solid domain has 477 tetrahedral second order Serendipity elements with 161 nodes. The cross-valley shape is rectangular and it remains unmodified as α varies. Fig.10 displays the hydrodynamic force, varying the angle 2α in the range $(0^\circ, 75^\circ)$. One can observe that the hydrodynamic load

decreases as α increases. In addition, the peak frequency value is practically unmodified with α . This is reasonable, since the peak frequency depends only on the reservoir depth.

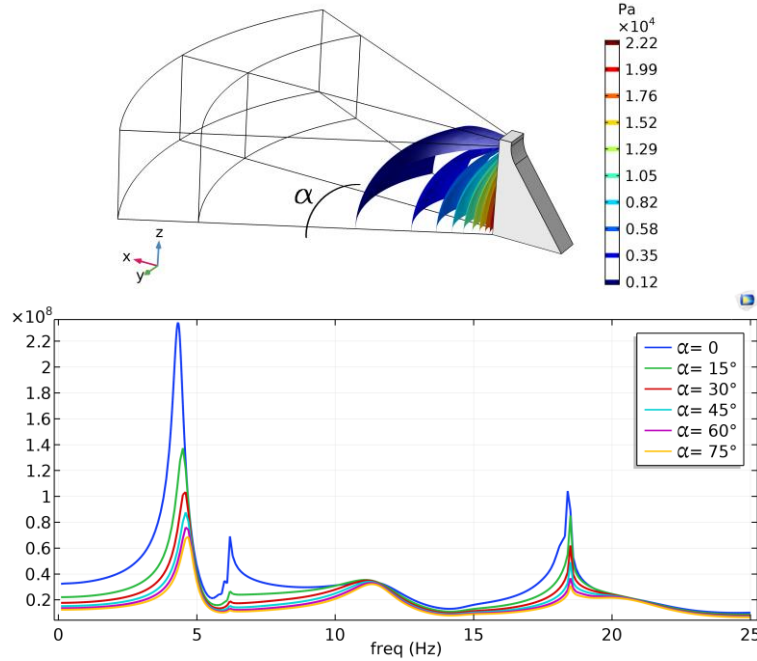


Figure 10: The 3D simplified model with pressure contours for 0.1 Hz frequency (upper figure); Frequency response of the idealized 3D model for different values of α - semi-logarithmic scale (lower figure).

3.2.2 Influence of canyon slope

The second simple FE 3D model is a trapezoidal prism, whose lateral walls have slope β . On the downstream side there is a rigid barrier and on the upstream side an auxiliary PML domain is added. The mesh of the fluid domain has at most 2261 tetrahedral quadratic Lagrange elements and 1600 nodes. Fig. 11 shows the effects of the variation of β on the total hydrodynamic force, while the barrier maintains constant area. The results demonstrate that even a small deviation of the side walls from the vertical ($\beta = 90^\circ$) produces a frequency shift to higher values. The peak values tend to infinite, because no sediment absorption is considered.

An attempt to obtain a quick estimation of reservoir natural frequency is performed by defining an equivalent depth \bar{h} . By averaging the reservoir depths at each monolith using the corresponding added mass as weight, one obtains

$$\bar{h} = \frac{\sum_i h_i \cdot m_{add_i}}{\sum_i m_{add_i}} \quad (6)$$

where h_i is the generic monolith height and m_{add_i} is the added mass calculated for h_i and width b , as illustrated in Figure 12. By doing so, the equivalent depth is found to be 50.75 m, which corresponds to a first frequency of 7.78 Hz, close to the FEM analysis result in the case of rigid barrier.

This simple calculation provides an insight on the nature of the phenomenon. The combined effect of these two parameters varying within a realistic range confirms the differences between the 2D model and the more accurate 3D model.

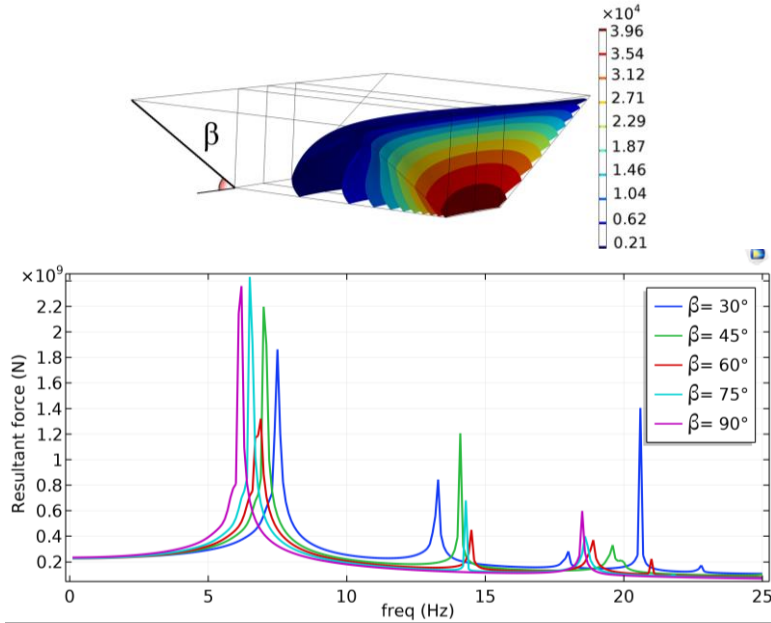


Figure 11. Pressure contours for 0.1 Hz frequency (upper figure), Frequency response of the idealized 3D model for different values of β (linear scale) (lower figure).

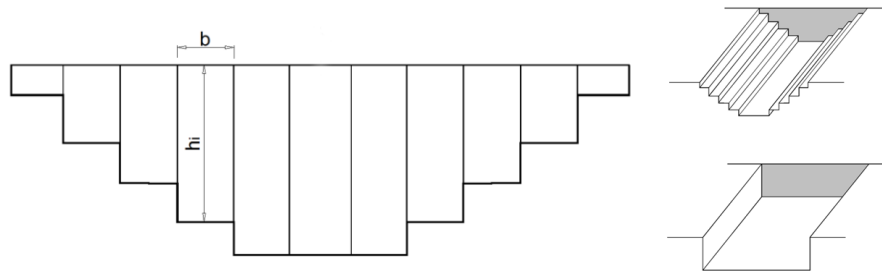


Figure 12: Equivalent reservoir depth definition.

3.3 Structural effects

As a last step, the global effects of hydrodynamic loads on the structural response are evaluated in term of base shear force. Results are displayed in Figure 13, for different modelling approaches, including the case of empty reservoir for reference purposes. It can be noted that, for very low frequencies, the hydrodynamic action increases the response of the dam only by nearly 30% in both 2D and 3D, while at the resonance condition the shear is nearly doubled. By comparing the different approaches, it can be observed that:

- the first peak frequency of the model of interaction is very close to the one of added masses, while higher frequency peaks differ greatly;
- in the 2D case the rigid barrier model displays a resonance at a higher frequency;
- the 3D model does not display an explicit reservoir resonance peak in the case of rigid barrier.

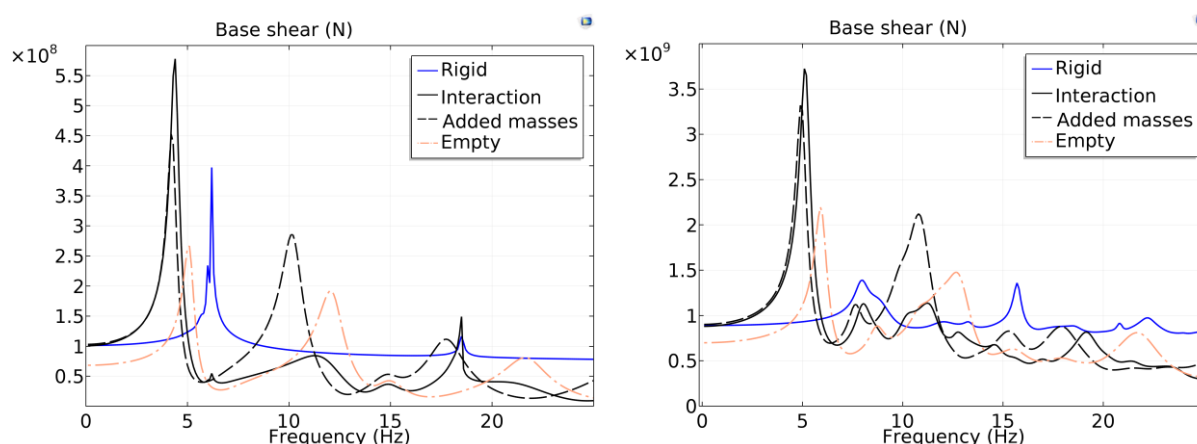


Figure 13: Base shear for 2D model for central monolith (left) and 3D model for the entire dam (right).

4 CONCLUSIONS

In this work, the problem of hydrodynamic pressure evaluation on existing gravity dams is addressed by using FE modelling. Such study allows us to overcome some of the rigid hypotheses of the analytic theories and efficiently evaluates fluid-structure interaction in complex 2D and 3D cases. The comparison between different approaches displays noticeable differences on the overall hydrodynamic response if the excitation frequency is higher than the first mode, both for a simple 2D model of the tallest monolith and the full dam 3D model of a case study. As a result of the subsequent parametric study, some differences between 2D and 3D have been observed. More specifically, in 3D the increase of the angle between lateral embankments induces a drastic reduction on the overall response amplitude, while the slope of reservoir walls modifies its frequency content. Such behaviour cannot be reproduced by a 2D plane model. Finally, it can be concluded that a realistic 3D geometry of the reservoir makes the fluid behaviour highly complex and, in 3D, the added mass model may produce even more different results from the full FSI modelling.

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