6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
1115 June 2018, Glasgow, UK

REDUCED ORDER MODELS FOR DYNAMIC ANALYSIS OF NONLINEAR ROTATING STRUCTURES

M. Balmaseda[1,2], G. Jacquet-Richardet[2], A. Placzek[1] and D.-M. Tran[1]

[1] ONERA, BP72, 92320 Châtillon, France
mikel.balmaseda@onera.fr, antoine.placzek@onera.fr and tran@onera.fr

[2] Université de Lyon, CNRS INSA-Lyon, LaMCoS UMR5259 F-69621 Villeurbane, France
Georges.Jacquet@insa-lyon.fr

Key words: Reduced Order Models, Rotating Structures, Nonlinear Dynamics, Stiffness Evaluation Procedure (STEP)

Abstract. In the present work autonomous reduced order models (ROM) of nonlinear rotating structures considering geometrical non linearities are proposed. The latter are not only taken into consideration in the geometrical pre-stressed stiffness matrix induced by the rotation, as in the classical linearised approach, but also in the relative dynamic response around the pre-stressed equilibrium. The reduced nonlinear forces are represented by a polynomial expansion obtained by the Stiffness Evaluation Procedure (STEP). The reduced nonlinear forces are corrected by means of a Proper Orthogonal Decomposition (POD) of the full order nonlinear forces. The solutions of the nonlinear ROM obtained with both time integration (HHT-α) and harmonic balance method (HBM) are in good accordance with the solutions of the full order model and are more accurate than the linearised solutions.

1 INTRODUCTION

Rotating structures are widely used in industrial applications such as turbo-machinery, helicopter blades and wind turbines. The design tendency to create more slender, more flexible and lighter structural components under greater excitations increases the nonlinear behaviour of these components. Thus, the need to accurately predict the dynamic response of geometrically nonlinear structures becomes essential for the designer.

To reduce the computational cost of the high fidelity large finite element nonlinear models, some investigators have developed the construction of nonlinear reduced order models (ROM) [1, 2]. However, without special techniques to create an efficient ROM able to evaluate the system matrices when the structure deforms, the computational cost of this process becomes equivalent to the time to carry the full order finite element analysis and the advantages of the reduction are counteracted. An efficient approach to nonlinear structural analysis was carried by [3, 4] representing the internal forces by a third order polynomial formulation of displacements. This method is known as the Stiffness Evaluation Procedure (STEP). The stiffness coefficients of the polynomial representation are obtained by a series of static results obtained with the full order finite element model. As an extension to STEP method “non intrusive” reduced order models have been reviewed by [5] and validated for the prediction of fatigue, nonlinear stochastic computations, post-buckling analyses and complex structures. The Element-wise Stiffness Evaluation Procedure (E-STEP) generalizes the STEP to optimization problems enabling the parametrization of the stiffness evaluation procedure.The hyper-reduction and the piecewise linearisation are alternative techniques to ease the system matrix computation issues.

In the framework of rotating structures, the vibration of linear rotating beams has been widely studied, extended to the study of nonlinear geometrical fixed beam models and adapted to rotating structures [6, 7]. The nonlinear effect due to rotation creates a coupling between the axial and transverse motions. Based on a von Karman formulation, a reduction model of a rotating beam is performed by nonlinear modes and invariant manifolds. The free vibrations of non-uniform fixed geometrically nonlinear beams with variable cross section and material properties along the axial direction are carried out in [8]. A comparative study between several models of a rotating cantilever beam in terms of accuracy and validity is presented in [9]. These models are mainly used in the study of helicopter and turbo-machinery blades, modelisations of slender beams or thin shells and structure-fluid interactions. A finite element formulation of the rotating problem is presented in [10] and the necessity to develop 3D finite element models for the study of rotordynamics is highlighted in [11].

In opposition to the classical FE formulation for geometrically nonlinear rotating structures [12] that considers small linear vibrations around the static equilibrium, the present work assumes nonlinear vibrations around the pre-stressed equilibrium state. Thus, as an extension to [13], an autonomous geometrically nonlinear reduced order model for the study of dynamic solutions of complex rotating structures is developed. For that purpose, the linear normal modes basis is used for the construction of the reduced order model, the stiffness evaluation procedure method (STEP) is applied to compute the nonlinear forces and the assumption of nonlinear perturbations around the static equilibrium is considered. The latter enhances the classical linearised small perturbations hypothesis to the cases of large displacements around the static pre-stressed equilibrium. The nonlinear forces are corrected by means of a filtering of the nonlinear forces so that only their components in a Proper Orthogonal Decomposition (POD) nonlinear force basis are kept. This POD basis is obtained by performing a singular value decomposition (SVD) of the snapshots constituted by the nonlinear forces previously computed on the full order model.

The proposed method is then applied for a 3D model of a rotating beam. Furthermore, a comparison between the periodic solution given by HHT-α and the Harmonic Balance Method (HBM) is carried out.

2 ROM FOR GEOMETRICALLY NONLINEAR STRUCTURES

The FOM (full order model) equation of motion of a rotating structure, discretized by FEM and geometrically non linear [10] is written in the rotating frame of reference as:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{u}}_p + [\mathbf{C} + \mathbf{G}(\Omega)\ ]\,\dot{\mathbf{u}}_p + \left[\mathbf{K}_c(\Omega)\ \ + \mathbf{K}_a(\dot{\Omega})\ \right]\mathbf{u}_p + \mathbf{g}(\mathbf{u}_p) = \mathbf{f}_e(t) + \mathbf{f}_{ei}(\Omega, \dot{\Omega}) \\ \mathbf{u}_p(t=0) = \mathbf{u}_{p,\,\mathrm{ini}} \\ \dot{\mathbf{u}}_p(t=0) = \dot{\mathbf{u}}_{p,\,\mathrm{ini}} \end{cases}, \quad (1)$$

where Ω and $\dot{\Omega}$ are the rotation speed and acceleration respectively, $\mathbf{u}_p$, $\dot{\mathbf{u}}_p$ and $\ddot{\mathbf{u}}_p$ are the physical displacements, velocities and accelerations of the structure of dimension n_{dof} equal to the number of FOM's degrees of freedom, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{G}(\Omega)$, $\mathbf{K}_c(\Omega)$, $\mathbf{K}_a(\dot{\Omega})$ are the mass, damping, gyroscopic coupling, centrifugal softening and centrifugal acceleration matrices respectively, $\mathbf{g}(\mathbf{u}_p)$ is the interior nonlinear force vector, $\mathbf{f}_e$(t) is the exterior force vector, $\mathbf{f}_{ei}(\Omega,\ \dot{\Omega})$ is the external inertial load vector. The initial conditions of the structure are defined by the initial displacements vector $\mathbf{u}_{p,\,\mathrm{ini}}$ and the initial velocity vector $\dot{\mathbf{u}}_{p,\,\mathrm{ini}}$. Hereafter a constant rotation speed is considered, thus, $\dot{\Omega} = 0$, $\mathbf{K}_a(\dot{\Omega}) = 0$ and $\mathbf{f}_{ei}(\Omega,\ \dot{\Omega}) = \mathbf{f}_{ei}(\Omega)$.

The nonlinear tangent stiffness matrix $\mathbf{K}_t(\mathbf{u}_p)$ and the elastic linear stiffness matrix $\mathbf{K}_e$ are defined as follows:

$$\mathbf{K}_t(\mathbf{u}_p) = \frac{\partial \mathbf{g}(\mathbf{u}_p)}{\partial \mathbf{u}_p}, \quad (2)$$

$$\mathbf{K}_e = \mathbf{K}_t(\mathbf{0}) = \left.\frac{\partial \mathbf{g}(\mathbf{u}_p)}{\partial \mathbf{u}_p}\right|_{\mathbf{u}_p=0}. \quad (3)$$

Under the effect of rotation, without considering the exterior force, the structure reaches the *static equilibrium state*. The static displacements $\mathbf{u}_s$ are calculated by an iterative procedure (i.e. Newton-Raphson method) solving the nonlinear static equation system (4) obtained from the equation (1).

$$\mathbf{K}_c(\Omega)\ \mathbf{u}_s + \mathbf{g}(\mathbf{u}_s) = \mathbf{f}_{ei}(\Omega)\,. \quad (4)$$

The tangent stiffness matrix with respect to the static displacements is defined hereunder:

$$\mathbf{K}_s = \mathbf{K}_t(\mathbf{u}_s) = \left.\frac{\partial \mathbf{g}(\mathbf{u}_p)}{\partial \mathbf{u}_p}\right|_{\mathbf{u}_p=\mathbf{u}_s}, \quad (5)$$

that is also written as:

$$\mathbf{K}_s = \mathbf{K}_e + \mathbf{K}_{nl}(\mathbf{u}_s)\ , \quad (6)$$

where:

$$\mathbf{K}_{nl}(\mathbf{u}_s) = \mathbf{K}_g(\mathbf{u}_s) + \ldots\ , \quad (7)$$

is the nonlinear part of the tangent stiffness matrix $\mathbf{K}_s$ including the geometrical pre-stressed stiffness matrix $\mathbf{K}_g(\mathbf{u}_s)$ related to the static equilibrium state $\mathbf{u}_s$.

The relative displacements $\mathbf{u}$ between the static displacements $\mathbf{u}_s$ and the physical displacements $\mathbf{u}_p$ are defined as follows:

$$\mathbf{u} = \mathbf{u}_p - \mathbf{u}_s \ . \tag{8}$$

Then, introducing the latter relation in the equation (1), the equation of motion of the structure around the static equilibrium state position is obtained:

$$\mathbf{M}\ddot{\mathbf{u}} + [\mathbf{C} + \mathbf{G}(\Omega)\ ]\,\dot{\mathbf{u}} + \mathbf{K}_c(\Omega)\ \mathbf{u} + \mathbf{g}(\mathbf{u}_s + \mathbf{u}) - \mathbf{g}(\mathbf{u}_s) = \mathbf{f}_e(t)\ , \tag{9}$$

where the external inertial forces $\mathbf{f}_{ei}(\Omega)$ are eliminated from (4).

The development of the nonlinear forces $\mathbf{g}(\mathbf{u}_s + \mathbf{u})$ in the neighbourhood of the static displacements $\mathbf{u}_s$ is the sum of a constant, a linear part and a purely nonlinear part:

$$\mathbf{g}(\mathbf{u}_\mathrm{s} + \mathbf{u}) = \mathbf{g}(\mathbf{u}_\mathrm{s}) + \left.\frac{\partial \mathbf{g}(\mathbf{u}_\mathrm{p})}{\partial \mathbf{u}_p}\right|_{\mathbf{u}_p=\mathbf{u}_s} \mathbf{u} + \mathbf{g}_{nl}\,(\mathbf{u}) = \mathbf{g}(\mathbf{u}_\mathrm{s}) + \mathbf{K}_s\mathbf{u} + \mathbf{g}_{nl}\,(\mathbf{u})\ . \tag{10}$$

The stiffness matrix function of the rotation speed Ω and the static displacements $\mathbf{u}_s$ is defined hereunder:

$$\mathbf{K} = \mathbf{K}(\Omega,\ \mathbf{u}_s)\ = \mathbf{K}_c(\Omega)\ + \mathbf{K}_s = \mathbf{K}_c(\Omega)\ + \mathbf{K}_e + \mathbf{K}_{nl}(\mathbf{u}_s)\ \ . \tag{11}$$

Then, from the equations (10), (11) and (1) the structure's FOM equation of motion as a function of the relative displacements $\mathbf{u}$ is written as follows:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{u}} + [\mathbf{C} + \mathbf{G}(\Omega)\ ]\,\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{g}_{nl}(\mathbf{u}) = \mathbf{f}_e(t) \\ \mathbf{u}(t=0) = \mathbf{u}_\mathrm{ini} = \mathbf{u}_{p,\,\mathrm{ini}} - \mathbf{u}_s \\ \dot{\mathbf{u}}(t=0) = \dot{\mathbf{u}}_\mathrm{ini} = \dot{\mathbf{u}}_{p,\,\mathrm{ini}} \end{cases} \ . \tag{12}$$

Notice that the form of the latter equation is general, thus, it is valid to represent the response of a linear or nonlinear structure with or without rotation and with or without pre-stressing. The classical linearised approach ignores the term $\mathbf{g}_{nl}(\mathbf{u})$ in (12) (see equation (14)).

2.1 Linear normal modes for a rotating structure

The natural frequencies and linear normal modes of the rotating structure are obtained under the hypothesis of small vibrations $\mathbf{u}$ around the static displacements $\mathbf{u}_s$. Thus, neglecting the purely nonlinear force $\mathbf{g}_{nl}(\mathbf{u})$, a first order development of the nonlinear force $\mathbf{g}(\mathbf{u}_s + \mathbf{u})$ around the static displacements $\mathbf{u}_s$ is performed:

$$\mathbf{g}(\mathbf{u}_s + \mathbf{u}) = \mathbf{g}(\mathbf{u}_s) + \left.\frac{\partial \mathbf{g}(\mathbf{u}_p)}{\partial \mathbf{u}_p}\right|_{\mathbf{u}_p=\mathbf{u}_s} \mathbf{u} = \mathbf{g}(\mathbf{u}_s) + \mathbf{K}_s\mathbf{u}\ . \tag{13}$$

Then, the linearised equation of movement of the structure around the static displacements $\mathbf{u}_s$ is obtained by substituting the equation (13) in the equation (9):

$$\mathbf{M}\ddot{\mathbf{u}} + [\mathbf{C} + \mathbf{G}(\Omega)\ ]\,\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_e(t)\ . \tag{14}$$

The natural frequencies and the normal linear modes of the rotating structure without considering the gyroscopic effect are the solution to the following eigenvalue and eigenvector problem:

$$\mathbf{K}\boldsymbol{\Phi} = \mathbf{M}\boldsymbol{\Phi}\boldsymbol{\omega}^2\ , \tag{15}$$

where $\boldsymbol{\omega} = diag\,[\omega_1, \dots, \omega_r]$ with $\omega_1 \leq \dots \leq \omega_r$ are the first r natural frequencies and $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_1, \dots, \boldsymbol{\Phi}_r]$ are the linear normal modes associated to those natural frequencies. The group of first r normal linear modes form the projection basis $\boldsymbol{\Phi}$.

2.2 ROM by projection

The order reduction by projection consists in representing the FOM displacements as a linear combination of the projection basis $\boldsymbol{\Phi}$ (i.e. linear normal modes):

$$\mathbf{u} \approx \boldsymbol{\Phi}\mathbf{q}\ , \tag{16}$$

where $\mathbf{q}$ is the generalized coordinates vector. The number of modes r used to build the projection basis is small in comparison to the FOM's dimension, $r \ll n_{dof}$.

Substituting the latter relation in the equation (12) and pre-multiplying the same equation by the transpose matrix of the projection basis $\boldsymbol{\Phi}^T$ (Galerkin projection), the reduced order model (ROM) equation of motion is defined as:

$$\begin{cases} \tilde{\mathbf{M}}\ddot{\mathbf{q}} + \left[\tilde{\mathbf{C}} + \tilde{\mathbf{G}}(\Omega)\right]\dot{\mathbf{q}} + \tilde{\mathbf{K}}\mathbf{q} + \tilde{\mathbf{g}}_{nl}(\mathbf{q}) = \tilde{\mathbf{f}}_e(t) \\ \mathbf{q}(\mathrm{t}=0) = \mathbf{q}_{\mathrm{ini}} = \left(\boldsymbol{\Phi}^T\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^T\left(\mathbf{u}_{p,\,\mathrm{ini}} - \mathbf{u}_s\right) \\ \dot{\mathbf{q}}(t=0) = \dot{\mathbf{q}}_{\mathrm{ini}} = \left(\boldsymbol{\Phi}^T\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^T\dot{\mathbf{u}}_{p,\,\mathrm{ini}} \end{cases}, \tag{17}$$

where $\tilde{\mathbf{M}} = \boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi}$, $\tilde{\mathbf{C}} = \boldsymbol{\Phi}^T\mathbf{C}\boldsymbol{\Phi}$, $\tilde{\mathbf{G}}(\Omega)\ = \boldsymbol{\Phi}^T\tilde{\mathbf{G}}(\Omega)\ \boldsymbol{\Phi}$, $\tilde{\mathbf{K}} = \boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi}$ are the generalized mass, damping, gyroscopic and stiffness matrices, $\tilde{\mathbf{f}}_e(t)$ is the generalized external force vector and $\tilde{\mathbf{g}}_{nl}(\mathbf{q})$ is the generalized purely nonlinear force vector. The generalized initial conditions $\dot{\mathbf{q}}_{\mathrm{ini}}$ and $\mathbf{q}_{\mathrm{ini}}$ are obtained from $\mathbf{u}_{p,\,\mathrm{ini}}$ and $\dot{\mathbf{u}}_{p,\,\mathrm{ini}}$ by a least-squares approximation.

3 COMPUTATION OF THE GENERALIZED NONLINEAR FORCES

3.1 Inflation method

The computation of the generalized nonlinear forces by the inflation method consists in the evaluation of the purely nonlinear force $\mathbf{g}_{nl}$ in the FOM from (10) and projecting the solution to the ROM:

$$\tilde{\mathbf{g}}_{nl}(\mathbf{q}) = \boldsymbol{\Phi}^T\mathbf{g}_{nl}(\boldsymbol{\Phi}\mathbf{q}) = \boldsymbol{\Phi}^T\mathbf{g}(\mathbf{u}_s + \boldsymbol{\Phi}\mathbf{q}) - \boldsymbol{\Phi}^T\mathbf{g}(\mathbf{u}_s) - \tilde{\mathbf{K}}_s\mathbf{q}\ , \tag{18}$$

where $\tilde{\mathbf{K}}_s = \mathbf{\Phi}^T \mathbf{K}_s \mathbf{\Phi}$.

This method is simple to implement, however, as the computation of the nonlinear forces is performed in the finite element FOM, the ROM directly depends on the size of the FOM. Thus, the ROM of the equation (17) is not autonomous and the computational cost remains important due to computation of the nonlinear forces.

3.2 STEP polynomial approximation

In order to get an autonomous ROM fully independent of the FOM, Muravyov and Rizzi [3, 4] developed a polynomial approximation to compute $\mathbf{g}_{nl}(\mathbf{q})$ as a function of $\mathbf{q}$. An extension to compute the nonlinear forces of structures under rotation is presented hereunder .

Each component p (with $p = 1, \ldots, r$) of the generalized purely nonlinear force vector $\tilde{\mathbf{g}}_{nl}(\mathbf{q})$ is expressed as a polynomial approximation of third degree in terms of the r number of variables that form the generalized coordinates $\mathbf{q} = [q_1, \ldots, q_r]$:

$$\tilde{g}^p_{nl}(q_1, \ldots, q_r) = \tilde{g}^p_{nl_{Quad.}} + \tilde{g}^p_{nl_{Cub.}} = \sum_{i=1}^{r}\sum_{j=i}^{r} A^p_{ij} q_i q_j + \sum_{i=1}^{r}\sum_{j=i}^{r}\sum_{m=j}^{r} B^p_{ijm} q_i q_j q_m \ . \tag{19}$$

Once the A^p_{ij} and B^p_{ijm} coefficients are calculated, the generalized purely nonlinear forces $\tilde{\mathbf{g}}_{nl}(\mathbf{q})$ are directly obtained by the equation (19). The ROM is independent of the FOM and the computational cost is considerably improved.

The polynomial coefficients A^p_{ij} and B^p_{ijm} are obtained by identification using a finite element software (i.e. NASTRAN, Code_Aster, Z-Set,...) to compute the nonlinear forces $\mathbf{g}(\mathbf{u}_s + \mathbf{u})$ associated to a given number of imposed displacements $\mathbf{u}_s + \mathbf{u}$. Then, the purely nonlinear part is identified from the equation (10):

$$\mathbf{g}_{nl}(\mathbf{u}) = \mathbf{g}(\mathbf{u}_s + \mathbf{u}) - \mathbf{g}(\mathbf{u}_s) - \mathbf{K}_s \mathbf{u} \ , \tag{20}$$

and its projection with respect to the p-th mode $\mathbf{\Phi}_p$ is:

$$\tilde{g}^p_{nl}(\mathbf{u}) = \mathbf{\Phi}^T_p \mathbf{g}_{nl}(\mathbf{u}) \ . \tag{21}$$

The imposed displacements vectors $\mathbf{u}$ used to obtain the polynomial coefficients are a linear combination of one, two and three linear normal modes. The computation of the polynomial coefficients is described in [13].

3.3 POD based correction of nonlinear forces

STEP method provides accurate solutions when the middle surface of the structure is submitted to stretching effects [3]. However, when structures exhibit other type of nonlinearities, such as cantilever beams, a correction of reduced nonlinear forces is needed. An approach to avoid the numerical issues in the identification of reduced nonlinear forces was proposed by [14], then extended to complex structures and generalised to a wing structure [15]. The present work presents an alternative method to minimize the numerical error of the projected forces inspired by [16, 17] mainly applied in computational fluid dynamics. The present approach is centered in the approximation of the nonlinear forces in the ROM.

First a POD is performed on the snapshots constituted by the nonlinear forces computed on the full order model obtaining a POD force basis, $\mathbf{\Phi}_f$, verifying $\mathbf{\Phi}_f^T\mathbf{\Phi}_f = \mathbf{Id}$. The full order model nonlinear forces, $\mathbf{g}_{nl}(\mathbf{u})$, are then approximated by their components in $\mathbf{\Phi}_f$:

$$\mathbf{g}_{nl}(\mathbf{u}) \approx \mathbf{\Phi}_f\mathbf{q}_{nl}^f = \mathbf{g}_{nl}^f(\mathbf{u})\,, \tag{22}$$

where the coefficients, $\mathbf{q}_{nl}^f$, are obtained by a least-square approximation:

$$\mathbf{q}_{nl}^f \approx \left(\mathbf{\Phi}_f^T\mathbf{\Phi}_f\right)^{-1}\mathbf{\Phi}_f^T\mathbf{g}_{nl}(\mathbf{u}) = \mathbf{\Phi}_f^T\mathbf{g}_{nl}(\mathbf{u})\,, \tag{23}$$

thus:

$$\mathbf{g}_{nl}^f(\mathbf{u}) \approx \mathbf{\Phi}_f\left(\mathbf{\Phi}_f^T\mathbf{\Phi}_f\right)^{-1}\mathbf{\Phi}_f^T\mathbf{g}_{nl}(\mathbf{u}) = \mathbf{\Phi}_f\mathbf{\Phi}_f^T\mathbf{g}_{nl}(\mathbf{u})\,. \tag{24}$$

The reduced order nonlinear forces are defined as:

$$\tilde{\mathbf{g}}_{nl}(\mathbf{q}) = \mathbf{\Phi}^T\mathbf{g}_{nl}^f(\mathbf{u}) = \mathbf{\Phi}^T\mathbf{\Phi}_f\left(\mathbf{\Phi}_f^T\mathbf{\Phi}_f\right)^{-1}\mathbf{\Phi}_f^T\mathbf{g}_{nl}(\mathbf{u}) = \mathbf{B}^T\mathbf{g}_{nl}(\mathbf{u})\,. \tag{25}$$

For the STEP polynomial approximation of the generalized nonlinear forces, the correction given by (24) and (25) is applied to the nonlinear forces associated with the imposed displacements given in (20) and (21) prior to computing the polynomial coefficients.

The POD nonlinear force basis, $\mathbf{\Phi}_f$, is obtained with a SVD decomposition from previously computed nonlinear forces data and then truncated to a number m of nonlinear basis vectors. This truncation simulates a filtering of nonlinear forces directions. As shown in Section 4.2, the accuracy of the ROM to predict the nonlinear response of the structure is strongly related with the choice of m.

4 NUMERICAL STUDY OF A ROTATING BEAM

4.1 Numerical model

The proposed method is validated with a 3D model of a rotating Titanium ($E = 104\,GPa$, $\nu = 0.3$) beam of dimensions $0.4\,m \times 0.03\,m \times 0.01\,m$. The beam is modeled by a set of $20 \times 2 \times 1$ quadratic hexahedral (Hexa20) finite elements. The model has 1179 degrees of freedom. The external loading, $\mathbf{F}_e(t)$, is applied at every node on the free end surface of the beam. The beam is clamped at one of its ends and rotates at a distance of $0.1\,m$ around the vertical z axis (see Figure 1). In the dynamic analysis a Rayleigh structural damping with inertial effect is considered ($\mathbf{C} = \beta_r\mathbf{M}$) where β_r is the Rayleigh damping coefficient equal to $\beta_r = 2\pi\omega_i\xi$.

Twenty external loading cases are considered in order to study the response of the structure under different external loading levels and rotating velocities. Each case is the

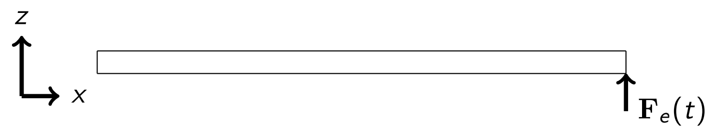
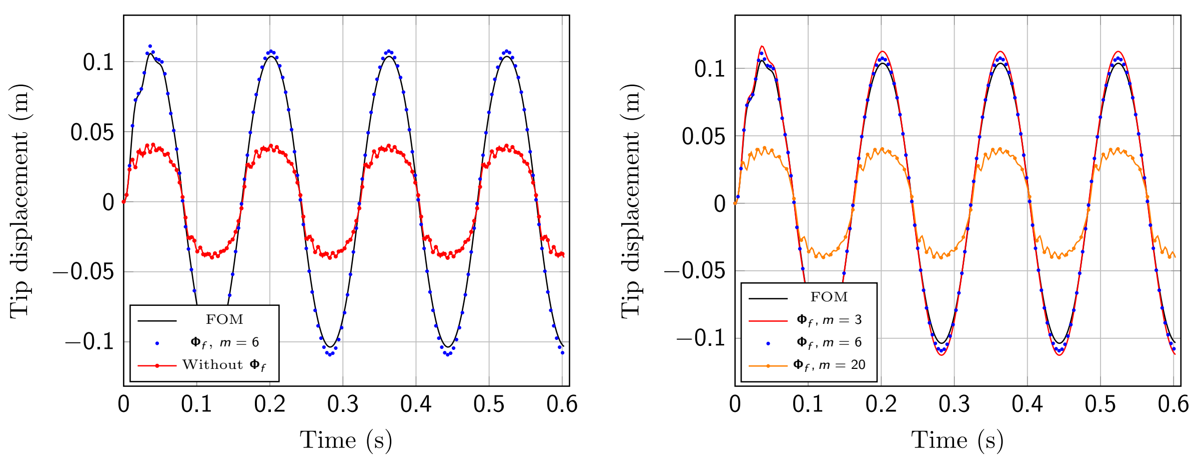


Figure 1: Clamped-free beam rotating around z axis.

combination of one sinusoidal, $\mathbf{F}_e(\mathsf{t}) = \alpha_f \sin(\omega_e t)$, loading level $\alpha_f \in \{0.1, 0.3, 0.5, 0.7\}$ and one rotating velocity $\Omega \in \{0\,rpm, 1000\,rpm, 2000\,rpm, 3000\,rpm, 4000\,rpm\}$. For rotating velocities greater than $4000\,rpm$ the linear and nonlinear ROM behave similarly as the external loading effects are small with respect to those of rotation. The displacements solution correspond to these loads range from linear to highly nonlinear responses.

4.2 Validating the influence of $\boldsymbol{\Phi}_f$

The SVD decomposition performed to obtain $\boldsymbol{\Phi}_f$ is carried out with the snapshot matrix, Q, obtained by computing the geometrically nonlinear FOM forces for the case with the largest displacements $\{\alpha_f = 0.7$ and $\Omega = 0\,rpm\}$. Thus, the computation of nonlinear forces is performed once and for all and remains valid for any loading level, α_f, smaller than the loading level considered to construct the snapshot matrix and for any rotating velocity, Ω.



(a) $\boldsymbol{\Phi}_f$ correction vs. Φ projection, $\alpha_f = 0.7$.

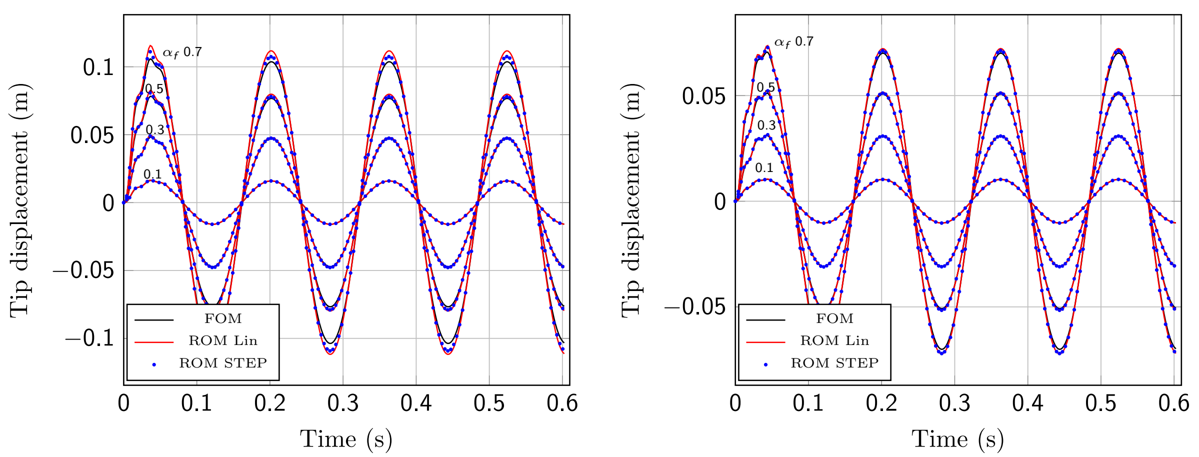
(b) Influence of m to construct $\boldsymbol{\Phi}_f$.

Figure 2: Influence of the POD based correction of nonlinear forces on the time response displacements at the tip of the rotating beam ($\Omega = 0$).

Figure 2a presents the need to correctly asses the nonlinear forces of the ROM. When a simple projection over Φ is performed to obtain the nonlinear forces, the response of the structure is stiffened, the maximum displacements of the FOM are not reached and higher order harmonics appear in the response. When the POD based nonlinear forces correction is performed, the quality of the response directly depends on the number of chosen modes to construct $\boldsymbol{\Phi}_f$ (see Figure 2b). However, there is a value of m for which the ROM accurately approaches the FOM response. In this case, the optimal value of m is 6. The optimum value of m is placed between the 99.99% and 100% of the cumulative participation factor, $\chi_m = \frac{\sum_{i=1}^{m} \sigma_i}{\sum_{j=}^{p} \sigma_j}$. Hereinafter, the results obtained with the nonlinear force projection matrix $\mathbf{B}$ constructed with the most nonlinear loading case and $m = 6$ are presented. Furthermore, when $m \to m_{max}$ the ROM response tends to the response obtained by a projection without Φ_f (see $\mathbf{\Phi}_f, m = 20$ curve in Figure 2b).

4.3 Dynamic response

The dynamic response of the structure is computed by two classical methods: (i) HHT-α time integration method and (ii) harmonic balance method (HBM) combined with the alternating frequency-time (AFT) technique to compute the geometrically nonlinear forces in time domain with the STEP method. As the results obtained with both methods are equivalent and just differ on a shift in time, just the time response computed by the HHT-α method are presented hereinafter. The inflation method for computing the generalised nonlinear forces gives almost the same results as the STEP method. Figure 3 shows the vertical displacement of the central node of the tip surface for different loading intensities and for $0\,rpm$ and $2000\,rpm$ rotating velocities.



(a) Time response for different loading levels without rotating velocity, $\Omega = 0 rpm$.

(b) Time response for different loading levels with constant rotating velocity, $\Omega = 2000 rpm$.

Figure 3: Tip displacements time response for a sinusoidal excitation.

Both linear and STEP methods present an accurate response when the loading intensity is small. Furthermore, without rotating velocity, $\Omega = 0\,rpm$, STEP method provides better results for important loading intensities. For a rotating velocity equal to $\Omega = 2000\,rpm$, both methods provide similar results for all loading intensities. Thus, when nonlinearities due to the external force are small with respect to the rotating effects, both methods provide accurate responses.

4.4 Accuracy and computational time consumption

In order to asses the accuracy of the proposed method and the classical linearised approach, the time average relative error with respect to the FOM solution is performed for all the loading cases and solution methods :

$$e_r(\%) = \frac{1}{n_t} \sum_{t_i=0}^{t_f} \frac{\|u_{ROM}(t_i) - u_{FOM}(t_i)\|}{\|u_{FOM}(t_i)\|} \cdot 100 \ . \tag{26}$$

The relative error for the HHT-α is computed considering all the time steps of the response. However, the relative error of the HBM method is computed for a single period once the periodic state of the FOM is reached. As shown in Figure 4, HBM method provides more accurate results than the HHT-α time integration method.

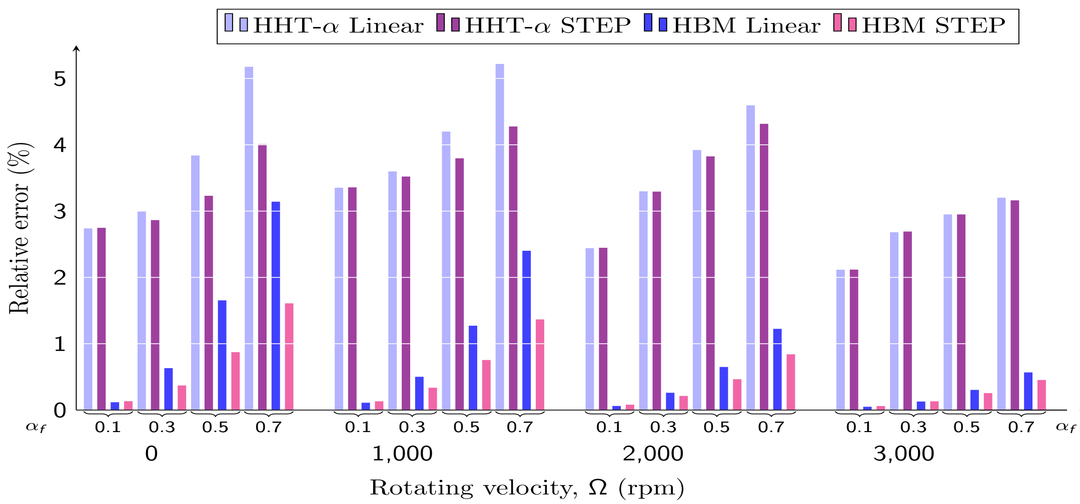


Figure 4: Comparative chart that presents the relative error with respect to different rotating velocities, loading intensities and time integration methods.

For any rotating velocity, the STEP model is more accurate than the linearised approximation in the nonlinear range. Furthermore, when the response behaves linearly, both methods provide similar results. The latter is specially highlighted in the HBM results. Thus, the proposed method is valid for any external loading intensity and for any rotating velocity.

Table 1: Computational cost in seconds for a single simulation with 3000 time steps, $\Omega = 0\ rpm$ and $\alpha_f = 0.7$.

	FOM	Linear	t_{Lin}/t_{FOM}	STEP	t_{STEP}/t_{FOM}
HHT-α	1147.18	34.76	3.03 %	53.76	4.68 %
HBM	-	24.48	2.13 %	32.09	2.79 %

As shown in Table 1 both ROM have a similar on-line computational cost equivalent to $\approx 3\%$ of the FOM computational time.

5 CONCLUSIONS

Reduced order models for the dynamic study of nonlinear rotating structures are presented in this work. As an extension to the classical linearised approach, the relative dynamic displacements around the pre-stressed equilibrium are considered as nonlinear. The geometrically nonlinear forces are computed by two different methods: (i) the inflation method that evaluates the nonlinear forces in the FOM and projects the solution to the ROM, and (ii) the STEP polynomial approximation where the nonlinear forces are a third order polynomial function of ROM's displacements. In order to improve the accuracy of the latter method a POD based correction of nonlinear forces is proposed. The proposed method is applied to a 3D model of a rotating beam. The solutions of the nonlinear ROM obtained with both time integration (HHT-α) and harmonic balance (HBM) methods are in good accordance with the solutions of the FOM and are more accurate than the linearised solutions. Further work will be focused in the application of the presented method on a complex structure. Then, the proposed ROM will be adapted to frictional contact problems and to aeroelastic coupling.

References

[1] J. J. Hollkamp et R. W. Gordon, *Reduced-order models for nonlinear response prediction: Implicit condensation and expansion*, Journal of Sound and Vibration, 318 (4), pp. 1139–1153, 2008.

[2] R. Sampaio et C. Soize, *Remarks on the efficiency of POD for model reduction in nonlinear dynamics of continuous elastic systems*, International Journal for numerical methods in Engineering, 72 (1), pp. 22–45, 2007.

[3] S. A. Rizzi et A. A. Muravyov, *Improved equivalent linearization implementations using nonlinear stiffness evaluation*, NASA/TM-2001-210838, L-18068, NAS 1.15:210838, 2001.

[4] A. A. Muravyov et S. A. Rizzi, *Determination of nonlinear stiffness with application to random vibration of geometrically nonlinear structures*, Computers & Structures, 81 (15), pp. 1513–1523, 2003.

[5] M. P. Mignolet, A. Przekop, S. A. Rizzi et S. M. Spottswood, *A review of indirect/non-intrusive reduced order modeling of nonlinear geometric structures*, Journal of Sound and Vibration, 332 (10), pp. 2437–2460, 2013.

[6] J.-D. Beley, Z. Shen, B. Chouvion et F. Thouverez, *Vibration non-linaire de poutre en grande transformation*, *Conference: CSMA 2017, At Giens, France*, 05 2017.

[7] H. Du, M. Lim et K. Liew, *A power series solution for vibration of a rotating Timoshenko beam*, Journal of Sound and Vibration, 175 (4), pp. 505–523, 1994.

[8] S. Kumar, A. Mitra et H. Roy, *Geometrically nonlinear free vibration analysis of axially functionally graded taper beams*, Engineering Science and Technology, an International Journal, 18 (4), pp. 579–593, 2015.

[9] O. Thomas, A. Sénéchal et J.-F. Deü, *Hardening/softening behavior and reduced order modeling of nonlinear vibrations of rotating cantilever beams*, Nonlinear Dynamics, 86 (2), pp. 1293–1318, Oct 2016.

[10] A. Sternchüss, *Multi-level parametric reduced models of rotating bladed disk assemblies*, Thèse de doctorat, Ecole Centrale Paris, 2009.

[11] G. Genta et M. Silvagni, *On centrifugal softening in finite element method rotordynamics*, Journal of Applied Mechanics, 81 (1), p. 011001, 2014.

[12] R. Henry, *Contribution a l'etude dynamique des machines tournantes*, Thèse de doctorat, 1981.

[13] F. A. Lülf, D.-M. Tran, H. G. Matthies et R. Ohayon, *An integrated method for the transient solution of reduced order models of geometrically nonlinear structures*, Computational Mechanics, 55 (2), pp. 327–344, 2015.

[14] K. Kim, V. Khanna, X. Wang et M. Mignolet, *Nonlinear reduced order modeling of flat cantilevered structures*, *50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference 17th AIAA/ASME/AHS Adaptive Structures Conference 11th AIAA No*, 2009.

[15] X. Wang, R. A. Perez et M. P. Mignolet, *Nonlinear reduced order modeling of complex wing models*, *54th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, 2013.

[16] K. Carlberg, C. Farhat, J. Cortial et D. Amsallem, *The GNAT method for nonlinear model reduction: effective implementation and application to computational fluid dynamics and turbulent flows*, Journal of Computational Physics, 242, pp. 623–647, 2013.

[17] S. Chaturantabut et D. C. Sorensen, *Discrete empirical interpolation for nonlinear model reduction*, *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on.* IEEE, 2009.