6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
1115 June 2018, Glasgow, UK

INTEGRATION SCHEMES FOR THE TRANSIENT DYNAMICS OF NONLINEAR CABLE STRUCTURES

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Key words: Integration Schemes, Transient Dynamics, Cables, IGA

Abstract. The recently developed time integration schemes are analysed and implemented to the transient dynamics of the space cable structures. The application of these methods to the selected examples of space cable structures, witch exhibit common properties regarding nonlinearity and involve low frequency transversal and higher frequency longitudinal modes, was used in evaluation. They are discretized with finite cable elements and B-splines. The time integration is performed for both models and issues related to the space discretization are additionally addressed.

1 INTRODUCTION

The direct time stepping integration schemes are commonly used in analysis of the transient dynamic problems for linear and nonlinear structures, [1, 2]. These schemes can be categorized by many criteria, however, they are typically first divided to implicit and explicit methods depending on weather factorization of the system dynamic (effective) stiffness matrix is required or not. The explicit methods that do not require factorization are more efficient on the time step level. However, these methods have hard limitation regarding the time step size due to stability issues. The implementation of the implicit time schemes is often more efficient amid the numerical effort necessary for the system matrix factorization. Many attempts of fusing the advantages of these methods while still keeping efficiency have been boiled out in a number of schemes.

The special attention is addressed to the numerical efficiency of the time integration schemes. The spectral radius based analysis for the corresponding time integration schemes performed in [4, 6, 8] is reliable in linear domain and it can only give guidance for nonlinear problems. The important issues: user controlled higher frequency damping, overshooting, amplitude decay and period elongations are typically monitored in simple benchmark examples. The focus of the paper is the efficient implementation of the time stepping algorithms with error monitoring mechanism and step size control. The objective of the paper is to put together desirable properties scattered over a many schemes in order to design reliable algorithm.

2 GENERAL DESCRIPTION OF DIRECT METHODS

The governing equation of the nonlinear structure reads:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{Q} \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are mass, damping and stiffness matrices, respectively. $\mathbf{Q}(t)$ and $\mathbf{q}(t)$ are force and position vector.

2.1 Generation of the integration time schemes

The general procedure of developing time integrating scheme can be summarized as follows: Relaying on finite differences, mean value theorems of calculus and approximation approach, it is possible to express the velocity, acceleration and displacement vectors at a certain specified instant as a function of nodal values and eventually some scalar parameters. Requirement for the dynamic balance at specific instant boils out the scheme: typically when that (specific) instant is at the beginning of the time step (where velocity and displacements are known) scheme is explicit, otherwise it is implicit. When the integration scheme depends on scalar parameter(s) this parameter(s) is optimized respect to accuracy and stability.

2.2 Considered methods

In this paper two specific methods (in what follows referred to as Bathe method and KDP G-α method) are assessed from the point of view: How they are suitable for the nonlinear transient dynamics of cables and cable structures. Additionally, an comparison to the simple explicit Euler with correction and implicit average acceleration method (e.g. Newmark δ method, $\delta = 0.5$, $\beta = 0.25$) is performed.

2.2.1 Composite Bathe method

Bathe method [4] is composite method designed for time integration of the nonlinear, second order in time differential equations. The time step is divided in two sub-steps. In the first sub-step trapezoidal rule and in the second sub-step 3 point Euler backward formula were employed. Both schemes were used independently for a long time before. The method is additionally analyzed by author in [5]. It demonstrates desirable and challenging property of energy and momentum conservation.

2.2.2 General–α scheme

The KDP G-α scheme [6] has been developed after generalized-α procedure for the first-order dynamic systems introduced in [7]. This scheme in class of the generalized-α

nominally depends on parameter α, however, it involves α_m, α_f and γ parameters that all depends on ρ_∞ where parameter ρ_∞ mast be chosen such that $0 \leq \rho_\infty \leq 1$ in order to fulfill stability and second order accuracy requirements. The numerical dissipation and dispersion analysis [6] provide guidance in selection of ρ_∞: The reduction in dissipation and dispersion errors are more pronounced as $\rho_\infty \to 0$.

2.2.3 Accuracy and stability analysis

The accuracy and stability analysis is performed essentially on the linear single degree of freedom (sdof) model and conclusion are deduced generalizing the conclusions extrapolating them from linear to the nonlinear problems. Generalization is founded in fact that accuracy and stability measures are generic properties of the integration operators related to the non-dimensional ratio of time step and natural period of considered problem ($T/\Delta t$). It looks like that this approach has limitations when nonlinear systems are involved.

Here, the schemes (Bathe [4] and KDP G-α [6]) are exposed to the specific class of the nonlinear problems that however assumes biased but more realistic practical assessment of the methods involved. The cable structures have been modeled as a catenaries with the isoparametric finite elements or B-spline as in IGA approach. The specificity of the models is that they do not sustain the compression of the catenary and in the case of the negative stretch the algorithm execution stops.

During the tests, Bathe method is always applied with double step in order to have equivalent numerical effort.

The performed tests are part of the wider investigation of the methods undertaken in order to establish efficient step size management based on the error and the nonlinear iteration control.

3 CATENARY EXAMPLES

3.1 Example 1: Catenary net

Double symmetrical cable net, Figure 1, is exposed simultaneously to the symmetrical step load of the discreet forces at four inner nodes (Figure 1, node 4 and tree symmetrical counterpart). The net properties are given Table 1 in consistent units.

Table 1: Example 1: Cable net properites in consistent units

$A = 1$	cross section area
$E = 29 \times 10^5$	elastic modulus
$\rho = 0.1$	density
$q = 1$	distributed load
$S = 0$	initial stress
$a = 40$	distance

The cable properties in Table 1 are borrowed from [9] where static analysis is performed for the structure that corresponds to the quarter of model used here.

Model: Only one quarter of the structure is modeled and corresponding nodes are marked in Figure 1. Boundary conditions at nodes 1,2,3 and 7 provide symmetry. A number of model possibilities are tested in order to evaluate IGA versus FEM efficiency. Each of 12 segments is modeled with catenary that is eider IGA or FEM substructure. In both cases substructures are 2 or 5 node mesh models. Finite elements involved are nonlinear isoperimetrical 2 and 3 node elements. IGA patches are 2 node (first order) or 5 nodes (3 order polynomials). The 2 nod FEM and 2 node IGA patch reduce practically to the same substructure.

Load: The structure is exposed to the gravity load first and then step forces of magnitude $F = -3000$ after $\Delta t = 0.015$ are applied simultaneously at four inner nodes in $-y$ direction.

Response: The time displacement of node 4 in y direction in consistent units is calculated with 7000 time steps $\Delta t = 0.005$ and it is given in Figure 2.

The increase of the time step to more then $\Delta t = 0.0125$ makes the KDP G-α scheme to stop execution due to nonphysical negative stretch, while the Bathe method gives acceptable results even with step $\Delta t = 0.2$.

During execution the monitoring of the number of iterations due to nonlinearity and square norms of the errors residuum has been performed. Monitored error norms are equation, energy and displacement residuum. The nonlinear iterations control is implemented as suggested in algorithm sources. It is observed that KDP G-α is considerably sensitive to the convergence criteria changes (transferring convergence to other then proposed measures) for the examples considered in this paper. Under same conditions, the Bathe method is proved to be less sensitive to manipulation with convergence criteria. In diagrams 3 and 4 are given monitored data during presented execution of the algorithms. It should be noted that convergence level for KDP G-α (set as 10^{-3}) has required 9 iterations as average and boiled out very low error norms. The Bathe scheme (with the same convergence criteria level of 10^{-3}) did required one nonlinear iteration per time step as maximum, however, the errors residuum are much higher.

3.2 Example 2: Three interconected catenary

The system of three interconnected catenary, Figure 5, previously statically analyzed in [9], [10], [11] and [12] is introduced to analyze transient response. The cable 1 in Figure 5 has properties according Table 2 and cables 2 and 3 have properties according Table 3.

The distances in Figure 5 are as follows: $a = 400$m and $b = 300$m. The stiffness of the spring is $K = 1000$ Nm^{-1} in y direction only. In the presented case, the additional concentrated mass at the meeting point M is zero.

The transient analysis is performed as follows: First the structure is exposed to the self weight and statical analysis is performed. This will be equilibrium position witch around the structure is going to vibrate. Then the force $F = 2000$N is applied at meeting node M in $-z$ direction and static equilibrium is reached. Then, the structure is released

Table 2: Example 2: Cable 1 properties

$AE = 2.9 \times 10^5$	N	stiffness coefficient
$\rho = 0.1$	kgm^{-1}	distributed mas
$q = 1$	Nm^{-1}	distributed load
$L_1 = 580$	m	initial length

Table 3: Example 2: Cables 2 and 3 properites

$AE = 2.9 \times 10^5$	N	stiffness coefficient
$\rho = 0.1$	kgm^{-1}	distributed mass
$q = 2$	Nm^{-1}	distributed load
$L_2 = L_3 = 510$	m	initial length

suddenly from the rest under self-weight (i.e. the applied force F disappears, $F = 0$). The response of the structure at meeting node M is given in Figures 6, 7 and 8. The response is calculated with KDP G-α ($\rho_\infty = 0$) scheme and Bathe method with time step $\Delta t = 0.01$s with 5000 time steps. The results agree well and discrepancies can be hardly observed for displacements. However, differences between velocities and accelerations are more easy to note.

It should be noted that the time step increase to $\Delta t = 0.1$s will bring new situation: the Bathe scheme still gives very good results while KDP G-α scheme stops execution due to nonphysical negative stretch sign. This deserves additional consideration and it is premature to make final conclusion.

4 CONCLUSIONS

The transient analysis of the cable structures is performed in order to evaluate time integration schemes regarding efficiency and the practical aspects in related implementations. Keeping in mind that these schemes can be implementation sensitive, a care has been payed to the original algorithm guidelines and some implementation sensitivity aspects are researched. The integration parameters are pushed over limits to provoke accuracy and stability issues.

- Both tested methods perform well when a small step size is involved. They give close results. With increasing step the Bathe method performs well while the KDP G-α scheme after certain level stops the execution due to nonphysical negative stretch (in all considered cases).

- Monitored accuracy and efficiency on the selected problem set give not clear advances: the Bathe method reaches more accurate results with relative less numerical effort. It is less sensitive to the extremely small steps. However, KDP G-α scheme that requires much more nonlinear iterations gives lower error residuals under same conditions in considered cases.

It should be noted that these tests may in general give biased answers, however, in specific implementation were appropriate representing structures are selected, these results lead to the reliable choices for specific implementation.

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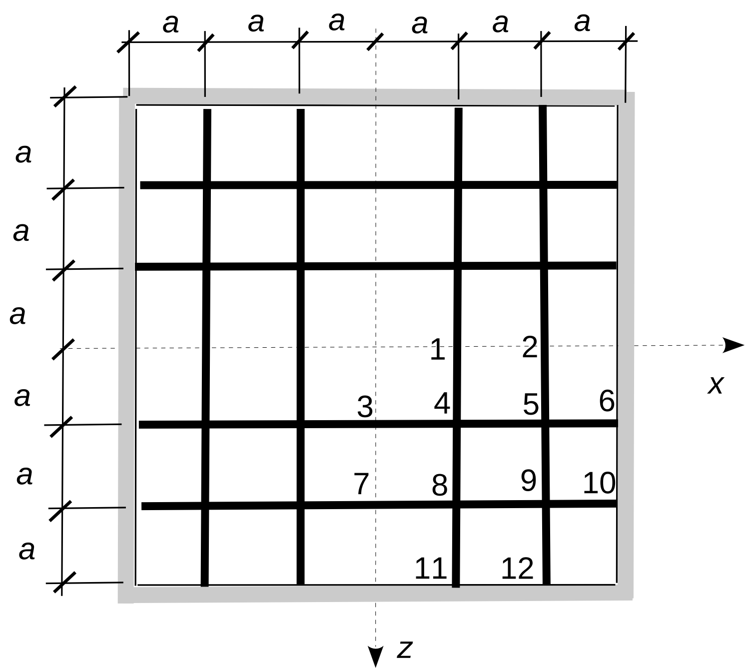

Figure 1: Example 1: Cable net structure

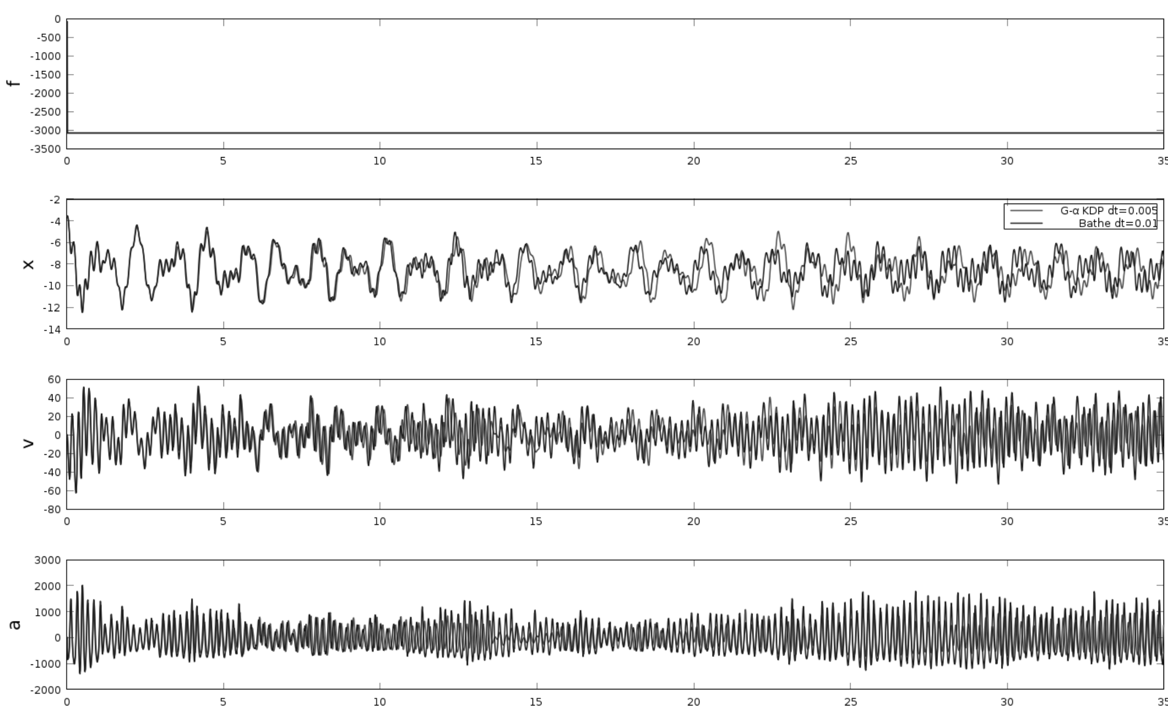

Figure 2: Example 1: Response of the net structure at node 4 in vertical direction

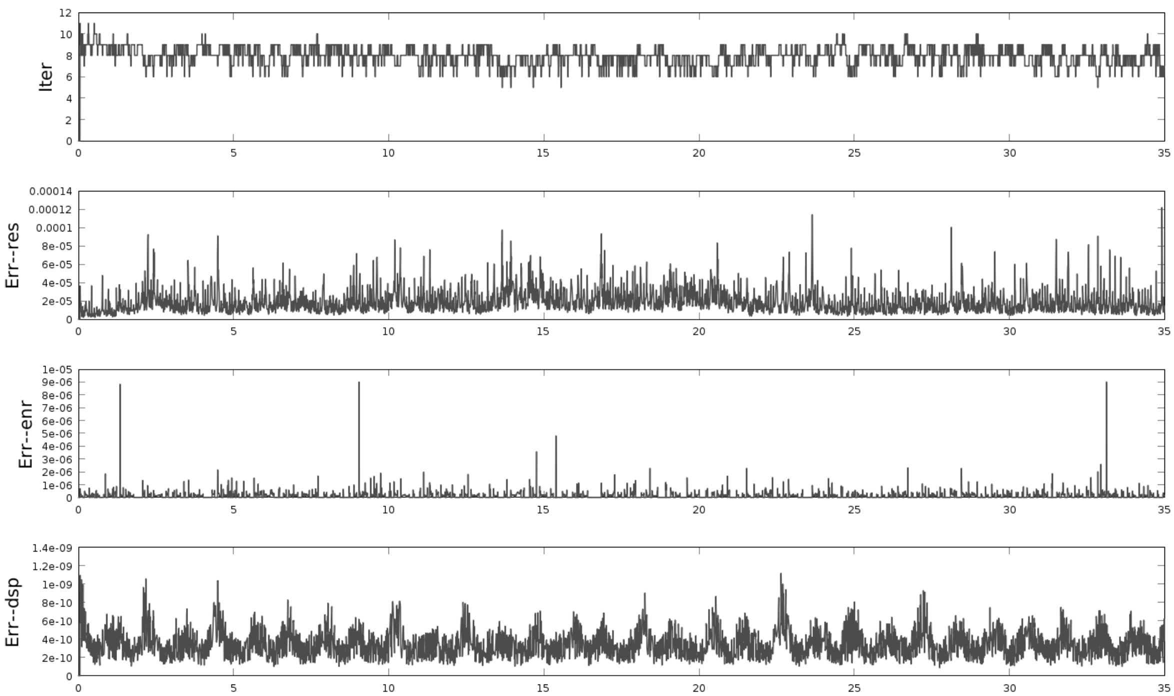

Figure 3: Example 1: Execution of the KDP G-α scheme: Iterations and error overview

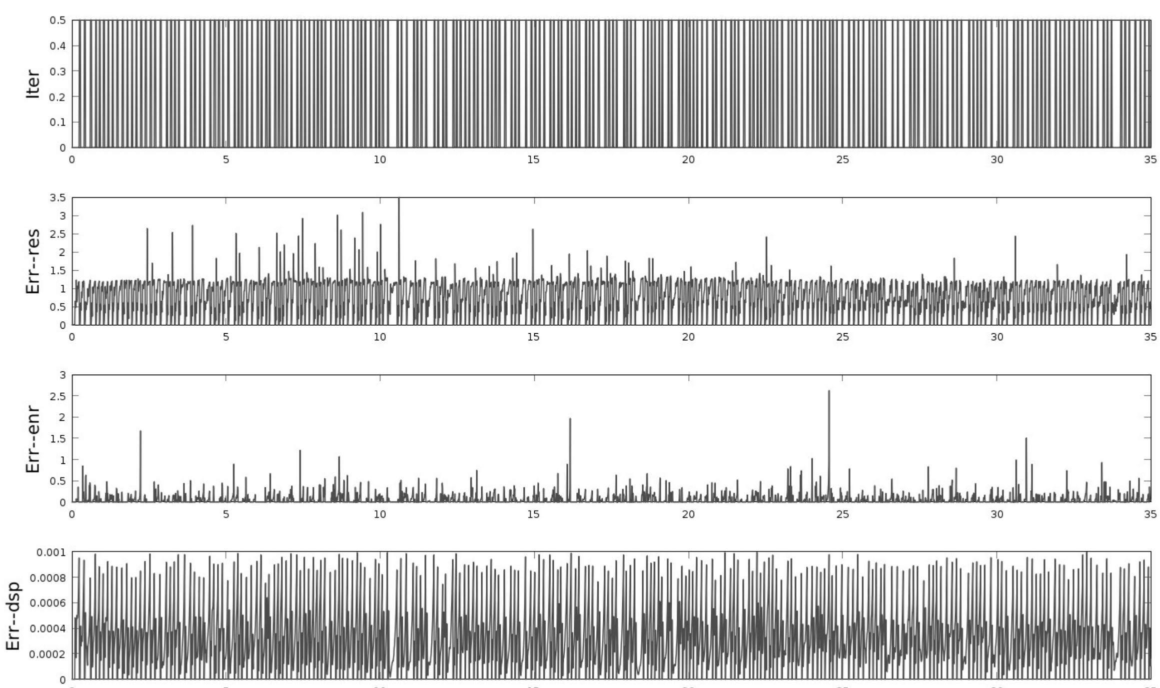

Figure 4: Example 1: Execution of the Bathe scheme: Iterations and error overview

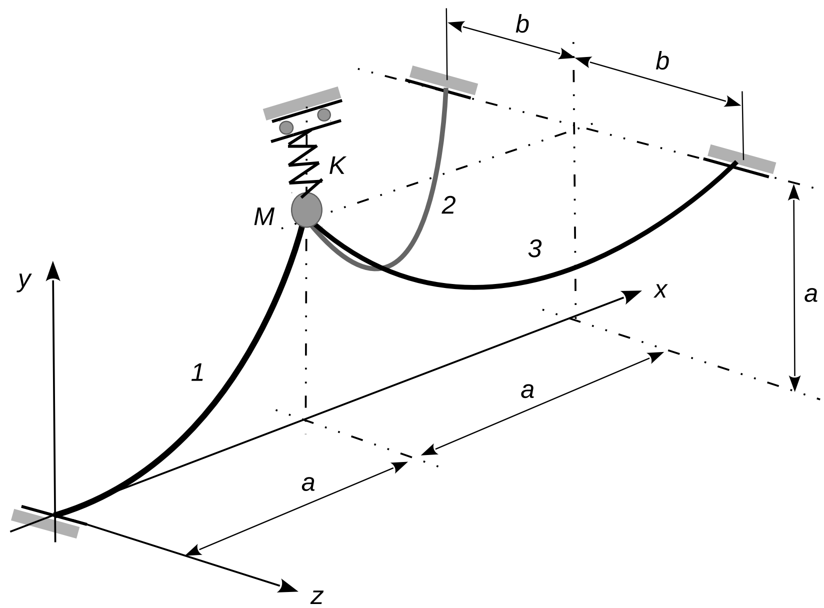

Figure 5: Example 2: Three catenary structure

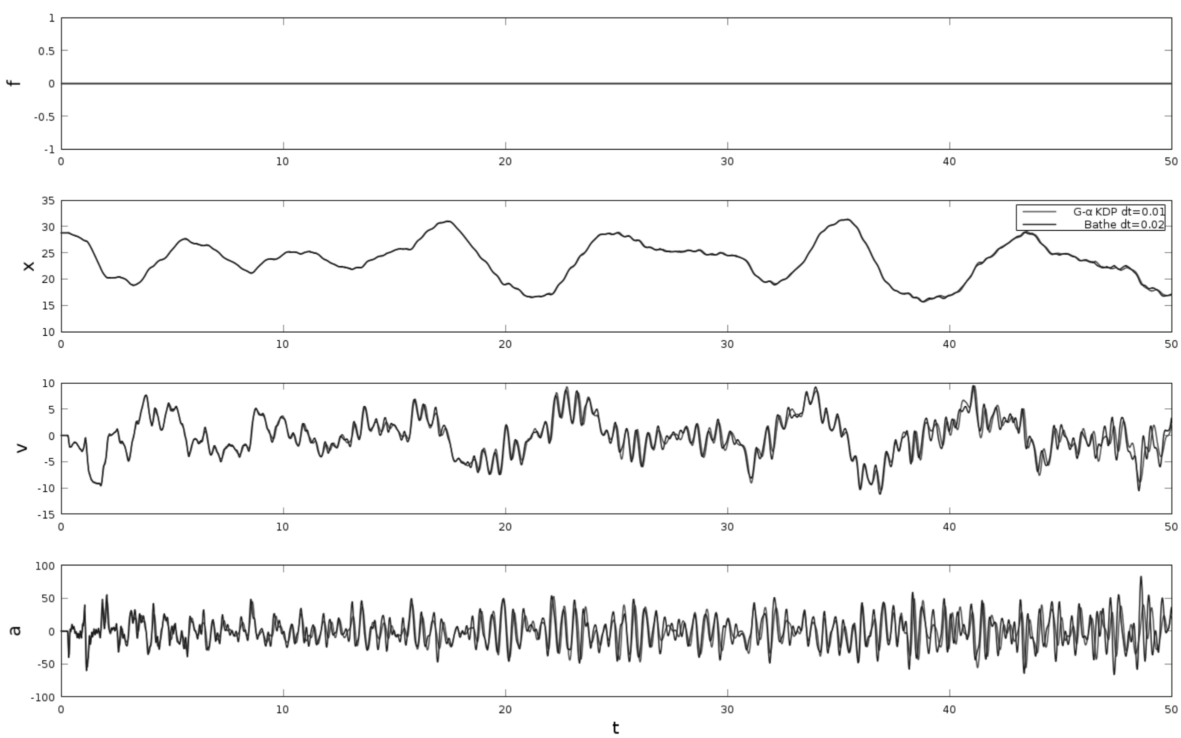

Figure 6: Example 2: Response of the three catenary system at meeting node M in x direction after releasing from initial position

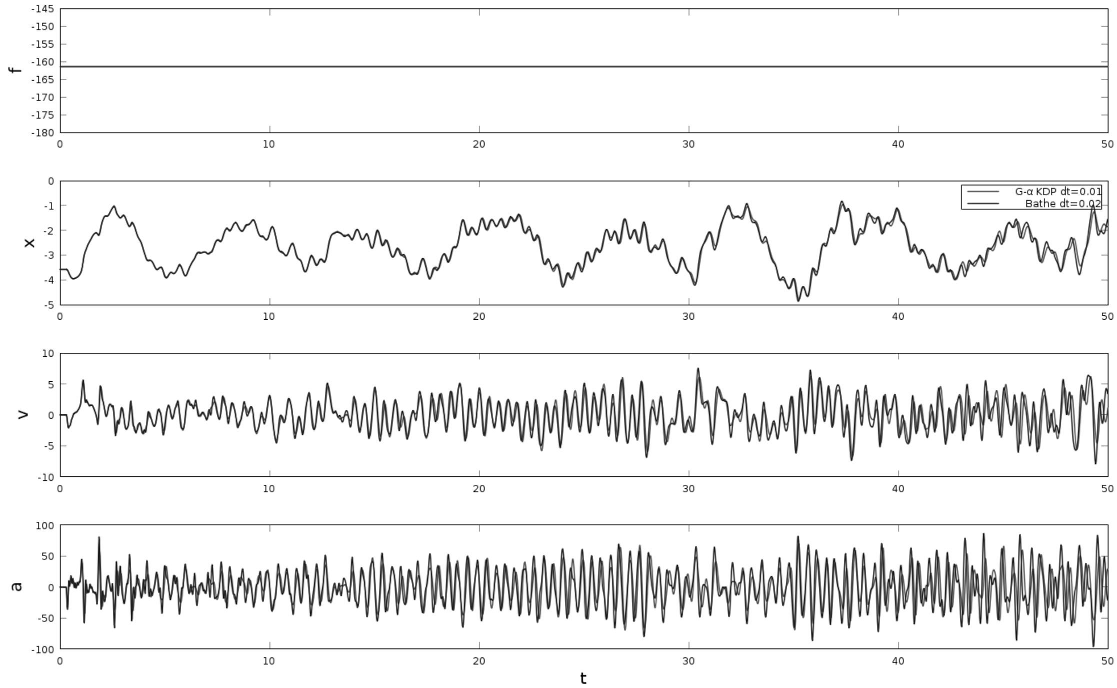

Figure 7: Example 2: Response of the three catenary system at meeting node M in y direction after releasing from initial position

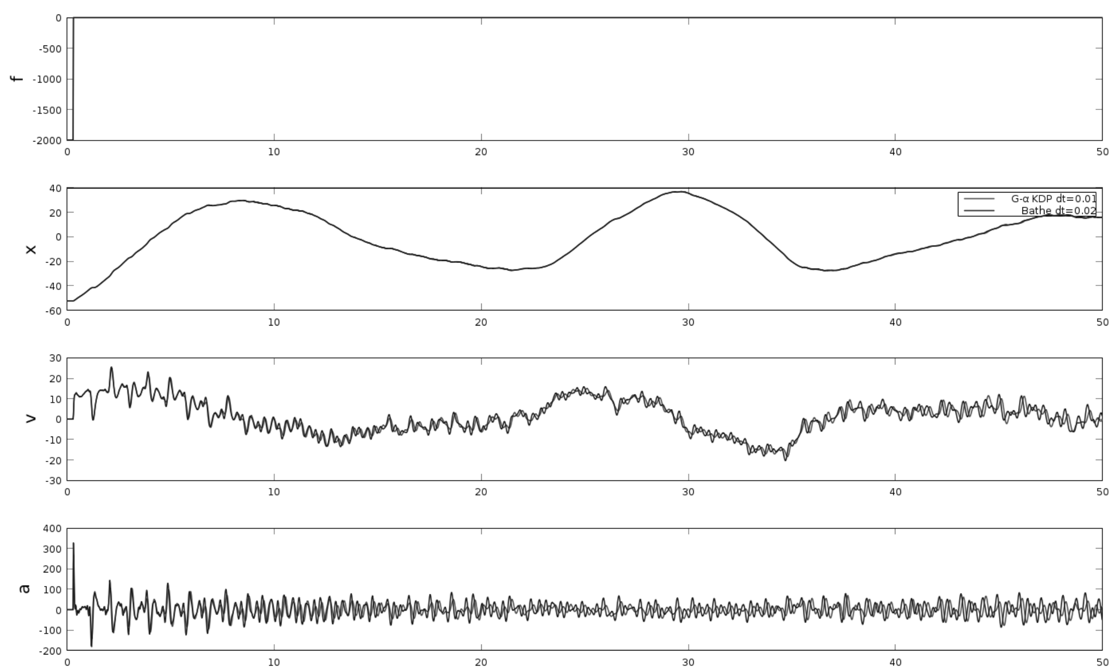

Figure 8: Example 2: Response of the three catenary system at meeting node M in z direction after releasing from initial position