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NEW APPROACH ON DISCRETIZATION METHODS FOR MESOSCOPIC STUDY OF CONCRETE STRUCTURES

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Abstract. This paper presents a study for modelling concrete in mesoscopic level by different discretization methods with in-house program CaeFem. Mesoscopic level has been chosen which let us to model different component of concrete like mortar and aggregate separately.

Discretization method will play significant role in analysis since we assumed concrete as a composite material. Finite element method (FEM) has been used as common discretization methods. However, usual FEM will bring dependency of the discretization on geometry of each component in model. This is only possible with irregular meshes with basically degraded accuracy. Furthermore, excessively fine discretizations occur to picture the random mesoscopic geometry. Such drawbacks are already evident in 2D but tighten in 3D. Thus, the paper aims at decoupling of mesoscopic geometry and discretization.

New discretization methods have been developed which are quite new with high potential of flexibility. Three methods, regardless of geometry and topology of inclusions, which are FEM method with regular mesh, Element Free Galerkin (EFG) method and Isogeometric (IGA) method are studied in this paper. These methods have new philosophy of solving problems with fast algorithms and help of different approaches. The resultant behaviours are compared and verified versus obtained results from a commercial FEM-program DIANA.

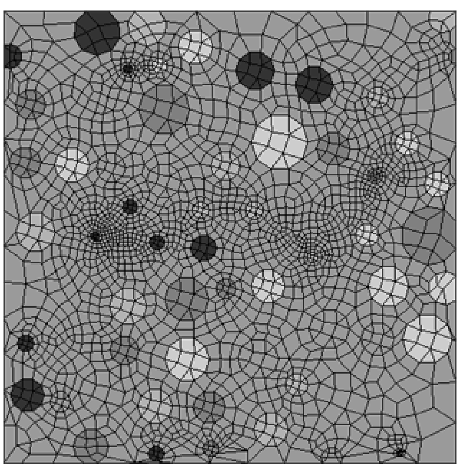
1 INTRODUCTION

The Finite Element Method has prevailed upon other numerical methods in calculation of mechanical behaviour of components and structural mechanics for many years [1, 2, 3], about applications in reinforced concrete structure see [4]. However, dependency on geometry of materials will bring lots of limitations specially when there are aggregates in the model in different range of sizes and locations which will need unstructured irregular mesh to cover all the edges properly. To overcome this difficulties, several discretization methods, regardless of geometry and topology of inclusions have been introduced. As an alternative to FEM new functions have been formulated which are no longer restricted to finite elements, e.g. EFG [5, 6, 7, 8, 9]. The omission of elements allows great flexibility in arrangement of nodes which facilitates adaptive mesh refinement and large deformation will appear further. Another method which allows high geometric adaptability with advantages for solving problems of structural mechanics is IGA. IGA used "Non-Uniform Rational B-Spilines"(NURBS) approach. NURBS are standard approaches in CAD for description of freeform surfaces, see [10, 11, 12, 13, 14].

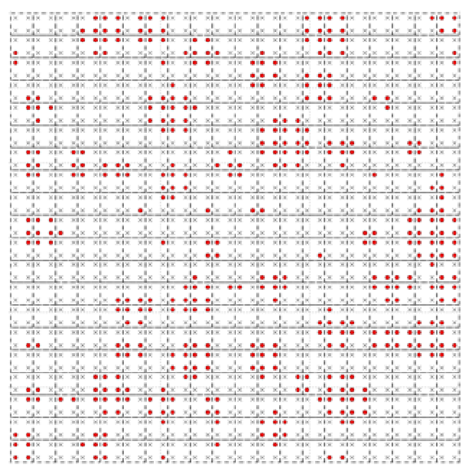
2 DISCRETIZATION APPROACHES

Discretization methods are classified in two different categories. First approach is irregular mesh which follows geometry of different materials and properties are presented by whole element. FEM is the most common method among numerical approaches which is used by most commercial software and using irregular mesh approach as discretization method, see Figure 1(a), shows irregular mesh model used for analysis with DIANA.

Another approach is regular mesh methods, whereby material properties are considered at integration point level, see Figure 1(b) which shows regular mesh model used with in-house program CaeFem. In this paper, models are analyzed with three different regular mesh methods which mentioned before and contains: Finite element methods with regular mesh, Element Free Galerkin method and Isogeometry method and compared with irregular mesh method result from two programs.



(a) Irregular mesh - DIANA model



(b) Regular mesh -CaeFem model

Figure 1: Discretized models

3 NUMERICAL SIMULATION

3.1 3D random mesoscopic model

A 3D random mesoscopic model has been generated by a program written in Matlab software. This model consists of aggregates and mortar while coarse aggregates are presented as spheres with randomly distributed size and location. Random distribution of aggregate is based on Fuller's curve. This curve can be described by a simple equation given below and presented in Figure 2.

$$P(d) = (\frac{d_i}{D})^n \tag{1}$$

Where as: $P(d)$ is Cumulative percentage passing a sieve with the diameter d_i. d_i is Diameter of aggregate, D is Maximum aggregate grain size and n is Exponent with a typical value between 0.45 and 0.70 (assumed as 0.5). In this study, the minimum and the maximum size of aggregate's diameter are considered to be 2 and 16 mm respectively and n as 0.5 with 19 sieves. Figure 3(a) shows aggregate distribution in a 3D mesoscopic model which is calculated with Equation 1 and based on Fuller's curve shown in Figure 2. In this model each of three dimensions of the model is equal to 60 mm.

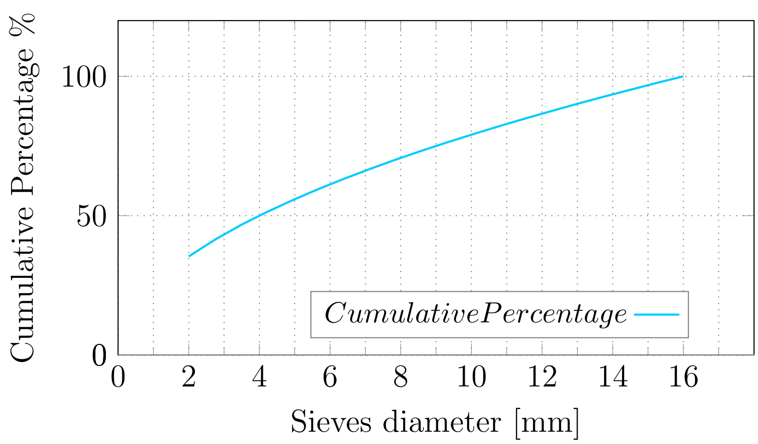
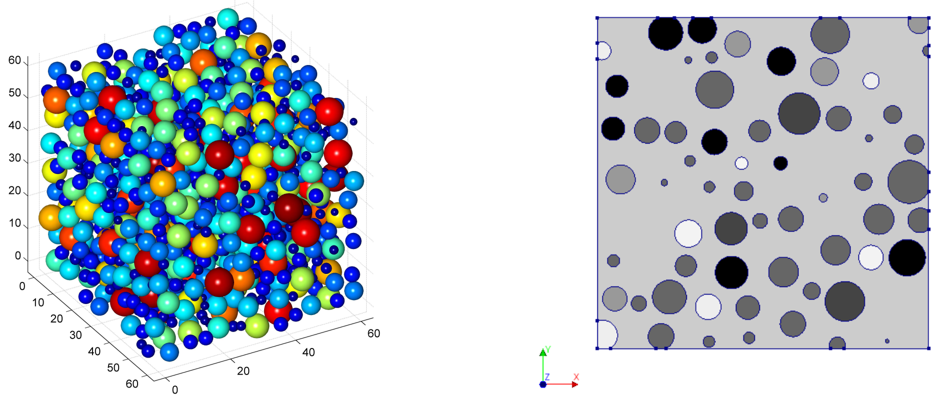


Figure 2: Grading of aggregate based on Fuller's curve gradation

3.2 2D Sections random mesoscopic model

As mentioned earlier, 2D simulation will provide reliable results with less complications in modeling. An example for 2D section out of 3D random mesoscopic model is shown in Figure 3(b).

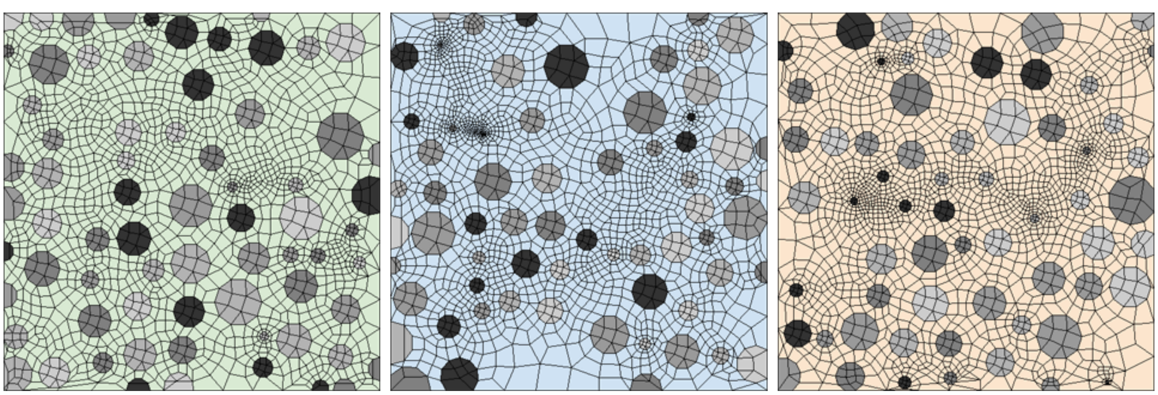


(a) (b)

Figure 3: (a) 3D random mesoscopic model generated with matlab code written by Tino Kühn [15] and (b) 2D section - random mesoscopic model consist of mortar and aggregates

4 IN-HOUSE PROGRAM CAEFEM

CaeFem has been developed in Python to solve numerical problems. It is under development to include different discretization approaches as well as various nonlinear material models. In this paper, three 2D sections have been selected in order to investigate the effect of different discretization methods on the final results. Figure 4 shows these 2D sections.



(a) Model 1 - z=0 mm (b) Model 2 - z=15 mm (c) Model 3 - z=30 mm

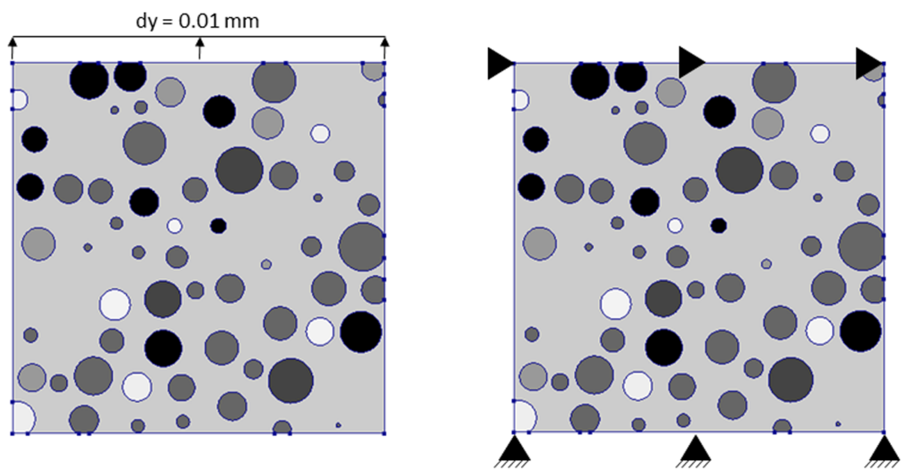
Figure 4: 2D sections generated from 3D-Random mesoscopic model

5 MODEL SPECIFICATION

These three models, are subjected to the same type of loading, boundary conditions and material properties. Table 1 shows the reference material properties and Figure 5 (a) and (b) show loading and boundary condition assigned to the 2D models. Plane stress material model is used and all materials are assumed to behave linearly. Model is fixed on bottom edge and tension load is applied on top edge as prescribed deformation equal to 0.01 mm in vertical direction.

Table 1: Material properties

	Aggregates	Mortar
Young's modulus	60000 MPa	20000 MPa
Poisson ratio	0.15	0.2

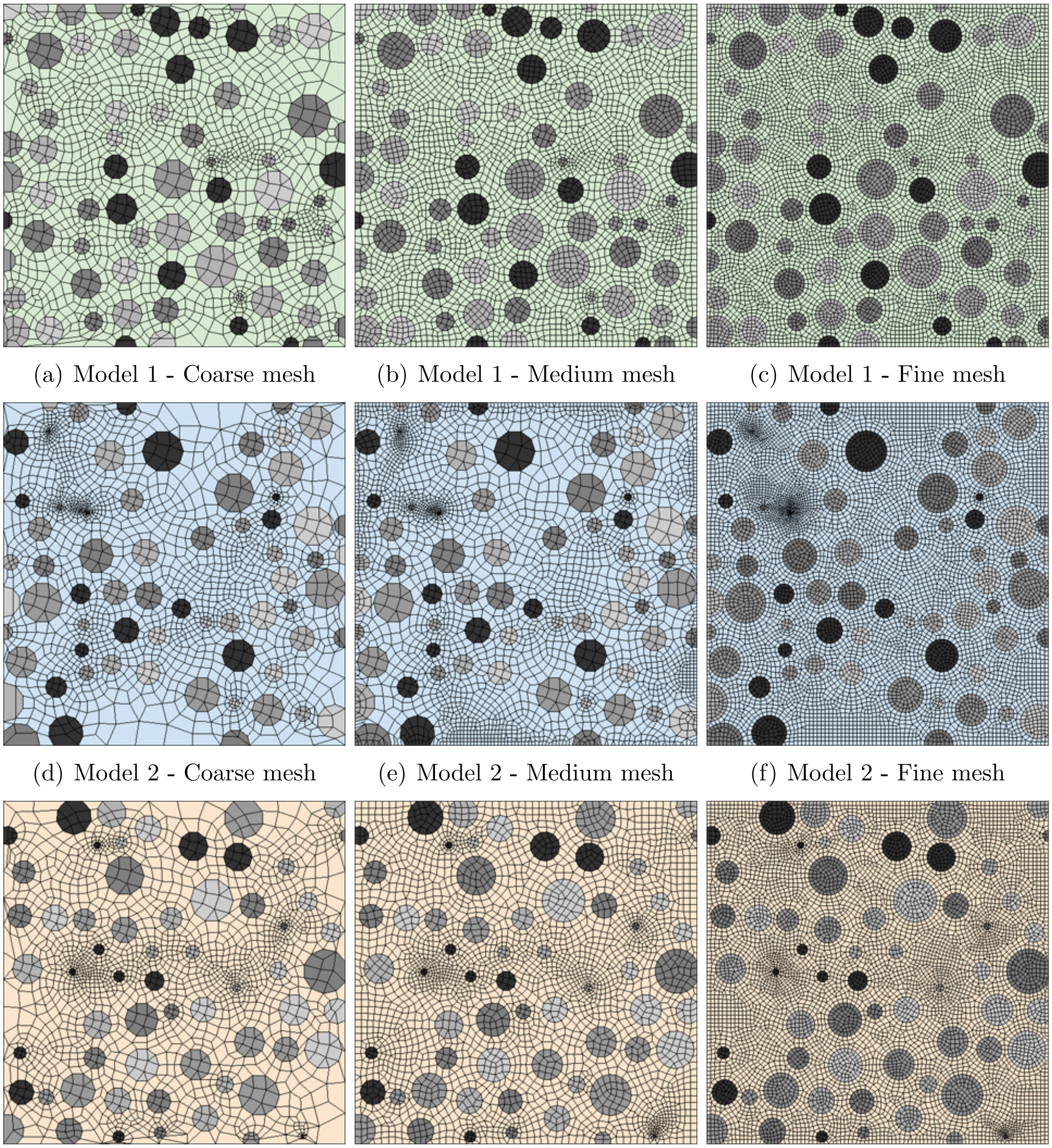


(a) Loading (b) Boundary condition

Figure 5: Schematic representation of 2D model, subjected to the prescribed displacement and static boundary conditions

6 ANALYSIS RESULTS AND PARAMETER STUDY

For parameter study and for each section, three different range of mesh sizes are checked, Figure 6 shows nine models used in sensitivity analysis. DIANA is a well-known commercial FEM software and used for validation of results [16]. Tables 2, 3 and 4 show results of each analysis in detail. Figures 7, 8 and 9 show comparison results for models with different numerical discretization approaches and with different number of degree of freedom. Total reaction force is the parameter used for results comparison in vertical direction. It is observed that there is a good agreement in the results obtained from different discretization methods.



(a) Model 1 - Coarse mesh (b) Model 1 - Medium mesh (c) Model 1 - Fine mesh

(d) Model 2 - Coarse mesh (e) Model 2 - Medium mesh (f) Model 2 - Fine mesh

(g) Model 3 - Coarse mesh (h) Model 3 - Medium mesh (i) Model 3 - Fine mesh

Figure 6: 2D section models used for sensitivity analysis

From Figures 7, 8 and 9 it is observed that IGA is most conservative method among different discretization methods. The area ratio of aggregates in 2D models are 0.32 , 0.28 and 0.26 for model 1 , model 2 and model 3 respectively. By using elastic material models in these three reference cases, it is observed that the total reaction forces are decreasing as the area ratio of aggregates is decreasing.

Table 2: Results comparison / Model 1 - $z = 0$ mm

	Model 1 Z = 0 mm	Number of Dof	Total reactions $F_x[N]$	Total reactions $F_y[N]$	Erorr Percentage
Coarse mesh	DIANA(Irregular mesh/FEM)	3660	-0.81	-380.89	-
	CaeFem(Irregular mesh/FEM)	3660	-0.84	-381.44	0.14%
	CaeFem(Regular mesh/FEM)	3872	-0.59	-397.93	4.47%
	CaeFem(Regular mesh/EFG method)	3872	-0.60	-397.95	4.48%
	CaeFem(Regular mesh/IGA method)	3872	-0.72	-432.47	13.54%
Medium mesh	DIANA(Irregular mesh/FEM)	7774	-0.88	-383.22	-
	CaeFem(Irregular mesh/FEM)	7774	-0.87	-383.51	0.08%
	CaeFem(Regular mesh/FEM)	7442	-1.06	-395.19	3.13%
	CaeFem(Regular mesh/EFG method)	7442	-1.06	-395.18	3.12%
	CaeFem(Regular mesh/IGA method)	7442	-1.00	-431.01	12.47%
Fine mesh	DIANA(Irregular mesh/FEM)	14692	-0.85	-384.17	-
	CaeFem(Irregular mesh/FEM)	14692	-0.85	-384.34	0.04%
	CaeFem(Regular mesh/FEM)	15138	-0.65	-392.00	2.04%
	CaeFem(Regular mesh/EFG method)	15138	-0.74	-392.47	2.16%
	CaeFem(Regular mesh/IGA method)	15138	-0.83	-427.99	11.41%

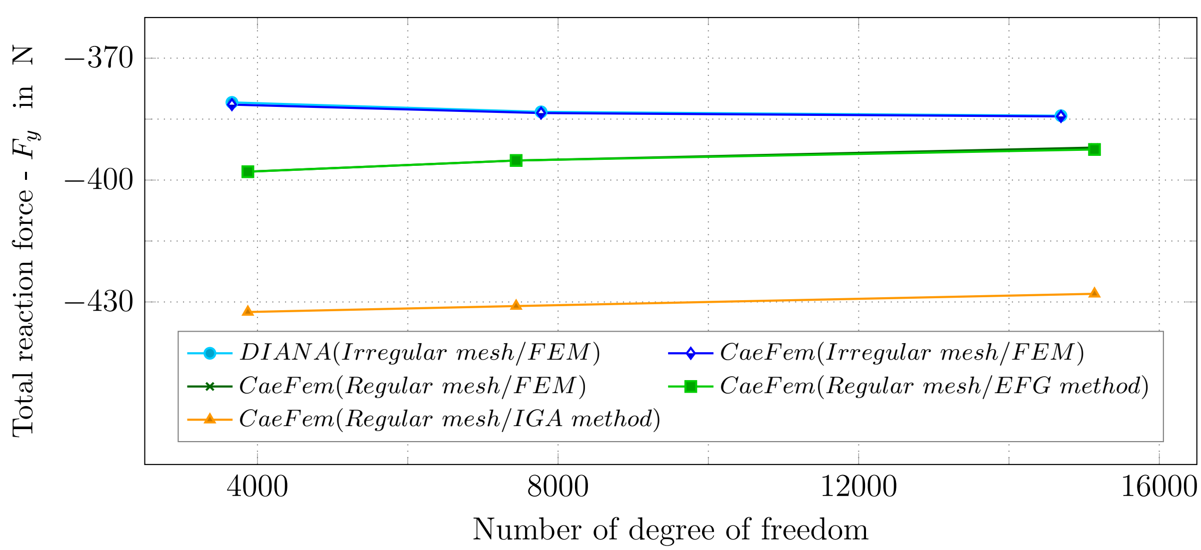


Figure 7: Results comparison / Model 1 - $z =$ 0 mm

Table 3: Results comparison / Model 2 - z = 15 mm

	Model 2 Z = 15 mm	Number of Dof	Total reactions $F_x[N]$	Total reactions $F_y[N]$	Erorr Percentage
Coarse mesh	DIANA(Irregular mesh/FEM)	4310	-4.37	-369.52	-
	CaeFem(Irregular mesh/FEM)	4310	-4.41	-370.02	0.13%
	CaeFem(Regular mesh/FEM)	3872	-5.82	-385.18	4.24%
	CaeFem(Regular mesh/EFG method)	3872	-5.84	-385.25	4.26%
	CaeFem(Regular mesh/IGA method)	3872	-6.03	-419.04	13.40%
Medium mesh	DIANA(Irregular mesh/FEM)	6776	-4.54	-368.92	-
	CaeFem(Irregular mesh/FEM)	6776	-4.56	-369.34	0.11%
	CaeFem(Regular mesh/FEM)	7442	-5.36	-381.63	3.45%
	CaeFem(Regular mesh/EFG method)	7442	-5.36	-381.62	3.44%
	CaeFem(Regular mesh/IGA method)	7442	-5.42	-415.89	12.73%
Fine mesh	DIANA(Irregular mesh/FEM)	15922	-4.80	-372.08	-
	CaeFem(Irregular mesh/FEM)	15922	-4.80	-372.24	0.04%
	CaeFem(Regular mesh/FEM)	15138	-5.03	-378.70	1.78%
	CaeFem(Regular mesh/EFG method)	15138	-5.03	-378.72	1.78%
	CaeFem(Regular mesh/IGA method)	15138	-5.53	-414.32	11.35%

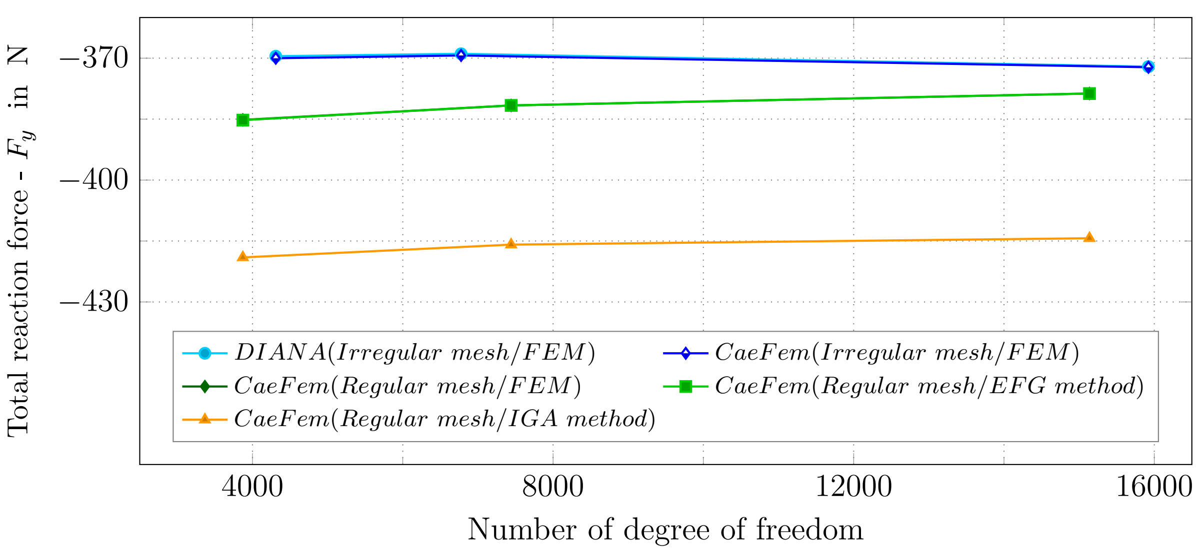


Figure 8: Results comparison / Model 2 - z = 15 mm

Table 4: Results comparison / Model 3 - $z = 30$ mm

	Model 3 Z = 30 mm	Number of Dof	Total reactions $F_x[N]$	Total reactions $F_y[N]$	Erorr Percentage
Coarse mesh	DIANA(Irregular mesh/FEM)	4348	-1.98	-367.49	-
	CaeFem(Irregular mesh/FEM)	4348	-1.97	-368.00	0.14%
	CaeFem(Regular mesh/FEM)	3872	-2.21	-380.91	3.65%
	CaeFem(Regular mesh/EFG method)	3872	-2.23	-380.96	3.66%
	CaeFem(Regular mesh/IGA method)	3872	-2.64	-414.98	12.92%
Medium mesh	DIANA(Irregular mesh/FEM)	6918	-1.99	-368.74	-
	CaeFem(Irregular mesh/FEM)	6918	-1.99	-369.03	0.08%
	CaeFem(Regular mesh/FEM)	7442	-1.93	-378.46	2.64%
	CaeFem(Regular mesh/EFG method)	7442	-1.93	-378.45	2.64%
	CaeFem(Regular mesh/IGA method)	7442	-2.36	-413.35	12.10%
Fine mesh	DIANA(Irregular mesh/FEM)	15576	-2.02	-369.96	-
	CaeFem(Irregular mesh/FEM)	15576	-2.02	-370.10	0.04%
	CaeFem(Regular mesh/FEM)	15138	-2.25	-376.73	1.83%
	CaeFem(Regular mesh/EFG method)	15138	-2.25	-376.75	1.84%
	CaeFem(Regular mesh/IGA method)	15138	-2.29	-411.43	11.21%

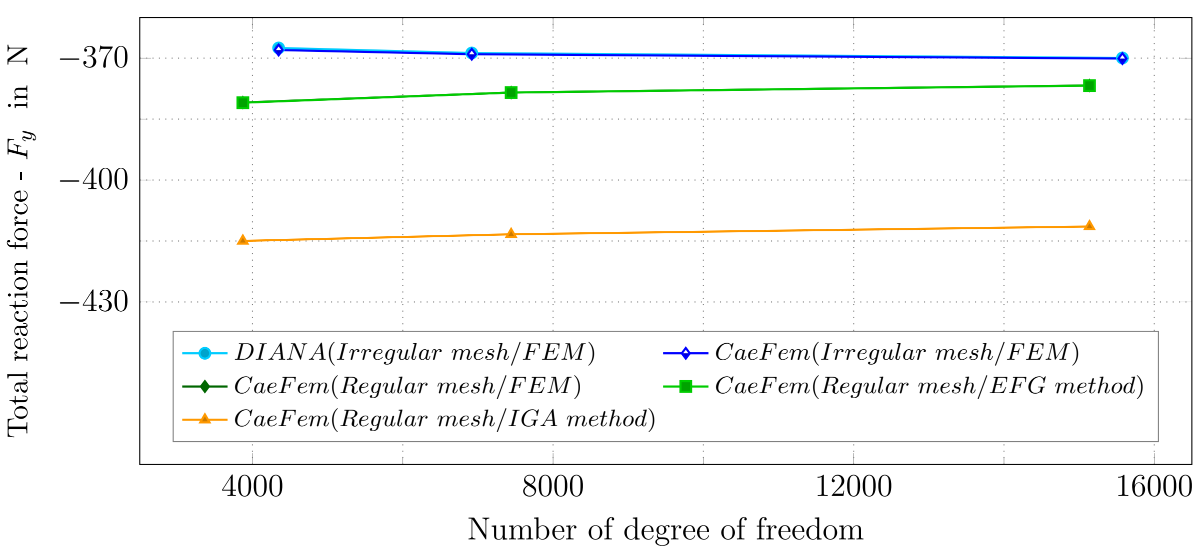


Figure 9: Results comparison / Model 3 - $z = 30$ mm

7 CONCLUSIONS

DIANA and CaeFem provide same result with FEM-Irregular mesh dicretization method which proves the accuracy of the program. EFG method has almost the same result as FEM with regular mesh, however, EFG needs more time to perform the analysis. While mesh becomes finer, results are converging and less error percentage in large NDOF. These 2D models are cross sections out of 3D mesoscopic model, total reaction forces in horizontal direction is small since deformation is applied in vertical direction and does not follow special pattern with respect to discretization methods and mesh sizes. In the next phase of this research, our focus will be towards introducing fiber elements in 2D models and upgrading our reference 2D sections to 3D models. Moreover, we will consider the effect of bonding between fiber elements and cementitious materials.

REFERENCES

[1] Zienkiewiczand O.C.and Taylor, R.L. *The Finite Element Method*. McGraw Hill, Vol. I, (1989).

[2] Bathe, K. *Finite Element Procedures*. MPrentice Hall, Englewood Cliffs, New Jersey (1996).

[3] Belytschko, T., Liu, W. and Moran, B. *Nonlinear Finite Elements for Continua and Structures*. John Wiley Sons, Chichester (2000).

[4] Häussler-Combe, U. *Computational Methods for Reinforced Concrete Structures*. Ernst Sohn / Wiley, Berlin (2014).

[5] Belytschko, T., Lu, Y.Y. and Gu, L. *Element-free Galerkin methods*. International Journal for Numerical Methods in Engineering (1994) **37**:229–256.

[6] Belytschko, T. et.al. *Meshless Methods: An overview and recent developments*. Comput. Methods Appl. Mech. Engrg (1996) **139**:3–46.

[7] Fleming, M., Chu, Y. and Belytschko, T. *Enriched element-free Galerkin methods for crack tip fields*. Int. J. Numer. Meth. Engng (1997) **40**:1483–1504.

[8] Lee, C. and Zhou, C. *On error estimation and adaptive refinement for element free Galerkin method: Part I: stress recovery and a posteriori error estimation*. Computers Structures (2004) **82**:413–428.

[9] Belytschko, T. Lu, Y.Y. and Gu, L. *Element-Free Galerkin Methods*. International Journal for Numerical Methods in Engineering (1994) **37**:229–256.

[10] Cottrell, J.A., Hughes, T.J. and Bazilevs, Y. *Isogeometric analysis: toward integration of CAD and FEA*. John Wiley Sons, Chichester (2009).

[11] Echter, R. and Bischoff, M. *Numerical efficiency, locking and unlocking of NURBS finite elements*. Computer Methods in Applied Mechanics and Engineering (2010) **199**:374–382.

[12] Bazilevs, Y. et.al. *Isogeometric analysis using T-splines*. Computer Methods in Applied Mechanics and Engineering (2010) **199**:229–263.

[13] Nguyen, V. P. et.al. *Isogeometric analysis: An overview and computer implementation aspects*. Mathematics and Computers in Simulation (2015)**117**:89-116.

[14] Hughes, T.J.R., Cottrell, J.A. and Bazilevs, Y. *Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement*. Comput. Methods Appl. Mech. Engrg (2005) **194**: 4135–4195.

[15] Nagadeepika, S. *Modeling concrete behavior at mesoscopic scale*. Technische Universität Dresden (2012).

[16] DIANA FEA. *Finite Element Analysis User's Manual*. The Netherlands (2018).