Model Reduction Methods for Quasi-real Time optimal Control Of Transfer Equations

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ABSTRACT

In this paper, the subject of investigations is the study of robustness, efficiency and gains in CPU time using model reduction techniques within the adjoint based optimal control methodology. Nonstationary transfer equations are considered for this purpose. The control parameter $\gamma \in \mathbb{R}^q$ appears in external boddy forces. Each of its componnents acts on a specefic subdomain, where the number of subdomains is exactly equal to q. The most used model reduction technique in optimal control is the Proper Orthogonal Decomposition (POD) [1]. Caracterized by its optimality, it has the property of representing the dynamic of problem in hand in the least number of modes. In other words, it consists in compressing information contained in a set of snapshots in just few modes forming an orthogonal basis. The solution reconstructed in this basis follows directly from the resolution of low order ordinary differential equations deduced from the Galerkin projection of transfer equations onto this basis. In optimal control, POD was already applied in previous works for instance in drag optimization of incompressible viscous flow past a circular cylinder [2], or for the control of temperature of an anisothermal flow [3]. Two approaches were considered, The first approach is active control, in which POD basis is determined once for all at the begining of optimization process. The second approach is adaptive control, where POD basis is updated whenever the existing basis runs out of its area of effictivness. For the first approach, a special care has to be involved in order to generate an adequate sampling that covers well parameters in optimizer's direction. It's worth mention that an optimal sampling with less parameters is generaly hard to generate. The latter approach is independant of any sampling, it requires determination of a new set of snapshots to build a new basis matching with the proposed parameters. Its main drawback is that new snapshots need the performance of high-dimensional costly simulations. In the present work, two additional approaches of basis adaptation are considered and discussed. The first one basicly acts as a basis enrichment tool allowing to cover new control parameters if the basis is out of its area of effectivness. This method is seen as a generalization of POD method and reffered to as Proper Generalized Decomposition (PGD) [4]. For the sake of CPU time saving, since this method requires high-dimensional simulations, it is carried out only if needed. The second approach is a totally different one from all aforementioned approaches. It's an unusual interpolation method based on calculus of geodesic path on a Grassmann manifold. It was firstly introduced for basis adaptation in the context of Aeroelasticity by Amsallem and Farhat [5]. Using the overall proposed approaches, results of numerical implementations obtained from controling a class of partial differential equations often encountred in transfer phenomena are presented and analyzed. Potential and limitations of these approaches are discussed in detail at the final stage.

References

[1] L. Sirovich, "Turbulence and the dynamics of coherent structures: I, ii and iii," *Quarterly of Applied Mathematics*, pp. 461–590, 1987.

[2] M. Bergmann and L. Cordier, "Optimal control of the cylinder wake in the laminar regime by trust-region methods and pod reduced-order models," *Journal of Computational Physics*, vol. 227, pp. 7813–7840, 2008.

[3] A. Tallet, C. Allery, and C. Leblond, "Optimal flow control using a pod based reduced-order model," *Numerical Heat Transfer, Part B*, vol. 170, 2016.

[4] A. Ammar, B. Mokdad, F. Chinesta, and R. Keunings, "A new family of solvers for some classes of multidimensional partial differential equations encountered in kinetic theory modelling of complex fluids: Part ii: Transient simulation using space-time separated representations," *Non-Newtonian Fluid Mechanics*, vol. 139, pp. pp 153–176, 2006.

[5] D. Amsallem and C. Farhat, "An interpolation method for adapting reduced-order models and application to aeroelasticity," *AIAA Journal*, pp. 1803–1813, 2008.