Mixed-Dimensional Problems in Elastic Media using an Iterative DtN Coupling Technique

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ABSTRACT

A situation often encountered in scientific computing in various applications, is where a mathematical model of high spatial dimension (say, three dimensional) has to be coupled with a mathematical model of lower spatial dimension (say, one dimensional). Typically, this situation occurs when the low-dimensional (lowD) model is employed as an approximation to the high-dimensional (highD) model in a partial region of the spatial domain. The highD region is the region of interest where the lowD approximation does not hold or does not provide the details required from the analysis. The lowD solution is possibly of less interest in its details to the analyzer, yet it affects the solution in the highD region significantly. In our present study, the coupling of two-dimensional (2D) and one-dimensional (1D) models in time-harmonic elasticity is considered. The hybrid 2D-1D model is justified in case that some regions in the 2D computational domain behave approximately in a 1D way. This hybrid model, if designed properly, is much more efficient than the standard 2D model taken for the entire problem. Our research focuses on the way the 2D-1D coupling is done, and the coupling error generated. An iterative Dirichlet-to-Neumann (DtN) coupling method is compared here to two other DtN-based methods – the DtN method in its original direct form and the Carke-Mear-Landis (CML) method. The iterative method is more general than the two other methods, and may have the benefit of higher precision, with the price of a larger computational effort, depending on the chosen stopping criteria. All three methods rely on a boundary stress calculation, i.e., the numerical computation of the first derivative of the solution on the interface. A boundary-stress-recovery (BSR) process, originally proposed by Hughes, was modified and implemented here, in order to improve the stress calculation accuracy.