Addressing the non-linearity and singularity phenomena of stress-based optimal design of material microstructures

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ABSTRACT

Material design is an active research field since composites have met increasingly interest, for instance, in lightweight construction as it happens in aerospace industry. Fully understanding the overall response of a composite demands greater insight about its microstructure behaviour. Micro-mechanical models detailing the interfaces of base constituents of a composite allow fine measure and control of local stresses. Homogenization models are often used to transfer data among different design scales. Multiscale analysis coupling macro, meso and micro scales can then be done.

For simplicity one assumes in the present work a given macroscopic stress field (one that may occur at a certain point of a macro-structure) and computes through homogenization the micro-stress distribution across the two (weaker and stronger) composite constituents mixed in a unit-cell domain Y, i.e., the representative volume element (RVE) of the material heterogeneous medium Ω.

Stress gradients depend a lot of design details but typically the stress function is highly non-linear. In the frame of finite element models for material microstructures one pursues here an investigation about mesh dependence. By means of ever-increasing mesh refinements and shape functions order one evaluates convergence of micro-stress measures and concludes about mesh quality. Besides investigating the quality of numerical approximation to the stress function, one also may address the quality of the homogenization stress predictions comparing them to the actual composite by repeating an unit-cell a limited number of times and subjecting the resulting numerical model to a battery of load tests for stress assessment furthering the work presented in [1]. The outcome indicates that it is sufficient to have a few repetitions of unit-cells to replace the non-homogeneous composite Ω by the equivalent homogeneous material with the stress field computed by homogenization.

Since stress distribution is strongly design dependent, that motivates one to pursue optimal design of the material microstructure to comply with admissible stress criteria. Stresses may be either minimized as the objective function or limited by an upper bound in the constraint function. The inverse homogenization method using density-based topology optimization is applied here for such purpose. This is quite a challenge not only because of the aforementioned non-linearity of the stress function but also due to the singularity phenomena which means that stresses tend to finite values as density design variables tend to zero due to degenerated regions of the problem feasible design domain. To overcome the singularity problem one may use the so-called relaxation approaches, known as ε-relaxation or *qp*-approach, which have been applied to the optimal design of structures with success. Bearing this in mind one extends the application of these relaxation techniques to material design in order to obtain some preliminary results and get some insight into the fine structure of composite materials and the influence of the stresses therein.

REFERENCES

[1] P.G. Coelho, R. Reis and J.M. Guedes, "Convergence analysis of stress fields to homogenization predictions in optimal periodic composite design", *ECCOMAS Congress*. Crete Island, Greece, 5-10 June, (2016).