Preconditioners for a full space model of coupled magma/mantle dynamics

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ABSTRACT

In this talk we consider numerical methods to efficiently solve the linear system arising from the discretization of models of coupled magma/mantle dynamics. The model we consider is based on a system of partial differential equations derived by McKenzie [1]. This system describes the creeping flow of high-viscosity mantle matrix and the porous flow of magma, a process which is described by a coupling of the Stokes equations for the mantle with Darcy's law for the magma.

The model describes mass and momentum conservation of the mantle and magma phases. Assuming the porosity field is given, different strategies exist for solving the model of coupled magma/mantle dynamics. For example, it is possible to eliminate the magma velocity and mantle pressure to obtain a system only for the mantle velocity and magma pressure. Preconditioners and iterative methods for this two-field system were developed in [3]. Although these preconditioners performed well in regions of high porosity, for porosities of interest, i.e., very small porosity, iterative methods in combination with these preconditioners failed to converge.

To deal with the complications that arise in the limit of small porosity, it has been advocated to eliminate only the magma velocity from the McKenzie equations, see e.g. [2]. Preconditioners for the resulting three-field formulation of the McKenzie equations were developed in [4]. It was shown that preconditioners developed for the three-field formulation are optimal in terms of problem size and less sensitive to model parameters compared to the two-field preconditioner. For small porosity, however, the iteration count to reach convergence is still unacceptably high.

In this talk we introduce a new formulation of the McKenzie equations in which we do not eliminate any of the unknown variables. Instead, we formulate the McKenzie equations in terms of a new set of variables, namely the mantle velocity, Darcy velocity, magma pressure and total pressure. We will discuss the development of a preconditioner for this "full space" formulation of the McKenzie equations and compare its performance to the two- and three-field preconditioners of [3, 4].

REFERENCES

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