The perturbation method in the problem on a nearly circular inclusion in an elastic body

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ABSTRACT

Stress concentration caused by different defects (such as holes and inclusions) existing in materials and structures is one of the reasons of devices failure. Apparently, it is not possible to obtain an exact analytical solution of an elastic boundary value problem for an arbitrary defect in an infinite plane. The real hole or inclusion has a shape which can't be usually described by a conformal image. For example, so-called circular defects have practically relief surface slightly deviated from a circle and a circular shape of them is nothing but idealization. In the works [1, 2] the perturbation method was used to solve the problem of an elastic infinity plane with a nearly circular hole at the macro- and nanolevel. The results obtained in [1], allow using boundary perturbation technique to solve a more complex problem of determining the stress-strain state in the area of the elastic inclusion.

In this work, we present solution of two-dimensional boundary value problem on a nearly circular inclusion in an infinity elastic solid. It is supposed that the uniform stress state takes place at infinity. Contact of the inclusion with the matrix satisfies to the ideal inseparability conditions. To solve the problem, we use Goursat – Kolosov complex potentials, Muskhelishvili representations and universal boundary perturbation technique applied recently to some problems of elasticity (see, for example [1]–[5]). Following Muskhelishvili [6], complex potentials are found out in terms of power series in small parameter. In each-order approximation, the problem is reduced to the solving to two independent Riemann – Hilbert's boundary problems. It is constructed an algorithm for finding any-order approximation expressed in elementary functions. For the periodic shape of the inclusions determined by the cosine function the first-order formulas of approximation are derived in an explicit form.

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