Shape Optimization Using CutFEM and CutIGA

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ABSTRACT

We present a shape optimization method based on the cut finite element method, see [1], [2], and [3], for the optimal compliance problem in linear elasticity. The elastic domain is defined by a level-set function, and the evolution of the domain is obtained by moving the level-set along a velocity field using a transport equation. The velocity field is defined to be the largest decreasing direction of the shape derivative that resides in a certain Hilbert space and is computed by solving an elliptic problem, associated with the bilinear form in the Hilbert space, with the shape derivative as right hand side. The velocity field may thus be viewed as the Riesz representation of the shape derivative on the chosen Hilbert space.

We thus obtain a coupled problem involving three partial differential equations: (1) the elasticity problem, (2) the elliptic problem that determines the velocity field, and (3) the transport problem for the levelset function. The elasticity problem is solved using a cut finite element method on a fixed background mesh, which completely avoids re–meshing when the domain is updated. The levelset function and the velocity field is approximated by standard conforming elements on the background mesh. We also employ higher order cut approximations including isogeometric analysis for the elasticity problem while the levelset function and the velocity field in this case are represented using linear elements on a refined mesh in order to simplify the geometric and quadrature computations on the cut elements. To obtain a stable method, stabilization terms are added in the vicinity of the cut elements at the boundary, which provides control of the variation of the solution in the vicinity of the boundary. We present numerical examples illustrating the performance of the method.

REFERENCES

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