A new framework for large strain electro-magneto-mechanics based on convex multi-variable strain energies: Conservation laws and hyperbolicity

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ABSTRACT

This paper exploits the new concept of multi-variable convexity, introduced by Gil and Ortigosa **[1-3]**, in a dynamic setting. Multi-variable convexity represents an extension of the concept of polyconvexity **[4]** to nonlinear electro-magneto-elasticity, where the internal energy functional is postulated as a convex combination of an electro-magneto-kinematic variable set comprising the deformation gradient tensor $\boldsymbol{F}$, its co-factor $\boldsymbol{H}$, its determinant J, the Lagrangian electric displacement field $\boldsymbol{D}_0$, the Lagrangian magnetic induction $\boldsymbol{B}_0$ and two additional spatial or Eulerian vectors $\boldsymbol{d}$ and $\boldsymbol{b}$, computed as $\boldsymbol{d}=\boldsymbol{F}\boldsymbol{D}_0$ and $\boldsymbol{b}=\boldsymbol{F}\boldsymbol{B}_0$, respectively.

This paper proves that the extended set of variables defining multi-variable convexity can be presented in the form of a system of first order conservation laws subjected to four involutions. Two completely new conservation equations for the spatial vectors $\boldsymbol{d}$ and $\boldsymbol{b}$ are presented in this work. A degeneration of the system of first order conservation laws to the case of low frequency scenarios is also described, where both Faraday and Ampère laws are replaced by curl–free constraints (different to the concept of involutions) over the material electric and magnetic fields, respectively. It is shown that, for the two scenarios considered, i.e. high and low frequencies, the expression of each of the convex multi-variable arguments as a first order conservation law leads to the fulfilment of the Legendre–Hadamard condition or, in other words, positive definiteness of the electro-magneto-mechanical acoustic tensor. The latter ensures that the speed of propagation of acoustic and electro-magnetic waves in the neighbourhood of a stationary point are real.

Crucially, the one-to-one and invertible relationship between the extended set of convex multi-variable arguments and its associated set of entropy variables enables to define a generalised convex entropy function and its associated flux. Following the work of Hughes et al. **[5]** in the context of Computational Fluid Dynamics, a symmetrisation of the hyperbolic equations in terms of the entropy variables is carried out for the first time in the context of nonlinear electro-magneto-mechanics. Finally, under a characteristic experimental set-up for electrostrictive dielectric elastomers, a study of the material stability of convex and non-convex multi-variable constitutive models is carried out.

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