Coupling CT images to non-homogeneous Timoshenko beams representing lumbar vertebrae

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ABSTRACT

3D Finite Element models are undoubtedly the golden standard in computational biomechanics. However, as a rule, they pose severe challenges in terms of computational power needed for model generation and for solving the corresponding systems of equations. This renders them cumbersome, particularly when the target should be a real-life, real-time use of such models in the clinical practice.

As a remedy, we here present a CT-image-to-short sandwich beam conversion technique. The theory of shear-compliant beams, pioneered by Timoshenko [1], is derived from the principle of virtual power [2], so as to account also for inhomogeneous mechanical property distributions across the beam cross sections. It is then applied to CT images of a human vertebra, whereby the attenuation coefficients are converted into elastic properties according to a recently developed method coupling X-ray physics and micromechanics [3]. Solution of the corresponding boundary value problem for bending, transverse, and axial loading of the short beam is based on the classical energetic approach of Cowpers [4] and the 2D stress function proposed by Kourtis et al [5]. The corresponding results agree very well with those obtained from classical 3D FE approaches, while the speed-up factor of the former as compared to the latter, is as large as ten for the model generation and four for the equation solving.

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