Mathematical modeling of magnetoelastic coupling

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ABSTRACT

Magneto-elastic coupling, often referred to as magnetostriction, is observed in most ferromagnetic materials and describes the reciprocal change of the mechanical and magnetic properties of the materials due to external magnetic and elastic influence. Although ferromagnets experience a measurable magnetostriction, its magnitude can be considerably increased by alloying certain materials. Such alloys can exhibit several thousand times greater magnitudes of magnetostriction and are thus called Giant Magnetostrictive materials.

In our work, we focus on the mathematical modeling of the magneto-elastic coupling using the Giant Magnetostrictive material Terfenol-D, which has the highest magnetostriction among all alloys. Based on a minimum energy principle, a fully-coupled system of PDEs is derived for a thin magneto-elastic plate. The linear-elastic, stationary, two-dimensional model is given in both strong and weak forms and allows the understanding of the mathematical foundation and the basic principles of the coupling.

Using two different potential approaches for the magnetic influence, we show that in the case of a magnetic scalar potential, the coupled system has the structure of a saddle-point problem, which can be avoided by using a magnetic vector potential. Furthermore, the properties of the resulting elastic and magnetic (bi-)linear forms are analyzed and features like coercivity and positive (semi-)definiteness are verified. Appropriate function spaces are introduced and the question of existence and uniqueness of the solutions is addressed for each of the derived models. Using the Finite Element Method, corresponding numerical simulations are finally carried out based on MATLAB- implementation and solutions of the coupled problem are obtained confirming the validity of the model.

LITERATUR

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