A Multirate Decoupling Scheme for Transient Coupled surface-subsurface flows

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We present an efficient multi-rate finite element decoupling scheme to approximate the solution to the time dependent coupled Stokes/Darcy problem. The coupled model describes a transient free surface flow modelled by the Stokes equations coupled with subsurface porous media flow following Darcy's law. The two flows are coupled by interface conditions specifying the continuity of the normal component of velocity between the free flow region and the porous medium, a balance of forces and the Beavers-Joseph-Saffman law. This coupled model is important in understanding the interactions between surface flow and ground flow. In addition, efficient numerical techniques for solving coupled problems are important as they can be applied to other couplings that may have the same general structure.

We discretize in space using a multi-numerics scheme combing the continuous finite element method in the free flow region and the discontinuous Galerkin method in the porous medium. This choice of finite element spaces is due to the discontinuities that are typical in the permeability of the porous medium.

The time descritization using a backward Euler scheme yields large fully coupled problems that must be solved at each time step. This can be computationally expensive for long time intervals and large domains. We propose a decoupling technique that takes advantage of the fact that the flow in the porous medium moves at a slower rate compared to the free flow. Therefore one can employ larger time steps in the porous medium compared to the free flow domain in addition the variables in each domain are time-lagged to achieve the decoupling.

We present convergence analysis of the decoupling scheme and demonstrate optimal convergence through numerical experiments. In addition, we compare the fully coupled scheme to the multi-rate decoupling scheme in-terms of CPU time and accuracy. We demonstrate long term stability of the scheme and robustness with respect to physical parameters through realistic physical problems.