An overlapping Domain Decomposition preconditioning method for monolithic solution of shear bands

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ABSTRACT

Metals subjected to high strain rate impact often exhibit a sudden and profound drop in the material's load bearing capability, a ductile failure phenomenon known as shear banding. Shear bands, characterized as material instabilities, are driven by shear heating and described as narrow regions that have sustained intense plastic deformation and high temperature rise. This coupled thermo-mechanical localization problem can be formulated as a nonlinear system with two balance equations, momentum and energy, and two constitutive laws for elasticity and plasticity.

In our formulation, mixed finite elements are used to discretize the equations in space and an implicit finite difference scheme is used to advance the system in time, where at every time step, a Newton type method is used to solve the system monolitically. To that end, a block Jacobian matrix is formed analytically using Gâteaux derivatives. The resulting Jacobian is sparse, nonsymmetric and its sparsity pattern and eigenvalue content vary with the different stages of shear bands formation, consequently posing a significant challenge to iterative solvers.

To address this issue, an overlapping Domain Decomposition preconditioner that takes into account the physics of shear bands and the domain in which it forms, is proposed. The key idea is to resolve the shear band domain accurately while only solving the remaining domain approximately, with selective updates when necessary. The preconditioner is formed on the basis of an additive Schwartz method that is applied to a Schur complement partition of the system.

The proposed preconditioner is implemented in parallel and tested on three different benchmark problems as compared against off-the-shelf solvers. We study the effect of h- and k- refinement that are obtained from Isogeometric descritizations of the system, the overlapping strategy and its parallel performance. Excellent performance is demonstrated in serial and parallel on all benchmark examples.

REFERENCES

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